WINSLOW'S

COMPREHENSIVE MATHEMATICS:

BEING AN EXTENSIVE CABINET OF

NUMERICAL, ARITHMETICAL, AND MATHEMATICAL FACTS, TABLES, DATA, FORMULAS, AND PRACTICAL RULES FOR THE GENERAL BUSINESS-MAN, MERCHANT, MECHANIC, ACCOUNTANT, TEACHERS OF SCHOOLS, GEOMETRICIANS, AND SCIENTISTS; APPROPRIATELY ARRANGED AND APPLIED.

BY

E. S. WINSLOW.

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PREFACE

TO THE COMPREHENSIVE MATHEMATICS.

On presenting this work to the public, it may be proper to state that it has been designed and written mainly for the practical man. It contains a vast array of Numerical, Arithmetical, and Mathematical facts, tables, data, formulas, and rules, pertaining to a great variety of subjects, and applicable to a diversity of ends, as well as much information of a more general nature, valuable to the artisan, and commercial classes; thus meeting the wants, in an eminent degree, of the lovers of the exact sciences, and the practical wants of students in the mathematics.

The facts and data alluded to have been gathered, with much care and patience, from a great variety of sources, or derived, often by toilsome investigations, from known and accredited truths. The care that has been taken in respect to these, it is thought, should secure for this particular department reliance and trust.

The tables, which are numerous, have, with few exceptions, been composed and arranged expressly for the work, and a confidence is felt that they may be relied on for accuracy.

From the valuable works of Dr. Ure, Adcock, Gregory, Grier, Brunton; from the publications of the transactions of London, Edinburgh, and Dublin Philosophical Societies; and from the publications by the Smithsonian Institute, much valuable information has been gained, relating mainly to machinery and the arts; and to these sources the author feels indebted.

The conciseness with which the work has been generally written would, perhaps, be found an objection, were it not that all the propositions and problems of intricacy are accompanied with examples and illustrations, and, in the matters of Geometry, additionally accompanied with diagrams. The whole, it is thought, will appear clear to him who consults it. A prominent feature in the design has been to produce a useful work, and one which in the way of price shall be readily accessible to all.

PREFACE

TO THE UNIVERSAL MODERN CAMBYST, AND FOREIGN AND DOMESTIC COMMERCIAL CALCULATOR.

This work is composed of the first five sections of the author's "COMPREHENSIVE MATHEMATICS." It was thought advisable to publish this portion of that work in a separate form on account of price; more especially as it contains all of a commercial nature treated of in that work. Indeed, the contents of that work were arranged expressly to this end. The Table of Contents in both works is the same: the work being stereotyped, this could not well be avoided. The Table of Contents, therefore, in either work, is that of the "COMPREHENSIVE MATHEMATICS;" and the first five sections thereof, that is, Section I., Section II., Section III., Section A., and Section B., is that of the "Universal Modern Cambyst, and Foreign and Domestic Commercial Calculator."

PREFACE

TO THE TIN-PLATE AND SHEET-IRON WORKERS' MONITOR.

This work is composed of Section VI. of the author's "COMPREHENSIVE MATHEMATICS," with portions of other sections of that work. It embraces all that is contained in the last-mentioned work of special interest to the Tinsmith, as such. It may be relied on for accuracy in all particulars, and is believed to be the first and only reliable work of the kind ever published. It is published in separate form on account of price, and with the view of affording apprentices and students every possible facility of obtaining it. It contains over 100 pages, nearly 50 diagrams, and step-by-step directions for constructing, *mechanically*, not less than 30 unlike and different patterns, embracing all of the more difficult and complicated in use, and several of new and beautiful designs.

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OF THE SIGNS USED IN THE FOLLOWING WORK.

$=$ *Equal to.* The sign of equality; as 16 oz. $=$ 1 lb.
$+$ *Plus*, or *More.* The sign of addition; as $8 + 12 = 20$.
$-$ *Minus*, or *Less.* The sign of subtraction; as $12 - 8 = 4$.
$\times$ *Multiplied by.* The sign of multiplication; as $12 \times 8 = 96$.
$\div$ *Divided by.* The sign of division; as $12 \div 4 = 3$.
$\backsim$ *Difference between the given numbers or quantities;* thus, $12 \backsim 8$, or $8 \backsim 12$, shows that the less number is to be subtracted from the greater, and the difference, or remainder, only, is to be used; so, too, *height* $\backsim$ *breadth*, shows that the difference between the height and breadth is to be taken.
: :: : *Proportion;* as 2 : 4 :: 3 : 6; that is, as 2 *is to* 4, *so is* 3 *to* 6.
$\sqrt{}$ *Sign of the square root;* prefixed to any number indicates that the square root of that number is to be taken, or employed; as $\sqrt{64} = 8$.
$\sqrt[3]{}$ *Sign of the cube root;* and indicates that the cube root of the number to which it is prefixed is to be employed, instead of the number itself; as $\sqrt[3]{64} = 4$.
^{2} *To be squared, or the square of;* shows that the square of the number to which it is affixed is the quantity to be employed; as $12^2 \div 6 = 24$; that is, that the square of 12, or $144 \div 6 = 24$.
^{3} Indicates that the cube of the number to which it is subjoined is to to be used; as $4^3 = 64$.
. *Decimal point*, or *separatrix.* See Decimal Fractions.
—— *Vinculum.* Signifies that the two or more quantities over which it is drawn, are to be taken collectively, or as forming one quantity; thus, $\overline{4 + 6} \times 4 = 40$; whereas, without the vinculum, $4 + 6 \times 4 = 28$; also, $12 - \overline{2 \times 3 + 4} = 2$; and $\sqrt{5^2 - 3^2} = 4$. So, also, $\sqrt{(5^2 - 3^2)} = 4$, and $(4 + 6) \times 4 = 40$.

$\frac{4^2}{2}$ $\left\{ \begin{array}{l} \text{half of } 4^2 \text{ or} \\ \text{half of the square of 4} \end{array} \right\} = 8.$

$\left(\frac{4^2}{2}\right)^2$ (the square of half the square of 4) $= 64$.

$\frac{1}{2}b^2$ or $\frac{1}{2}(b)^2$ (half the square of b.)
$(\frac{1}{2}b)^2$ (the square of half b.)
$(2b)^2$ (the square of twice b.)

SECTION A.

FOREIGN MONEYS OF ACCOUNT, COINS, WEIGHTS, AND MEASURES,

REDUCED TO THEIR VALUES IN THE MONEY, WEIGHTS, AND MEASURES OF THE UNITED STATES.

THE many changes that have been made in the moneys of account, coins, weights, and measures of different countries, by their respective governments, within the last few years, chiefly, though not in all cases, by the adoption of the *Metric System*, or systems bearing aliquot relations thereto, have compelled the author to re-write this section of the work, in a great measure, since the first edition was published; and it is the intention that this edition, and subsequent editions that may be published, shall contain this section strictly correct in all particulars at the time of going to the press.

The Federal units of comparison in the following tables, unless otherwise expressed, are as follows; viz., the *dollar* of 100 cents, in gold; the commercial or avoirdupois *pound*, of 7000 grains; the commercial *yard*, of 36 inches; the commercial or wine *gallon*, of 231 cubic inches; the commercial or Winchester *bushel*, of $2150\frac{42}{100}$ cubic inches; the standard *foot*, of 12 inches; the statute *mile*, statute *acre*, &c.

The value in Federal money, therefore, affixed to any particular denomination of a foreign money of account in the following tables, is the equivalent, or intrinsic par, of that denomination in United-States gold coins. It is predicated upon the standard weight and purity of the coins coined especially to represent that denomination, or conventionally held to be the measure of its value, compared with the standard weight and purity of the gold coins of the United States, that represent the dollar or its multiples.

Thus, in respect to those countries in which gold is made the measure of value and chief legal tender, it is the *intrinsic* par, gold for gold; and, in respect to those countries in which silver is made the measure of value and chief legal tender, it is the par value of that denomination in United-States gold coins, based upon the almost constantly prevailing relative commercial values for many years past, of gold to silver, as $15\frac{3}{8}$ to 1, for equal weights. It is, therefore, in a commercial point of view, the *intrinsic par* of that denomination, in Federal gold coins, in all cases.

The denomination itself, to which the Federal value is immediately affixed, is usually the integer, or ultimate money of account, of the country especially referred to. It is a money of account in that country always, but not always the name of a circulating coin. Occasionally, even, its value is not represented by any known single circulating coin.

From the foregoing remarks, it will be perceived that, when the *mintage* relative values of gold to silver, in any particular country, are maintained at rates nearer to each other than $15\frac{3}{8}$ to 1 for equal weights, the gold coins of that country are commonly worth more, as commercial material, than its silver coins of the same denominations, or same prescribed values; and, conversely, that when the mintage relative values are limited to rates more remote from each other than $15\frac{3}{8}$ to 1, the gold coins are commonly worth less than the silver coins.

Thus, the Federal *dollar*, in standard *silver* coins, is ordinarily worth, as commercial material, but $14.88372 \div 15.375 = 96\frac{4}{5}$ cents in Federal gold coins. But, since most Governments make silver the chief measure of value, the *mint* value of gold is usually purposely placed above its commercial worth. Thus, twenty francs in French gold coins are ordinarily worth, as commercial material, but $15.375 \times 20 \div 15.5 = 19.8387$ francs in French silver coins.

It is true that the silver coins of the United States, in small sums, for immediate use, in limited localities at home, may occasionally sell in exchange for the gold coins at their nominal values, or even at a premium, according to the local demand and supply; and the same may happen with regard to the gold coins in exchange for the silver, in France, and those other countries where gold is purposely over-valued in the mintage; but these conditions do not affect the general commercial relations of the metals: they are due only to a slight derangement in the required distributions of the two kinds of coins.

In Germany and Austria, the mint relative values of gold and silver for the Zollverein money, are as $15\frac{3}{8}$ to 1, for equal weights.

FOREIGN MONEYS OF ACCOUNT AND COINS REDUCED TO THEIR VALUES IN FEDERAL MONEY.

Foreign.	*U. States.*
ABYSSINIA, (E. AFRICA).—*Massuah:* The old Venetian zechino (*sequin*) is current here at 50 harfs; and 23 harfs = 1 pataka, or old Spanish dollar, - - - - -	= $1.01385
Austrian rix-dollars and Spanish dollars are current here at 1 pataka each.	

NOTE.—The old peso duro colonato, or Carolus silver dollar of Spain, contained, at mint usage, 415 grains of mint silver $\frac{43}{48}$ fine = $1.041353; but it is no longer struck at the mint, and those in circulation are more or less abraded. It is now valued, throughout the British Possessions in North America, and generally, wherever it circulates by tale, or is made the integer of the moneys of account, at 50 pence sterling in gold = $1.0138542. The Austrian rix-dollar (tallaro), scudo, or crown, which, by the way, has not been coined since 1858, except on orders for foreign circulation, contains $\frac{1}{12}$ Vienna mark of fine silver, or 361.11 grains = $1.0114911. This is often called the German dollar; and the Venetian dollar is of the same value.

ALGERIA (N. AFRICA).—*Algiers, Bona, &c.:* 100 centimes = 1 Franc - - - -	= 0.19452
ARABIA.—*Muscat:* 20 goz = 1 mamooda, 20 m. = 1 current Spanish dollar - - - -	= 1.01385

NOTE.—1 goz = 2 paras, and 1 mamooda = 1 piastre of Egypt. See EGYPT.

Mocha, Hodeida: 2 crats = 1 commasse; 60 c. = 1 Mexican or Spanish dollar by tale = 360 grains of fine silver, - - - - -	= 1.00838
160 crats = 1 wakega or troy ounce (*gold and silver weight*).	
Jidda: Same as at Alexandria, Egypt.	
Aden: 80 caveers = 1 piastre of account = $\frac{5}{6}$ current Spanish dollar, - - - -	= 0.84488
Also, as at Calcutta. Official, as in Great Britain.	
AUSTRALIA. — *Sidney, Melbourne, Hobart Town, and Australasia generally:*	
Standard of purity, denominations, values, and relative values, since 1855, same as in Great Britain.	
AUSTRIA. — *Vienna, Prague, Trieste, Ragusa, &c. Zollverein money:* Standard for gold and silver coins = $\frac{9}{10}$ fine, each; relative values, gold to silver as 15.375 to 1.	

Foreign.	*U. States.*
4 pfenninge = 1 kreuzer; 60 k. = 1 gulden, or florin = $\frac{1}{45}$ Zollverein pfund ($11\frac{1}{9}$ grams) of fine silver, or 171.471 grains, - - - =	$0.4803
1½ gulden = 1 Zollverein thaler; 1107 gulden = 80 Zoll. krones; 81 gulden = 200 francs; U. S. Customs value of gulden =	
AZORE ISLANDS.— *Fayal, Terceira, Corvo, St. Michael, &c.:*	
1000 reis = 1 milreis of account = $\frac{5}{6}$ old current Spanish dollar, - - - - =	0.84488
U. S. Customs value = $83\frac{1}{3}$ cents.	

NOTE.—In 1834, English sovereigns and Spanish dollars were made legal tender here and at the Madeiras; the former at the rate of 4120 reis, and the latter at 870 reis, each, which corresponds very nearly with their intrinsic values in standard Portuguese gold coins.

BALEARIC ISLANDS. — MAJORCA, *Palma*; MINORCA, *Port Mahon:* Same as new system in Spain, see SPAIN.	
BELGIUM. — *Brussels, Antwerp, Ostend, &c.*. Standard for gold and silver coins = $\frac{9}{10}$ fine, each. Relative values, gold to silver as 15.8228* to 1.	
100 centimes = 1 Franc = 4½ grammes of fine silver, - - - - - - =	0.19452
BERBERA (E. AFRICA.): Same as at Mocha, ARABIA.	
BERMUDA ISLANDS.— Official, as in Great Britain. In trade, 100 cents = 1 dollar = 1 old current Spanish peso, or 50 pence sterling in gold, - - - - - - =	1.01385

NOTE.—At the Bermudas, in British America, and other British foreign possessions generally, official or government accounts are kept, and duties to the government are assessed, in sterling money; but until 1842 this class of accounts were kept at the Bermudas and Jamaica, in pounds, shillings, and pence, at 12 shillings sterling to the pound, when it was ordered that hereafter they be kept in sterling money, and that all existing contracts in those colonies be settled at the rate of $\frac{3}{5}$ pound sterling per colonial pound.

BOURBON ISLAND. — *St. Denis:* 100 centimes = 1 franc - . - - - - - =	0.19452

* Although the silver coins of Belgium, both by law and general usage, have the same intrinsic values as those of France of the same denominations, yet this rule does not hold good with regard to the gold coins. The mint standard for 25 francs of France is $8\frac{2}{31}$ grammes of mint gold $\frac{9}{10}$ fine, while that for 25 francs of Belgium is only $7\frac{9}{10}$ grammes of mint gold $\frac{9}{10}$ fine; and so in proportion for the other gold coins. 25 francs, French mint, are worth $4.823816, while the 25-franc piece, Belgic mint, is worth only $4.72541; in other words, the Belgic gold coins are less in value two centimes per franc than the French gold coins.

Foreign. *U. States.*

CANADA, DOMINION OF, *and British America generally:* Standard for silver coins (20-cent pieces or Colonial shillings) = $\frac{37}{40}$ fine; for gold coins (British sovereigns) = $\frac{11}{12}$ fine. Relative values, gold to silver as 14.341 to 1.

100 cents = 1 dollar colonial, - - - = $1.00

Also, 4 farthings = 1 penny; 12 p. = 1 shilling; 20 s. = 1 pound colonial, - - - = 4.00

NOTE.—The standard 20-cent piece, or colonial silver shilling, of British America, contains 66$\frac{3}{5}$ grains of fine silver, and is, therefore, worth only 18.655 Federal cents in Federal gold coins, or 19.2708 Federal cents in Federal silver coins. But the money of account shilling in British America is equal to 20 Federal cents in gold; thus, the British pound sterling in gold, the British sovereign, is equal to $4.8665 in gold; and the shillings in that sovereign are equal to 24.3325 Federal cents, each, in gold; therefore, $\frac{4.8665 \times 20}{24.3325}$ = $4.00, the value of the colonial pound in Federal money (gold), measured by, or payable in, British standard gold.

CANARY ISLANDS. — *Teneriffe, Palma, Grand Canary, Fuerteventura, &c.:* Official as in Spain; in trade, occasionally, 8 reáls (antiquas) of 34 maravedes each = 1 piastre, or peso of exchange, - - - - - = 0.75623

CANDI ISLAND. — Same as in Turkey.

CAPE OF GOOD HOPE (S. AFRICA). — *Cape Town, &c.:* Same as in Great Britain.

CAPE VERDE ISLANDS. — ST. VINCENT, *Mindello;* ST. JAGO, *Porto Praya, &c.:*

1000 reis = 1 milreis. Old Spanish dollars are current here at 870 reis.

Central and South America.

CENTRAL AMERICA. — HONDURAS, *Truxillo, Port San Lorenzo, Omoa, &c.;* NICARAGUA, *Realejo, Greytown, &c.;* SAN SALVADOR, *La Union, Sonsonate, &c.;* COSTA RICA, *Puntas Arenas, Matina, &c.:* GUATEMALA, *Ystapa, &c.:* Standard for silver coins (dollars) = $\frac{20}{171}$ Castilian marco of silver $\frac{171}{200}$ fine, or $\frac{1}{10}$ marco of fine silver to the dollar; for gold coins (double escudos of 32 reáls) = $\frac{5}{177}$ Castilian marco of gold $\frac{171}{200}$ fine. Relative values, gold to silver as 16.5614 to 1.

100 centavos or 8 reáls = 1 dollar = 355.08 grains of fine silver, - - - - = 0.9946

Spanish dollars and U. S. gold coins circulate here, dollar for dollar.

1 *

Foreign.	*U. States.*
BALIZE, *Balize:* Official accounts are kept here in sterling money.	
SOUTH AMERICA.—PERU, *Callao, Islay, Truxillo, Arica, &c.;* CHILI, *Valparaiso, Concepcion, Coquimbo, &c.;* NEW GRANADA, *Cartagena, Santa Martha, Savanillo, Buenaventura, &c.;* ECUADOR, *Guayaquil, &c.* The prescribed standard for the mintage of these States is now in conformity with that of France, and there is strong probability that Brazil, and the other States in South America having mints, will soon adopt the same standard. Standard for gold and silver coins = $\frac{9}{10}$ fine, each. Relative values, gold to silver as $15\frac{1}{2}$ to 1.	
100 cents = 1 sol, or dollar = $22\frac{1}{2}$ grammes of fine silver, - - - - - =	$0.97461
Also, 100 centesimas = 1 duro or old Spanish dollar, - - - - - - =	1.01385
BRAZIL.—*Rio Janeiro, Maranham, Bahia, Para, Pernambuco, &c.:* Standard for silver coins = $\frac{1}{18}$ Castilian marco of silver $\frac{11}{12}$ fine per milreis; for gold coins = $\frac{4}{103}$ Castilian marco of gold $\frac{11}{12}$ fine per 10 milreis. Relative values, gold to silver as 14.30556 to 1.	
1000 reis = 1 milreis = 180.8278 grains fine silver, - - - . - - =	0.50651
A current Spanish dollar passes for 2 milreis of account, and the modern gold coins (10 milreis and 20 milreis) = $0.544375 per milreis.	
BOLIVIA.—*Cobija, &c.:* Standard for silver coins (dollars) = $\frac{2}{23}$ marco of silver $\frac{11}{12}$ fine; for gold coins (doubloons) = $\frac{2}{17}$ marco of gold $\frac{7}{8}$ fine. But little gold is coined in Bolivia, and that in circulation has, at present, no nominal mint relation to the silver coins.	
100 centavos = 1 dollar = 283.03478 grains of fine silver, - - - - - =	0.79292

NOTE.—The gold coins of Bolivia are often light of weight, and seldom range above $\frac{87}{100}$ fine. The silver dollars are usually minted at full weight, but are often but little if any above $\frac{9}{10}$ fine. The fractional silver coins are worth less, relatively, than the integer.

VENEZUELA.—*La Guayra, Maracaybo, Cumana, Puerto Cabello, &c.:* In Venezuela, as in Peru, &c.,

Foreign. *U. States.*

the silver 5-franc piece of France is made the measuring unit of value, and is divided into 100 centavos. In the moneys of account, however, the peso macuquins of 80 centavos is sometimes used; and one dollar in United-States gold coin is assumed to be worth 7 cents more than 5 silver francs, which is the case at the metal ratio of gold to silver as 16.0794 to 1, for equal weights. Hence, 1 peso fuerte Americano de premio = $\frac{107}{80}$ = $1.3375, measured by the peso macuquins.

ARGENTINE REPUBLIC.—*Buenos Ayres, Parana, &c.;* URUGUAY, *Montevideo, &c.;* PARAGUAY, *Assumption, Neembucu, &c.:*

Foreign gold and silver coins circulate here measured generally by the Spanish dollar: 10 decimos = 1 reál; 8 r. = 1 peso.

GUIANA, *Cayenne:* Same as in France.

Paramaribo: Same as in Holland.

Georgetown: Same as in Great Britain.

FALKLAND ISLANDS.— Same as in Great Britain.

CHINA.— *Canton, Shanghai, Amoy, Ningpo, Foochoo, Hongkong I., Macao:*

Standard for gold and silver ingots = 94 touch, or $\frac{94}{100}$ fine, each. Gold is not treated as money, but as merchandise.

10 cash or le = 1 candarine or fun; 10 c. = 1 mace or tsëen; 10 m. = 1 tael or leäng = 583⅓ grains of silver $\frac{94}{100}$ fine, or 548⅓ grains of fine silver, - - - - - =$1.53591

NOTE.—Mexican dollars, which are about the only coins, except copper cash, that circulate in China, on account of their convenience generally, bear a premium well laid on, upon their intrinsic worth. They commonly pass for about 72 candarines each, or $\frac{72}{100}$ of the tael in ingots of standard purity. A tael weight, or leäng as it is called by the natives, of wan-yin or sysee (see-sze) silver, that is, of silver as fine as silk, or that may be drawn into a thread as fine as silk (pure silver), is valued by the East India Company at 80 pence sterling. In London, it is commonly valued by the price per ounce paid at the mint for the fine silver in foreign silver coins, which is about 66 pence sterling, or $\frac{66 \times 583\frac{1}{3}}{480}$ = 80.2 pence sterling per tael. In 1799, the Congress of the United States fixed the value of the tael, in ingots of standard purity, for customs' purposes, at $1.48; but this was when silver was the measure of value in the United States, and when the dollar contained 371¼ grains of fine metal. The law, it is believed, has not been repealed.

CORSICA ISLAND.— *Ajaccio:* 100 centimes = 1 franc, - - - - - - = 0.19452

Foreign.	*U. States.*
CYPRUS ISLAND. — Same as in Turkey.	
DENMARK. — *Copenhagen, Elsinore, Odense, &c.:* Standard for silver coins = $\frac{877}{1000}$ fine.	
16 skillinge = 1 mark; 6 m. 1 rigsdaler = $\frac{75}{1216}$ Cologne mark of mint silver = $\frac{1}{2}$ former speciesdaler = 195.1233 grains fine silver, -	= $0.546552
U. S. Customs value of speciesdaler = $1.05.	

NOTE. — Denmark coins ducats and 5-thaler and 10-thaler gold pieces; but these and the speciesdaler have no proposed mint-relation to each other. The ducat is of the ordinary value of that denomination of coins; and the 10-thaler piece is designedly and practically of the same intrinsic value as that of North Germany; viz., equal to ten Bremen thalers in silver at the metal ratio of 15.409 to 1; or equal, at the common par of exchange, to $7.90005 in gold.

EGYPT (N. AFRICA). — *Cairo, Alexandria, Suez, &c.:* 40 paras = 1 piastre; in current gold coins = $0.04996647; in current silver coins = $0.05054979.	

NOTE. — In 1836, British sovereigns were made legal tender throughout Egypt at 97 pi., 20 pa., each; Spanish doubloons or onzas at 313 pi., 29 pa.,; Napoleons (20 francs in gold) at 77 pi., 6 pa.; Venetian sequins at 46 pi., 13 pa.; Dutch ducats at 45 pi., 26 pa.; tallaros (German dollars) at 20 pi.; colonatos (Spanish dollars) at 20 pi., 28 pa.; 5-franc pieces at 19 pi., 10 pa. The measure of value is $\frac{3}{8}$ oka-drachmas of fine silver to the piastre = $2\frac{2}{9}$ Abyssinian dirhem or 18 troy grains = $0.050419109.

FRANCE. — Standard for gold and silver coins = $\frac{9}{10}$ fine, each. Relative values, gold to silver as $15\frac{1}{2}$ to 1.	
10 centimes = 1 decim; 10 d. = 1 franc = $4\frac{1}{2}$ grammes of fine silver, - - -	= 0.19452
In practice, 100 centimes = 1 franc. U. S. Customs value = $0.186.	
GERMANY (*The Zollverein States, or North German Confederation*): PRUSSIA, SAXONY, MECKLENBURG, OLDENBURG, *the* HANSE CITIES, *&c.* *Zollverein money:* Standard for gold and silver coins = $\frac{9}{10}$ fine, each. Relative values, gold to silver as 15.375 to 1.	
12 pfennige = 1 silber groschen; 30 s. g. = 1 thaler = $\frac{1}{30}$ Zollverein pfund ($16\frac{2}{3}$ grams) of fine silver, or 257.2058 grains, - -	= 0.72045
369 Zollverein thalers = 40 Zollverein krones; 1 Zoll. krone = $\frac{1}{50}$ Zollverein pfund (10 grams) of fine gold, or 154.3235 grains = $6.64614; 27 thalers = 100 francs.	
U. S. Customs value of thaler =	

NOTE. — Saxony reckons 10 pfennige to the groschen.

Foreign. *U. States.*

Bremen (special): 5 schwaren = 1 groot; 72 g. = 1 thaler = $\frac{1}{5}$ old Frederic d'or, - - = $0.79619

U. S. Customs value = 78$\frac{3}{4}$ cents.

Hamburg, Lubec, Altona (special): 12 pfennige = 1 schilling; 16 s. = 1 mark.

1 marc current = $\frac{1}{35}$ Cologne mark of fine silver, = 0.28869

1 marc banco, at par (London rate of 13*m.* 12*s.* to the £.), - - - - - = 0.35393

U. S. Customs value of mark current = 28 cents; of mark banco = 35 cents.

Southern States. — BAVARIA, WURTEMBURG, BADEN, *&c.*: Standard for silver coins = $\frac{9}{10}$ fine.

4 pfennige = 1 kreuzer; 60 k. = 1 gulden or florin = $\frac{2}{105}$ Zoll. pfund of fine silver (9$\frac{11}{21}$ grams) or 146.975 grains, - - - = 0.41169

7 gulden = 4 Zollverein thalers; 189 gulden = 400 francs.

U. S. Customs value of gulden = 40 cents.

GREAT BRITAIN. — *Sterling money:* Standard for silver coins = $\frac{37}{40}$ fine; for gold coins = $\frac{11}{12}$ fine. Relative values, gold to silver as 14.287891 to 1.

4 farthings = 1 penny; 12 p. 1 shilling; 20 s. = 1 pound, - - - - - = 4.86656

U. S. Customs value = $4.84.

NOTE. — Since 1816, the Government of Great Britain has estimated the value of gold compared with that of silver as 14$\frac{1393}{4840}$ to 1, for equal weights. In theory, by the mint regulations, a *pound sterling* is equal to 1614$\frac{6}{11}$ grains of fine silver, or 113$\frac{1}{623}$ grains of fine gold. A pound sterling in silver, therefore, measured by the Federal standard of 345$\frac{3}{5}$ grains of fine silver to the dollar, is equal to 4\frac{133}{198}$ in Federal silver coins; or a standard silver shilling, measured by this measure, is equal to 23$\frac{71}{198}$ cents; and at the ordinary par of exchange it is worth $0.226122, very nearly, in Federal gold coin. But a pound sterling in British gold (a sovereign) measured by the Federal standard of 23$\frac{11}{50}$ grains of fine gold to the dollar, is equal to 4\frac{626788}{723303}$, or $4.866563529, very nearly, in Federal gold coins. At the former relative values of the two currencies, or relative values before the Federal standard was changed, in 1834, viz., 4 *shillings* and 6 *pence sterling* to the *dollar*, or 4.44\frac{4}{9}$ to the *pound*, the par of exchange is .09498, or 9$\frac{1}{2}$ per cent, practically, in favor of sterling money. Gold is the measure of value in Great Britain, as well as in the United States; and the silver coins of that government are not legal tender at home in sums exceeding £2.

GREECE. — *Athens, Patras, the Ionian Islands, &c.:* Standard for gold and silver coins = $\frac{9}{10}$ fine, each. Relative values, gold to silver as 15.549 to 1.

100 lepta = 1 dramia = 1 dramia weight of fine silver, or 62$\frac{1}{8}$ grains, - - - - = 0.17401

Foreign.	*U. States.*
5 dramias = 1 taliron. 20 dramias (gold) = 1 othonion = \$3.44137.	
100 oboli =1 Spanish, German, or Venetian dollar = \$1.01385.	

NOTE.—Greece is a party to the present movement (1869) to legalize and enforce the use of the metric system of moneys, weights, and measures, throughout Continental Europe.

HOLLAND (NETHERLANDS).—*Amsterdam, Rotterdam, The Hague, &c.:*	
Standard for silver coins = $\frac{118}{125}$ fine; for gold coins = $\frac{9}{10}$ fine. Relative values, gold to silver as 15.7333 to 1.	
100 centimes = 1 guilder or florin = 10 grams of mint silver, - - - - -	= \$0.40806
Gouden Willem (10 guilders) = 6⅔ grams of mint gold = \$3.98769.	
U. S. Customs value of guilder = 40 cents.	
HAWAIIAN OR SANDWICH ISLANDS.—*Honolulu, &c.:*	
100 cents = 1 dollar, - - - -	= 1.00

India and Malaysia, or East Indies.

HINDOSTAN.—BENGAL, MADRAS, BOMBAY, *Presidencies of:* Standard for gold and silver coins = $\frac{11}{12}$ fine, each. Relative values, gold to silver as 15 to 1.	
Calcutta, Madras, Rangoon, &c.: 12 pice = 1 anna; 16 a. = 1 rupee = 1 tola, or 180 grains of mint silver, = $\frac{15}{16}$ old sicca rupee, - - -	= 0.46217
1 mohur, or gold rupee (E. I. Co.), = 15 silver rupees in theory = 1 tola of mint gold, or 165 grains of fine gold, = \$7.10594.	
Bombay, Surat, &c.: 100 reas = 1 quarter; 4 q. = 1 rupee, - - - - -	= 0.46217
U. S. Customs value of rupee = 44½ cents.	
Madras, &c. (old usage): 48 jittas = 1 fanam; 36 f. = 1 star pagoda = 4 arcot, or Company rupees, - - - - - -	= 1.84869
48 jittas = 1 fanam; 36 f. = 1 India pagoda = 8 shillings sterling in gold, - - -	= 1.9466
U. S. Customs value of star pagoda = \$1.84; of Indian pagoda, = \$1.94.	
Goa: Official, same as in Portugal.	
Pondicherry: Same as in France.	

Foreign.	*U. States.*
CEYLON ISLAND. — *Colombo, Trincomalee, &c.:* Official, same as in Great Britain; also, 1 rixdaalder = 18*d.;* 1 Spanish dollar = 50*d.;* 1 rupee = 22*d.*, sterling.	
MALAYA. — *Malacca, Pahang, Perak, &c.:* Same as at Singapore.	
PENANG ISLAND. — Same as in Calcutta; also, 100 cents = 1 Spanish dollar.	
SIAM. — *Bangkok, &c.:* 4 prangs, or clams, = 1 sompay; 4 s. = 1 salung; 4 salungs = 1 tical = 1 tical weight of silver 93 touch, or $\frac{93}{100}$ fine, - =	$0.61659
U. S. Customs value of tical = 61 cents.	
SINGAPORE ISLAND. — *Singapore:* 100 cents = 1 dollar (old Spanish), - - - - =	1.01385
BANCA ISLAND. — Same as at Batavia (JAVA I.).	
BORNEO ISLAND. — *Banjermassin, Sarawak, &c.:* Generally as at Batavia.	
CELEBES ISLAND. — *Macassar*, and the other Dutch settlements: Same as at Batavia.	
JAVA ISLAND. — *Batavia, Samarang, &c.:* 100 cents = 1 guilder, - - - - - =	0.40806
Mexican dollars are current here at 2$\frac{3}{5}$ guilders, each.	
MOLUCCA ISLANDS. — *Amboyna, &c.:* Same as at Batavia (JAVA I.).	
PHILIPPINE ISLANDS. — LUZON, *Manila, Mindano, &c.:* Official, as in Spain; also, 34 maravedis = 1 reál; 8 r. = 1 peso, or dollar, - - =	1.00465
SUMATRA ISLAND. — *Bencoolen, Padang:* Official, as at Batavia; also, 8 satellers = 1 soocoo; 4 s. = 1 dollar, or rial, = 1 pardow of Acheen.	
Acheen: 16 copangs = 1 mace; 10 mace of fine silver, or 374.306 troy grains = 1 pardow, - =	1.04845
Mexican and Spanish dollars pass for 1 pardow, each.	

ITALY. — The recent formation of the several States in Italy into one kingdom, Rome and in its immediate vicinity only excepted, has had the effect to unify the moneys and moneys of account in that portion of Europe also; and now, throughout Italy proper, or the Kingdom of Italy, including the Island of Sicily and that of Sardinia, the official monetary system, with a slight difference in nomenclature, is identically the same as that of France.

Standard for gold and silver coins = $\frac{9}{10}$ fine, each.
Relative values, gold to silver as 15$\frac{1}{2}$ to 1.

Foreign.	*U. States.*
100 centesimi = 1 lira, or franco, = $4\frac{1}{2}$ grams of fine silver, - - - - -	= $0.19452
JAPAN. — *Yeddo, Miaco, Osaka, Simoda, Hakodadi, Nagasaki, Yokohama, Matsmay, Napa, &c.: New System:* Standard for silver coins (assay) = $\frac{89}{100}$ fine; for gold coins = $\frac{143}{250}$ fine. Relative values, gold to silver (assuming the cobang to represent 10 silver itzebous) as 15.01877 to 1.	
50 sen = 1 itzebou, or itchibou (often called *boo*) = 5 monme of mint silver, or $\frac{89}{20}$ monme of fine silver = 119.1888 grains, - - -	= 0.333855
1 cobang contains $\frac{518}{100}$ monme of mint gold, or 79.35992 grains of fine gold, - - -	= 3.417744
U. S. Customs value of itzebou =	

NOTE. — Mexican dollars circulate in Japan, commonly at 2.87 to 2.90 itzebou, each; but they are intrinsically worth over 3 itzebous, each, by tale.

LIBERIA (W. AFRICA). — *Monrovia, &c.:* Same as in the United States.	
MADEIRA ISLANDS. — *Funchal, &c.:* 1000 reis = 1 milreis of account, - - -	= 1.01385
See Azore Islands, note relative to.	
U. S. Customs value of milreis = $1.00.	
MALTA ISLAND. — *Valetta, &c.:* Official, as in Great Britain; also, 20 grani = 1 taro; 12 t. = 1 scudo = $\frac{1}{2}$ ducat of Naples, - -	= 0.41371
U. S. Customs value of scudo = 40 cents.	
MAURITIUS ISLAND. — *Port Louis, &c.:* Official, as in Great Britain; also, 100 cents = 1 dollar. Tallaros of Austria and silver Napoleons (5-franc pieces of France) are current here at 1 dollar each; Spanish pesos and Mexican dollars, at 52 pence sterling each.	
MEXICO. — *Acapulco, Mazatlan, San Blas, Campeachy, Tampico, Sisal, Vera Cruz, &c.:*	
Standard for silver coins (dollars, average by assay) = 416.15 grains $\frac{9}{10}$ fine; for gold coins (doubloons, average by assay) = 416.4 grains $\frac{433}{500}$ fine. Relative values, gold to silver as 16.618192 to 1.	
100 cents = 1 dollar; also, 6 grani = 1 cuarto; 2 c. = 1 medio; 2 m. = 1 reál; 8 r. = 1 dollar = 374.535 grains of fine silver, - -	= 1.0491

NOTE. — The Mexican marco = 3549.81 grains; and the standard silver dollar should weigh $\frac{3549.81 \times 40}{341}$ = 416.4 grains.

Foreign.	*U. States.*
MOROCCO (N. AFRICA). — *Morocco, Fez, Tangier, &c.:* 24 fluce = 1 blankeel; 10 b. = 1 metical; 4 m. = 1 oncia or ducat = 4 meticals weight of fine silver, or 295.3846 grains = 1 old standard ducat of Naples, - - - -	= $0.82739
MOZAMBIQUE (E. AFRICA). — *Mozambique, Quilimane, Sofala, Delagoa Bay, &c.:* 1000 reis = 1 milreis of account.	

NOTE. — A variety of foreign coins are current here, and many of them wide of their true values; viz., Spanish and Mexican dollars at 1000 reis, each; silver 5-franc pieces of France and United-States gold coins per dollar at 900 reis, each; Spanish doubloons at 17120 reis, patriot doubloons at 16000 reis; and British sovereigns at 4500 reis, each.

NORWAY. — *Christiania, Bergen, &c.:* Standard for silver coins = $\frac{877}{1000}$ fine.	
20 skillinge = 1 mark; 6 m. = 1 speciesdaler = 2 rixdalers = $\frac{75}{608}$ Cologne mark of mint silver or 390.2465 grains of fine silver, - -	= 1.0931

NOTE. — Norway and Sweden are about to coin gold of 10 and 20 francs.

NUBIA (E. AFRICA). — *Suakin, &c.:* Same as at Alexandria, Egypt.	
PERSIA. — *Bushire, Gombroon, Astrabad, &c.:* 100 mamoodis, or 50 abasse, = 1 toman = $\frac{3}{4}$ miscal weight of fine gold, or 49.23077 grains, -	= 2.12019
5 dinars = 1 kasbequis; 2 k. = 1 dinars-biste; 2 d.-b. = 1 shatree, shafree, or shahis; 2s. = 1 mamoodi; 2 m. = 1 abasse; 2½ a. = 1 papabat; 2 p. = 1 saheb-keran.	

NOTE. — Foreign gold and silver coins in great variety circulate in Persia, but at fluctuating prices; the toman, however, is commonly valued at two Mexican dollars.

PORTUGAL. — *Lisbon, Oporto, St. Ubes, &c.:* Standard for gold coins (coroas) = $\frac{1}{24}$ marco of gold $\frac{11}{12}$ fine; for silver coins (crusados) = $\frac{1}{16}$ marco of silver $\frac{9}{10}$ fine. Relative values, gold to silver as 14.72524 to 1.	
1000 reis = 1 milreis = 2 crusados = 398.4237 grains of fine silver, - - - -	= 1.11601
1 milreis in gold = $1.16525.	
Method of writing and reading monetary quantities: *Example.* — Rs. 5 : 600 ⊕ 750 = 5,600 milreis and 750 reis.	
U. S. Customs value of milreis = $1.12.	

NOTE. — In commercial transactions, lately, Mexican dollars commonly pass for 1 milreis each, and United-States gold coins at 970 reis per dollar.

Foreign.	*U. States.*
ROME *and Civita Vecchia:* Standard for gold and silver coins $= \frac{9}{10}$ fine, each. Relative values, gold to silver as $\frac{24.2 \times 125}{196} = 15.43367$ to 1.	
5 quatrini = 1 baiocho; 10 b. = 1 paolo; 10 p. = 1 scudo, or crown, = $24\frac{1}{5}$ grams of fine silver	= $1.04609
The gold coins are $2\frac{1}{2}$, 5, and 10 scudo pieces, and contain $\frac{196}{125}$ grams of fine gold per scudo.	
U. S. Customs value of scudo = $1.05.	
RUSSIA.— *St. Petersburg, Riga, Cronstadt, Odessa, &c.:* Standard for silver coins $= \frac{7}{8}$ fine; for gold coins (assay) $= \frac{229}{250}$ fine. Relative values, gold to silver as 15.17295 to 1.	
10 kopecks = 1 grieven; 10 g. = 1 rublyu (*rouble*) $= \frac{20}{451}$ funt of fine silver, or 280.19 grains	= 0.78483
In practice, 100 kopecks = 1 rouble.	
100 silver roubles = 360 bank or paper roubles. Bank rouble varies from par to 4 per cent premium.	
U. S. Customs value of silver rouble = 75 cents.	
SENEGAMBIA (W. AFRICA).— *Gambia, Bathurst, Sierre Leone, &c.:* Official, as in Great Britain.	
St. Louis: Official, as in France.	
SOCOTRA ISLAND.— Same as at Muscat, Arabia.	
SPAIN.— *Madrid, Malaga, Cadiz, Santandre, Bilboa, Barcelona, &c.: New system,* legalized Oct. 19, 1868, and its use made obligatory to the exclusion of all other systems after Dec. 31, 1870. Standard for gold coins $= \frac{9}{10}$ fine; for silver coins (5-peseta pieces, duros, or pesos) $= \frac{9}{10}$ fine; 1-peseta pieces and less $= \frac{167}{200}$ fine. Relative values, gold to silver (duros) as $15\frac{1}{2}$ to 1.	
100 centesimas = 1 peseta = $4\frac{1}{2}$ gramos of fine silver = 1 franc of France, - - -	= 0.19452

NOTE.—In this system the peseta of account is equal to 1 silver franc of France; and the 5-peseta silver coin is equal to 5 francs in silver of France; but the 1-peseta coin is worth but $0.18047. The gold coins are worth peseta for franc in gold.

Prevailing system last preceding the foregoing: Standard for silver coins (escudos of 10 reáls vellón) $= \frac{9}{10}$ fine; for gold coins (10 escudos and over) $= \frac{112}{125}$ fine. Relative values, gold to silver as 15.5555 to 1.	
1000 milesimas = 1 escudo $= \frac{5}{99}$ marco of fine silver, or $179\frac{1}{3}$ grains, - - - -	= 0.50232

NOTE.—The silver coins of less denominations than 4 reáls, and the gold coins of less denominations than 10 escudos, have less purity than those mentioned above; the 80-reáls piece, even, is but $\frac{1739}{2000}$ fine = \$3.85446, while the 100-reáls piece (double de Isabel) is worth \$4.96493.

Foreign.	*U. States.*
Gibraltar: 16 quartos = 1 reál; 12 r. = 1 peso duro or old Spanish dollar, - - - -	= \$1.01385
SWEDEN.—*Stockholm, Gothenburg, Carlscrona, Gefle, &c.:* Standard for silver coins = $\frac{3}{4}$ fine; for gold coins (ducats) = $\frac{39}{40}$ fine. Relative values, gold to silver as 15.18517 to 1.	
100 öre = 1 riksmynt, or riksdollar riksgäld = $\frac{1}{4}$ speciesdollar, - - - - -	= 0.27622
1 speciesdollar = $\frac{15}{124}$ mark of fine silver, or 393.15 grains, - - - - -	= 1.10124
U. S. Customs value of speciesdollar = \$1.06.	
1 ducat = 8 riksmynts, nominally, = $\frac{3}{205}$ marks of fine gold = \$2.23738.	
Sweden is about to coin gold coins of 10 and 20 francs.	
SWITZERLAND.—*Basel, Bern, Geneva, Lausanne, Lucerne, Neufchatel, Zurich, &c.:*	
Standard for gold and silver coins = $\frac{9}{10}$ fine, each. Relative values, gold to silver as $15\frac{1}{2}$ to 1.	
10 rappen = 1 batzen; 10 b., or 100 centimes, = 1 franc = $4\frac{1}{2}$ grams of fine silver, - -	= 0.19452
TRIPOLI (N. AFRICA).—*Tripoli, &c.:* 100 paras = 1 piastre, or ghersch; value of piastre same as that of Tunis.	
TUNIS (N. AFRICA).—*Tunis, Soosa, Cabes, &c.:*	
Standard for silver coins = $\frac{11}{12}$ fine; for gold coins = $\frac{9}{10}$ fine. Relative values, gold to silver as 15.8125 to 1.	
2 burbine = 1 asper; 52 a., or 16 karob = 1 piastre = $\frac{11}{12}$ drachma of fine silver, or 44 grains,	= 0.123247
TURKEY.—*Constantinople, Smyrna, Aleppo, Trebizonde, &c.:* Standard for silver coins (assay) = $\frac{83}{100}$ fine; for gold coins (assay) = $\frac{183}{200}$ fine. Relative values, gold to silver as 15.15625 to 1.	
40 paras, or 100 aspers, = 1 piastre = $\frac{1}{320}$ checki of fine silver, or 15.3856 grains, - -	= 0.043096
1 piastre in gold = $\frac{1}{4850}$ checki of fine gold = \$0.0437181. 100 piastres in gold = 1 medjdie.	
1 purse of silver = 500 piastres; 1 purse of gold = 30,000 piastres. U. S. Customs value of piastre = 5 cents.	

West Indies.

CUBA ISLAND. — *Havana, Matanzas, Santiago, Manzanillo, Baracoa, Cardenas, Cienfuegos, Nuevitas, Trinidad, &c:* Official as in Spain; also, 12 dineros, or 16 quartos, = 1 reál; 8 r., or 100 centesimas = 1 duro, peso, piastre, or dollar.

NOTE. — The full weight Spanish duro colonato is made the unit, or measure of value, = $1.04872: but the Spanish onza, or doubloon, passes for 17 dollars, and the Mexican and S. American, for 16 dollars.

HAYTI ISLAND. — HAYTI, *Port au Prince, Aux Cayes, Cape Haytien, Gonaives, &c.:*
DOMINICA, *San Domingo, Porto Plate, Samana, &c.:*
100 centesimas = 1 dollar, or gourda (old Spanish).

NOTE. — Spanish doubloons pass for 16 dollars of account; and the Haytian silver gourda is worth about ½ Spanish dollar.

PORTO RICO ISLAND. — *San Juan, Guayama, Ponce, &c.:* Official, as in Spain; also, 100 centesimas = 1 dollar.

JAMAICA ISLAND. — *Kingston, Falmouth, Savana la Mar, &c.:* Official, as in Great Britain; also, 100 cents = 1 dollar = 1 old peso, or duro of Spain = 50 pence sterling in gold, - - = $1.01385

CARIBBEE ISLANDS. *LEEWARD ISLANDS.* — ANTIGUA, — *St. John, Falmouth;* DOMINICA, MONTSERRAT, TORTOLA, VIRGIN GORDA, ST. CHRISTOPHER, ANGUILLA, BARBUDA, NEVIS, SABA: Same as in Jamaica.

ST. EUSTACIUS: Official, as in Holland.

GUADELOUPE, ST. MARTIN,* MARIE GALANTE, DESIRADE, LES SAINTES: Official, as in France; also, 100 centimes = 1 dollar; the colonial *livre* of these islands = $\frac{1}{10}$ standard duro colonato of Spain.

ST. THOMAS, — *Charlotte Amalie;* SANTA CRUZ, ST. JAN: Official, as in Denmark; also, 100 cents = 1 dollar.

ST. BARTHOLOMEW: Official, as in Sweden; also, 1 Spanish dollar = 9 shillings currency.

* The southern portion of the island of St. Martin is owned and settled by the Dutch; and the moneys of account, weights, and measures of Holland are in general use there.

Foreign. *U. States.*

WINDWARD ISLANDS. — TRINIDAD, *Port Spain*; BARBADOES, *Bridgetown*; GRENADA, ST. VINCENT, ST. LUCIA. TOBAGO: Same as at Jamaica Island.

MARTINIQUE, *St. Pierre*, *Port Royal*: Official, as in France.

BAHAMA ISLANDS. — NEW PROVIDENCE, *Nassau*; TURKS, ABACO, ANDROS, GREAT BAHAMA, &c.: Same as at Jamaica Island.

LITTLE ANTILLE ISLANDS. — CURACOA, BUEN AYRE, ORUBA: Official, as in Holland.

MARGARITA, TORTUGA, BLANQUILLA: Same as in Venezuela.

ZANGUEBAR (E. AFRICA). — *Zanzibar* (Island and Town), *Quiloa*, *Mombas*, *Magadoxo*, *Brava*, *Socotra Island*, *&c.*: Accounts are now kept in Zanzibar and the Sultan of Muscat's dominions on the east coast of Africa generally in *dollars* of 100 *cents*; and by the exertions of William E. Hines, Esq., of New York, late resident consul of the United States at Zanzibar, the present standard dollar in gold of the United States is made the measure of value, or, in other words, is officially rated at par. This was accomplished in 1865. Other foreign gold and silver coins circulate at conventional rates, some of them above and others below their intrinsic values. Thus, the Spanish dollar = $1; the Austrian rix-dollar, scudo, or crown, sells for $1.01 to $1.03; the English sovereign is rated at 4¾ Austrian rix-dollars; the French franc in gold, at 18¾ cents; the silver 5-franc piece, at 94 cents; and the Indian silver rupee, at 47 cents.

The prices of United-States Bonds, and of American stocks generally, are quoted in Paris in cents per dollar, payable in French silver coins, and upon the conventional Bourse rate of 5 francs to the dollar; whereby the rate of exchange affects the prices. From the Paris quotations, therefore, 2.8172 per cent. must be deducted to express the true prices, when exchange is at par, or when a dollar in United-States gold is quoted at *fr.* 5.14086.

FRANKFORT. — At Frankfort, the prices of United-States Bonds, and of American stocks generally, are quoted in cents per dollar, payable either in United-States gold coins, or in guilders at the conventional rate of 2½ guilders to the dollar. To these quotations, therefore, when payable in guilders, 2.674 per cent. must be added to express the true prices. See page 50 *a*.

FOREIGN LINEAL AND SURFACE MEASURES,

REDUCED TO THE LINEAL AND SURFACE MEASURES OF THE UNITED STATES.

Foreign.		*U. States.*
ABYSSINIA. — *Massuah:* 8 robi = 1 derah, or pic, - - - - - =	0.682	yard.
ALGERIA. — 10 decimetres = 1 metre, - =	1.0936	"
8 robi = 1 pic. Pic, *Moorish*, for linens, - =	0.519	"
Pic, *Turkish*, for silks, &c., - =	0.692	"
ARABIA. — *Muscat:* 8 gheria = 1 covid; 8 c. = 1 kassaba, - - - - =	$12.86\frac{2}{3}$	feet.
500 kassaba = 1 coss, - - - =	1.2185	miles.
Aden: 8 robi = 1 yard or pic, - - =	0.95	yard.
Jidda: 8 robi = 1 pic, - - - =	0.743	"
Mocha: 8 robi = 1 gez, - - - =	0.694	"
AUSTRALIA. — Same as in Great Britain.		
AUSTRIA. — *Imperial and Legal:* 12 zollen = 1 fuss, - - - - - =	1.04	feet.
$29\frac{1}{2}$ zollen = 1 elle, - - - =	0.85216	yard.
6 fusse = 1 klafter; 10 fusse = 1 ruthe, - =	10.39924	feet.
2,400 ruthen = 1 meile, - - - =	4.7269	miles.
192 square ruthen = 1 metzen; 3 m. = 1 joch, - - - - - =	1.43	acres.
AZORE ISLANDS. — Same as in Portugal.		
BALEARIC ISLANDS. — 3 pie, or 4 palmi, = 1 vara.		
MAJORCA: 2 varas = 1 cana, - - =	1.711	yards.
MINORCA: 2 varas = 1 cana, - - =	1.754	"
BELGIUM. — 10 streep = 1 duim; 10 d. = 1 palm; 10 p. = 1 el = 1 metre, - - =	1.0936	"
10 elen = 1 roed; 100 r. = 1 mijl, *or* kilometre, - - - - =	0.6214	mile.
100 square roeds = 1 bunder = 100 ares - =	2.471	acres.
The old Brabant el = 0.76006 yard.		
BERMUDA ISLANDS. — Same as in Great Britain.		
BOURBON ISLAND. — Same as in France.		
CANARY ISLANDS. — Same as in Spain.		
CANDIA ISLAND. — 8 rob = 1 pic, - - =	0.697	yard.
CAPE VERDE ISLANDS. — Same as in Portugal.		

Foreign.	U. States.	
CANADA, DOMINION OF.— Same as in the United States.		
LOWER CANADA (*special*): 1 arpent, - =	0.8475	acre.
CAROLINE ISLANDS.— Same as in Spain.		

Central and South America.

CENTRAL AMERICA.— COSTA RICA, GAUTEMALA, HONDURAS, NICARAGUA, SAN SALVADOR: Same as Spain,— *standard of Castile.*		
SOUTH AMERICA.— ARGENTINE REPUBLIC: 36 pulgadas = 3 pes = 1 vara, - =	34.1	inches.
150 varas = 1 cuadra; 40 c. = 1 legua, - =	3.229	miles.
27,000 square varas = 1 suertes des estancia.		
BRAZIL: 24 pollegadas = 2 pes = 1 covado = $\frac{66}{100}$ metre, - - - - =	0.7218	yard.
40 pollegadas = 5 palma = 1 vara = $\frac{1}{2}$ bracio = $1\frac{1}{10}$ metre, - - - =	1.203	"
250 varas, *or* 125 braccios = 1 estadio; 8 e. = 1 milha; 3 m. = 1 legoa, - - =	4.101	miles.
4,840 square varas = 1 geira, - - =	1.447	acres.
CHILI: 36 pulgadas = 1 vara (*customs*), - =	1.000	yard.
In all other respects as in Spain,—*Castilian standard.*		
VENEZUELA, NEW GRENADA, ECUADOR, PERU, BOLIVIA, PARAGUAY, URUGUAY: Same as Spain,— *Castilian standard.*		
GUIANA: *Cayenne:* Same as in France.		
Paramaribo: Same as in Holland.		
Georgetown: Same as in Great Britain.		
FALKLAND ISLANDS: Same as in Great Britain.		
CHINA.— 10 tsun, *or* fan, = 1 punt; 10 p. = 1 chih, covid, *or* cobre (*mercers'*), - - =	1.226	feet.
$17\frac{1}{2}$ punts, *or* 10 tac, = 1 thuoc (*tradesmen's*) =	0.7152	yard.
Chih (*mathematical*), - - - =	13.122	inches.
Chih (*engineers' and surveyors'*), - - =	12.059	"
Chih, *or* kong-pu (*architects'*), - - =	12.709	"
10 chih = 1 chang *or* cheung; 10 chang = 1 yan; 18 y. = 1 li, - - - =	0.3425	mile.
100 square chih = 1 mow; 240 m. = 1 fu, *or* king, - - - - - =	0.5564	acre.
CYPRUS ISLAND.— 8 robi = 1 pic, - =	0.696	yard.

Foreign.		*U. States.*	
DENMARK. — 24 tomme, *or* 2 fod, = 1 aln, -	=	0.6862	yard.
10 fod = 1 rode; 2,400 r. = 1 mül, -	=	4.6785	miles.
280 square rodes = 1 skiepper; 2 s. = 1 toende, - - - - -	=	1.362	acres.
6 fod = 1 favn; and 1 fod = 1 Prussian Rhein-fuss.			
EGYPT. — 3 kirat = 1 rob; 8 r. = 1 pic *or* endrasi (*for silks and woolens*), - -	=	27.06	inches.
1 halebi *or* archin = 27.9 in.; 1 derah = 25.264 inches.			
2 derah = 1 fedan; 3 f. = 1 gasab; 420 g. = 1 berri; 3 b. = 1 parasang, - -	=	3.015	miles.
400 square gasab = 1 fadden al risach, -	=	1.465	acres.
FRANCE. — 100 centimètres, *or* 10 decimètres, = 1 mètre, - - - -	=	39.37	inches.
100 mètres, *or* 10 decamètres, = 1 hectomètre; 100 hectomètres, *or* 10 kilomètres, = 1 myriamètre, - - -	=	6.2137	miles.
100 square metres = 1 are; 100 a. = 1 hectare, - - - - -	=	2.471	acres.
1 old aune de Paris = 1.29972 yard; 1 aune *usuelle*, or *metrique*, = $1\frac{1}{5}$ metre.			
FRIENDLY ISLANDS (Feejee and Tonga Groups). — Same as in England.			
GERMANY. — Prussia, and the Zollverein längenmaasse for all Germany:			
12 linie = 1 zoll; 12 z. = 1 fuss (Rheinfuss), - - - - -	=	12.3514	inches.
$25\frac{1}{2}$ zollen (Rhein-zollen) = 1 elle = $\frac{2}{3}$ metre, - - - - -	=	$26.24\frac{2}{3}$	"
10 land-fusse, *or* 12 Rhein-fusse, = 1 ruthe; 2,000 r. = 1 meile, - - -	=	4.67855	miles.
180 square ruthen = 1 morgen, - -	=	0.6304	acre.
Special and local: —			
Saxony: 12 zollen = 1 fuss; 2 f. = 1 elle, -	=	0.6196	yard.
1 Lubec-Brabant elle, - - -	=	0.7498	"
Bavaria: 120 zollen, *or* 10 fusse, = 1 ruthe,	=	9.579	feet.
$34\frac{1}{4}$ zollen = 1 elle = $\frac{5}{6}$ metre, - -	=	$32.808\frac{1}{3}$	inches.
Wurtemburg: 100 zollen, *or* 10 fusse = 1 ruthe, - - - - -	=	9.37326	feet.
$21\frac{1}{2}$ zollen = 1 elle, - - -	=	0.67175	yard.
Baden: 20 zollen, *or* 2 fusse, = 1 elle = $\frac{3}{5}$ metre, - - - - -	=	23.622	inches.

Foreign.	*U. States.*
HESSE DARMSTADT: 10 zollen = 1 fuss = $\frac{1}{4}$ metre, - - - - -	= 0.822$\frac{1}{12}$ foot.
24 zollen = 1 elle = $\frac{3}{5}$ metre, - -	= 0.6561$\frac{2}{3}$ yard.
MECKLENBURG: Same as at Hamburg.	
OLDENBURG: 12 zollen = 1 fuss; 2 f. = 1 elle,	= 0.63528 "
BREMEN: 24 zollen, *or* 2 fusse, = 1 elle, -	= 0.63276 "
1 Bremen-Brabant elle = 1$\frac{1}{5}$ Bremen elle.	
HAMBURG: 12 zollen = 1 fuss; 2 f. = 1 elle,	= 0.62681 "
1 Hamburg-Brabant elle, - - -	= 0.75615 "
LUBEC: 24 zollen, *or* 2 fusse = 1 elle, -	= 0.6294 "
GREAT BRITAIN.—Same as in the United States.	
GREECE.—60 onué = 5 pes = 1 passo = 1$\frac{3}{4}$ metre.	
22 onué = 1 pichi, *for silks*, - -	= 0.701734 "
23$\frac{1}{2}$ onué = 1 pichi, *for woolens, &c.*, -	= 0.749579 "
HOLLAND.—10 streep = 1 duim; 10 d. = 1 palm; 10 p. = 1 el = 1 metre, -	= 39.37 inches.
10 el = 1 roed; 100 roed = 1 mijl = 1 kilometre.	
100 square roede = 1 bunder = 1 hectare.	
1 old Brabant el - - - -	= 0.75931 yard.

India and Malaysia, or East Indies.

ANAM: Same as in China.	
BURMAH, PEGU: 5$\frac{1}{2}$ pulgaut = 1 taim; 4 t. = 1 sadang; 7 s. = 1 sha, *or* bambou, -	= 154 inches.
1,000 dhas = 1 dain, - - -	= 2.4306 miles.
CEYLON ISLAND: 2 covids = 1 guz, *or* yard, -	= 36 inches.
HINDOSTAN.—*Calcutta:* 16 tussoos = 8 gheira = 1 haut, *or* covid; 2 h. = 1 ghes, *or* guz,	= 36 "
2,000 ghes = 1 coss, - - -	= 1.1364 miles.
20 square covids = 1 chattack; 16 c. = 1 cottah; 20 cottahs = 1 biggah, - -	= 0.3306 acre.
Bombay: 16 tussoos = 8 gheira = 1 haut *or* covid; 1$\frac{1}{2}$ h. = 1 guz; 2 hauts, *or* 1$\frac{1}{3}$ guz, = 1 imperial yard, - - -	= 36 inches.
Madras: 16 tussoos = 8 gheira = 1 haut *or* covid; 2 h. = 1 guz, - - -	= 36 "
Goa: Same as in Portugal.	
Massulipatam: 2 palms = 1 span; 3 s. = 1 cubit *or* covid, - - - -	= 19$\frac{1}{8}$ "
Surat: 18 tussoos = 1 haut, *for matting*, -	= 21 "
84 tussoos, *or* 20 wiswūsa, = 1 wūsa, -	= 98 "

Foreign.	*U. States.*
MALABAR COAST. — *Mangalore, Cananore, Calicut, Cochin, Quilon, Trivandrum:* 3 gheria = 1 ady; 2½ a. = 1 haut; 2 h. = 1 guz *or* gujah, - - - -	= 38½ inches.
COROMANDEL COAST, generally, same as at Madras.	
SIAM. — 48 nions = 4 keubs = 2 soks = 1 ken,	= 37.836 "
2 kens = 1 vouah; 20 v. = 1 sen; 4 s. = 1 jod; 25 j. = 1 roëneng, - -	= 2.3886 miles.
MALACCA. — Same as at Calcutta.	
SINGAPORE ISLAND. — Same as at Calcutta.	
PHILIPPINE ISLANDS. — Same as in Spain, — *standard of Castile.*	
JAVA ISLAND. — 1 voot (Rhyn-voot), - -	= 1.029 feet.
1 el, *or* covid (old Amsterdam el), - -	= 0.7522 yard.
English measures also, as at Sumatra.	
SUMATRA ISLAND. — 2 tempohs = 1 jancal; 2 j. = 1 etto; 2 e. = 1 hailoh, - -	= 36 inches.
ITALY. — The metric system of weights and measures is now the official standard, and is in general commercial use throughout the kingdom.	
Special and local: —	
Milan: 10 atomi = 1 dito; 10 d. = 1 palmo; 10 p. = 1 metro *or* braccia, - -	= 39.37 inches.
1,000 metri = 1 miglio (kilometre).	
100 metri quadrata = 1 tavolo (are); 100 t. = 1 tornatura (hectare).	
2½ metri = 1 trabucco; 4 t. = 1 decametre.	
Florence, Leghorn, Pisa, &c.: 2 palmi = 1 braccia; 4 b. = 1 canna = 2⅓ metres.	
Carrara: 1 palm for marble = 9.06 U. S. inches.	
JAPAN. — 10 rin = 1 bun; 10 b. = 1 tsun; 10 t. = 1 sasi *or* sjak.	
4 tsun-sasi = 5 kani-sasi = 1 hiro; 5 tsun-sasi = 1 ink *or* tattamy, - - -	= 2.07103 yards.
7 kani-sasi = 1 kan, kian, *or* ikje, - -	= 2.31955 "
60 inks = 1 ting *or* masti; 36 t. = 1 ri, -	= 2.54172 miles.
MADEIRA ISLANDS. — Same as in Portugal.	
MAURITIUS ISLAND. — Official as in Great Britain; also, —	
1 aune (old aune de Paris), - -	= 46.79 inches.
MEXICO. — Same as in Spain, *standard of Castile.*	

Foreign.	*U. States.*	
MOROCCO. — 8 pollegadas = 1 palmo da craveira (Portuguese).		
$18\frac{2}{3}$ pollegadas, *or* $2\frac{1}{3}$ palmi da craveira, = 1 covado *or* cadee, - - -	= 20.21	inches.
8 rob = 1 pic (Turkish in theory), -	= 26.03	"
NORWAY. — Same as in Denmark.		
NEW ZEALAND ISLANDS. — Same as in England.		
PERSIA. — *Monkelser or royal:* 12 fingers = 1 foot; $1\frac{1}{2}$ f. = 1 cubit; 2 c. = 1 guz *or* gueza (*for silks, &c.*), - - -	= 36.92	"
1 guz shah (royal), *for woolens*, - -	= 40	"
1 guz tabree (of Tabreez), - -	= 40.4	"
6,000 guz (monkelser) = 1 parasang, -	= 3.496	miles.
PORTUGAL. — 8 pollegadas = 1 palmo da craveira = $\frac{22}{100}$ metre.		
12 pollegadas = 1 pé; 2 p. = 1 covado = $\frac{66}{100}$ metre.		
40 pollegadas = 1 vara = $1\frac{1}{10}$ metre, -	= 43.307	inches.
$24\frac{3}{4}$ pollegadas = 1 covado avantejado (retail).		
$13\frac{1}{3}$ pollegadas = $\frac{1}{3}$ vara = 1 terça.		
60 pollegadas = 1 passo; $1\frac{1}{3}$ p. = 1 braça.		
780 pés = 1 estado; 8 e. = 1 milha; 3 m. = 1 legua, - - - -	= 3.8386	miles.
4,840 square varas = 1 geira, - -	= 1.4471	acres.
ROME. — 12 onze = 1 palmo; $1\frac{1}{3}$ p. = 1 pié;		
5 piédi = 1 passo, - - -	= 58.6104	inches.
10 palmi, *or* $7\frac{1}{2}$ piédi, = 1 canna architectona, - - - - -	= 87.9156	"
$3\frac{3}{4}$ canne arch. = 1 catena (*chain*); 1 catena = 10 stajoli.		
8 palmi mercantile = 1 canna merc., -	= 78.43932	"
9 palmi d'are = 1 canna d'are = $1\frac{1}{8}$ metre,	= $44.29\frac{1}{8}$	"
1,000 passi = 1 miglio, - - -	= 0.92504	mile.
$3\frac{1}{2}$ square catene = 1 quartuccio; 2 q. = 1 scorzo; 4 s. = 1 quarta; 4 quarte (7 pezze) = 1 rubbio, - - -	= 4.559	acres.
RUSSIA. — $6\frac{6}{7}$ verschok = 1 foot, - -	= 12	inches.
16 verschok = 1 archine *or* halebi, -	= 28	"
3 archines (7 feet) = 1 sachine, - -	= 7	feet.
500 sachines = 1 verst, - - -	= 0.6629	mile.
2,400 square sachines = 1 deciatene, -	= 2.7	acres.
SANDWICH ISLANDS. — Same as in the United States.		

Foreign. *U. States.*

SOCIETY ISLANDS.—Official, as in France.

SPAIN.—*Standard of Castile:* 36 pulgadas = 6 sesma = 4 quarta *or* palmos = 3 tercia *or* pie = 1 vara, - - - - = 33.385 inches.

2 varas = 1 estado, braza, brazada, *or* toesa.

2 estados = 1 estadale; 2,000 estadales = 1 legua, - - - - - = 4.2153 miles.

560 square estadales = 1 fanegada, - = 1.592 acres.

$1\frac{1}{2}$ pie = 1 codo; 5 pies = 1 passo.

Gibraltar: As above; also, 12 inches = 1 foot; 3 f. = 1 yard, - - - - = 1 yard.

Alicante: 36 pulgadas = 1 vara, - - = 0.8319 "

Barcelona: 1 vara *or* matja-cãna, - - = 0.8641 "

Santander: 8 octava, *or* 4 palma, = 1 vara, - = 0.9142 "

Valencia: 36 pulgadas = 4 palmos = 3 pie = 1 vara, - - - - = 1.0044 yards.

$2\frac{1}{4}$ varas = 1 braza-reale.

NOTE.—In 1849, Spain legalized the use of the metric system of weights and measures in all her dominions; and now, since she is to enforce the employment of the money system of France, to the exclusion of all others (see Foreign Moneys of Account, Spain), it may be expected that she will unify her weights and measures by enforcing the employment of the metric system generally.

SWEDEN.—10 linier = 1 tum; 10 t. = 1 fot; 10 f. = 1 stäng; 10 s. 1 ref, - - = 97.406 feet.

20 tumen = 2 fus = 1 aln = $\frac{1}{2}$ old aune de Paris, - - - - - = 0.64937 yard.

6 fus = 1 famn; 6,000 f. (3,600 stänger) = 1 mil, - - - - - = 6.6413 miles.

560 square stänger (14,000 square alner) = 28 kannland = 16 kappland = 2 spannland = 1 tunnland, - - - = 1.21975 acres.

SWITZERLAND.—Official and general for the 22 cantons forming the Republic;—

10 zollen = 1 fuss; 2 f. = 1 elle = $\frac{6}{10}$ metre, = 23.622 inches.

4 fusse = 2 ellen = 1 stab = 1 aune metric.

6 fusse = 1 klafter; 10 fusse = 1 ruthe *or* toise, - - - - - = 9.8425 feet.

1,600 ruthen = 1 meile *or* hour's way = $4\frac{4}{5}$ kilometres, - - - - = 2.9826 miles.

400 square ruthen = 1 juchart *or* feld-aker = 36 ares, - - - - = 0.8896 acre.

TRIPOLI.—8 rob = 3 palmi = 1 pic *or* dra, - = 26.42 inches.

The pic for silks = 1 Arabic covid, - - = 19.03 "

Foreign.	*U. States.*	
TUNIS. — 8 robi = 1 pic; pic for woolens, -	= 26.5	inches.
Pic for silks, - - - - -	= 24.83	"
Pic for linens, - - - -	= 18.62	"
TURKEY. — *Constantinople and Smyrna:* 1 halebi *or* archin (Russian archine of 28 U. S. inches in theory), - - - -	= 27.9	"
7,500 halebi = 1 agatch, - - -	= 3.3142	miles.
1 pic, draa, *or* endrasi, *for silks and woolens,*	= 27.06	inches.
1 indise, *or* endese, *for cottons, &c.,* - -	= 25.688	"
Bassorah, Bagdad: 8 robi = 1 guz, - -	= $31\frac{2}{3}$	"

West Indies.

In the Islands of *Cuba*, *Hayti*, *Porto Rico*, and *Isle of Pines*, the measures of length and of surface are the same as in Spain, *Castilian standard*, except that in Port au Prince and the French portion of Hayti, generally, the old piede du roy of 12.78918 U. S. inches, and the old aune de Paris of 1.29972 U. S. yards, is used.

In *Jamaica*, *St. Kitts*, *Antigua*, *Montserrat*, *Tortola*, *Anguilla*, *Dominica*, *Barbuda*, *Nevis*, *Virgin Gorda;* in *Trinidad*,* *Grenada*, *St. Vincent*, *St. Lucia*, *Barbadoes*, *Tobago*, *Grenadines;* and in *New Providence*, *Great Bahama*, *Turks*, *Abaco*, and the Bahamas generally, the lineal and surface measures are the same as in Great Britain and the United States.

In *Gaudeloupe*, *Martinique*, *Desirade*, *Les Saintes*, *Marie Galante*, the lineal and surface measures are the metric; but the pied américaine (United-States foot) is chiefly used in measuring timber.

In *Santa Cruz*, *St. Thomas*, *St. John*, the official measures of length and of surface are the same as in Denmark, but those of the United States are much used.

In *St. Bartholomew:* Official, as in Sweden; also as in the United States.

In *St. Eustatius*, *Curacoa*, *Buen Ayer*, *Oruba:* Official, as in Holland.

In *Margarita*, *Blanquilla*, *Tortuga:* Official, as in Venezuela.

ZANGUEBAR (EAST AFRICA): Same as at *Muscat*, Arabia.

* The Spanish (Castilian) vara is still used to some extent by the merchants in Trinidad.

FOREIGN COMMERCIAL WEIGHTS,

REDUCED TO THEIR EQUIVALENT VALUES IN THE UNITED STATES.

Foreign.	*U. States.* Avoirdupois pounds.
ABYSSINIA (E. AFRICA).—*Massuah:* 10 dirhem = 1 wakea; 12 w. *or* 10 mocha = 1 rotl, rotolo *or* litre = 10 troy ounces,* - - =	0.68571
ALGERIA (Barbary, N. AFRICA). — *Algiers, Bona, Oron:* The metric weights, under French denominations, are in official and common use here.	
ARABIA. — *Mocha:* 233⅓ krat = 1 wakea, wakega, *or* vacia = 700 troy grains.	
15 wakea = 1 rotl; 2 r. = 1 maund *or* maon, - =	3.—
10 maunds = 1 frazil; 15 f. = 1 bahar, - =	450.—
At the bazaar, 14½ frazils = 1 bahar for coffee, =	435.—
Jidda: Official, as at Alexandria, Egypt; also, 3 5/9 drachmas musr (of Egypt) = 1 wakea; 15 w. = 1 rotl; 5 r. = 1 maund; 10 m. = 1 frazil; 10 f. = 1 bahar = 66⅔ okes of Egypt, - =	182.8571
Hodeida, Beit-el-fakih; 15 wakea = 1 rotl; 2 r. = 1 maund; 10 m. = 1 frazil; 40 f. = 1 bahar, =	815.2381
16½ rotl = 1 tomaun of rice, - - - =	168.14286
Muscat, Hasek, &c.: 233⅓ krat = 1 wakea; 10 w. = 1 rotl; 9 r. = 1 maund; 200 m. = 1 bahar, =	1800.—
Aden: Official, same as in Great Britain.	
AUSTRALASIA (OCEANICA). — AUSTRALIA, NEW ZEALAND, TASMANIA: Same as in Great Britain.	
AUSTRIA (*legal for the Empire*): 32 lothe = 16 unzen = 4 vierdinge = 2 marken = 1 pfund; 20 p. = 1 stein; 5 s. = 1 centner, - - =	123.468
4 centners = 1 karch; 5 k. = 1 last; 2¾ centners = 1 saum; 1¼ centners = 1 lagel; 2 l. = 1 saum for steel.	
Ragusa, (DALMATIA): 2½ pfunde = 1 oka, - =	3.0867
AZORE ISLANDS (N. Atlantic Ocean): Same as in Portugal.	

* See Turkey, weights of, and note relative to.

Foreign.	*U. States.* Avoirdupois pounds.
BALEARIC ISLANDS (Mediterranean Sea): 25 rotoli ($22\frac{1}{2}$ Castilian libras) = 1 aroba; 4 a. = 1 quintal, - - - - =	91.3063
3 quintals = 1 carga; 110 rotoli = 1 oder; 100 libra menor = 87 Castilian libra = 1 cantaro grosso = 88.2627441 av. lbs.	
BELGIUM.— Same as in France (metric system).	
BERBERA (E. Africa): Same as at Mocha, Arabia.	
BERMUDA ISLANDS (N. Atlantic Ocean): Same as in Great Britain.	
BOURBON ISLAND (Mascerene group, Indian Ocean): The metric system, French nomenclature, is in use here.	
CANADA, DOMINION OF.— Same as in the United States.	
CANARY ISLANDS (Atlantic Ocean, W. coast N. Africa): Same as in Spain, Castilian standard.	
CANDIA ISLAND, or CRETE (E. Mediterranean Sea): 100 rottoli = 44 okes = 1 cantaro, - - - - - - =	116.565
CAPE OF GOOD HOPE (S. Africa): Same as in Great Britain.	
CAPE VERDE ISLANDS (Atlantic Ocean, near W. African coast): Same as in Portugal.	

Central and South America.

Guatemala, Honduras, San Salvador, Nicaragua, Costa Rica: 16 onzas = 2 marcos = 1 libra; 25 l. = 1 arroba; 4 a. = 1 quintal, - - - - - =	101.4514
$2\frac{1}{2}$ quintals = 1 carga; 8 c. = 1 tonelada, - =	2029.0285
Balize.— Official, same as in Great Britain.	
BRAZIL.—16 onças = 2 marcos = 1 arratel, - =	1.01187
32 arratels = 1 arroba; 4 a. = 1 quintal, - =	129.5193
Argentine Republic, *or* La Plata: 16 onças = 2 marco = 1 libra; 25 l. = 1 arroba; 4 a. = 1 quintal, - - - - =	101.274

Note.—The Argentine Republic has established independent standards of weights and measures, which are now in practice, and which vary more or less in each department from those of Castile.

Foreign.	*U. States.* Avoirdupois pounds.
PERU, CHILI, BOLIVIA, ECUADOR, NEW GRANADA, VENEZUELA, URUGUAY, PARAGUAY: Generally as in Central America.	
Montevideo: 1 pesado of dry hides (fresh or salted) contains 1½ arrobas; 1 pesado of wet salt hides contains 2⅓ arrobas.	

NOTE.—In Peru, Chili, New Granada, Bolivia, Venezuela, and Surinam, the use of the metric system of weights and measures is sanctioned by law; but as yet (1869) is very little employed in either of the States.

GUIANA.—*Cayenne:* Same as in France.	
Paramaribo, or *Surinam:* Same as in Holland; also as in France.	
Georgetown: Same as in Great Britain.	
FALKLAND ISLANDS.—Same as in Great Britain.	
CHINA.—10 tsëen = 1 tael *or* leäng; 16 t. = 1 catty *or* kan; 100 c. = 1 pecul *or* tam,	= 133.3333
22¾ chu = 1 leäng; 2 catties = 1 yin; 15 y. = 1 kwan; 3⅓ k. = 1 tam; 1⅕ t. (60 yin) = 1 shik,	= 160.—
CORSICA ISLAND (Mediterranean Sea): Same as in France, of which it forms a department.	
CYPRUS ISLAND (Mediterranean Sea): Same as in Turkey, and forming a part of Turkey in Asia.	
DENMARK.—32 lod = 16 unze = 2 marken = 1 pund = ½ kilogram; 100 p. = 1 centner,	= 110.2311
16 pund = 1 lispund; 20 l. = 1 shifpund; 16¼ s. = 1 last.	
12 punds = 1 bismerpund; 3 b. = 1 waag *or* vog.	
EGYPT (N. AFRICA): 4 gran = 1 kara; 16 k. = 1 drachma (oka-drachma) = $\frac{1}{10}$ troy ounce, or 48 grains.	
400 drachmas = 4 oka,	= 2.74286
144 drachmas = 1 rottolo; 100 r. (36 okes) = 1 cantaro (customs),	= 98.7429
FRANCE.—1000 milligrammes = 100 centigrammes = 10 decigrammes = 1 gramme = 15.43235 troy grains.	
1000 grammes = 100 decagrammes = 10 hectogrammes = 1 kilogramme,	= 2.20462
10 kilogrammes = 1 myriagramme; 100 m. = 10 quintals = 1 tonneau,	= 2204.62143

Foreign.	*U. States.* Avoirdupois pounds.
GERMANY. — Zollverein gewighte for all the States of the tariff-alliance:	
10 quentchen = 1 loth; 30 l. = 1 pfund; 100 p. = 1 centner = 50 kilograms, - - =	110.2311
Special and local, or domestic: —	
512 pfennige = 128 quentchene = 32 lothe = 16 unzen = 2 marken = 1 pfund.	
PRUSSIA: 100 pfunde = 1 centner, - - =	103.1194
BAVARIA: 100 pfunde = 1 centner = 56 kilograms, =	123.4588
BREMEN: 116 pfunde = 1 centner, - - =	127.4887
BRUNSWICK: 100 pfunde = 1 centner, - - =	103.0656
HAMBURG: 112 pfunde = 1 centner, - - =	119.6044
HESSE DARMSTADT: 100 pfunde = 1 centner = 50 kilos, - - - - - =	110.2311
LUBEC: 112 pfunde = 1 centner, - - - =	119.6813
MECKLENBURG: 14 pfunde = 1 liespfund; 8 l. = 1 centner, - - - - - =	119.5164
OLDENBURG: 100 pfunde = 1 centner; 3 c. = 1 pfundschwer, - - - - =	317.7241
10 pfunde = 1 liespfund; 29 l. = 1 schiffpfund.	
WURTEMBERG: 100 pfunde = 1 centner, . - =	103.1153
BADEN: 10,000 as = 1000 dekas = 100 centas = 10 zehnling = 1 pfund.	
100 p. = 10 stein = 1 centner = 50 kilograms, - =	110.2311
SAXONY: 10,000 as = 1000 dekas = 100 hektas = 10 kilas = 1 pfund; 100 p. = 10 halbstein (half-stones) = 1 centner = 50 kilos, - =	110.2311
Leipsic (domestic): 100 pfund = 1 centner, - =	103.0734
GREAT BRITAIN. — Same as in the United States.	

NOTE. — In Great Britain, in addition to the denominations of weights used in the United States (the values of which are the same), the

Clove of wool, - - - =	7 lbs.	Stone of butchers' meat or flesh =	8 lbs.
Stone " " iron, flour, - =	14 "	Stone of cheese, - - - - =	16 "
Tod " " - - - =	28 "	Stone of glass, - - - - =	5 "
Weigh " " - - - =	182 "	Seam of glass, - - - - =	120 "
Sack " " - - - =	364 "	Stone of hemp, - - - - =	32 "
Last " " - - - =	4368 "	Fother of lead, - - - - =	19½ cwt.

GREECE. — 72 cocos = 1 dramia; 8 d. = 1 oung-hia; 8 o. = 1 imilitron; 2 i. = 1 litra, - =	1.136
400 dramias = 3⅛ litras = 1 oka, - - =	3.55
137½ litras = 44 okas = 1 cantaro, - - =	156.2
HOLLAND. — 10,000 korrel = 1000 wigtje = 100 lood = 10 onz = 1 pfund = 1 kilogram, - =	2.20462

Foreign.	*U. States.* Avoirdupois pounds.
HAWAIIAN ISLANDS (Sandwich Islands, Polynesia, N. Pacific Ocean): Same as in the United States.	

India and Malaysia, or East Indies.

ANNAM. — *Kesho* (Tonquin): 100 catties = 1 pecul,	= 132.—
Hue, Sai-gon, &c. (Cochin China): 16 leäng = 1 can; 10 c. = 1 yen; 5 y. = 1 binh; 2 b. = 1 ta; 5 t. = 1 quan, - - - -	= 688.76
BURMAH (Farther India), *Prome, Patanago, Ava, &c.:* 100 tical *or* kiat = 8 abucco = 4 agito = 3 catties = 1 vis *or* visay, - - -	= 3.39286
CEYLON ISLAND (Indian Ocean): 500 pond = 1 bahar *or* candy, - - -	= 500.—
HINDOSTAN. — *Bombay:* 72 tanks = 30 pice = 1 maund, - - - - -	= 28.—
Also, 80 tipprees = 40 seers = 1 maund, -	= 28.—
20 maunds = 1 candy = 5 cwt.	
Calcutta, Bengal *(factory weight)*: 5 siccas *or* rupees = 1 chattac; 16 c. = 1 seer; 40 s. = 1 maund; 3 m. = 2 cwt., - - -	= 224.—
4 chattacs = 1 pouah; 5 seers = 1 pussaree.	
1 maund (*bazaar weight*) = 100 troy lbs., nominally, - - - - -	= 82.133
11 factory maunds = 10 bazaar maunds.	
Madras (Carnatic, Coromandel coast): 10 pagodas *or* varahuns = 1 polam; 8 p. = 1 seer; 5 s. = 1 vis *or* visay; 8 v. = 1 maund *or* maon; 20 m. = 1 candy *or* baruay, - -	= 500.—
Goa: 32 seers = 1 maund; 20 m. = 1 bahar, -	= 495.—
Pondicherry: 10 varahuns = 1 poloin; 40 p. = 1 vis; 8 v. = 1 maund; 20 m. = 1 candy, -	= 588.—
Surat: 16 pice = 2 tipprees = 1 seer; 40 s. = 1 maund; 20 m. = 1 candy, - - -	= 300.—
3 candies = 1 bahur.	
Tatta: 4 pice = 1 anna; 16 a. = 1 seer = 72¼ tola; 40 s. = 1 maund, - - -	= 72.32
Mysore, Seringapatam, and Malabar coast generally: 40 polams = 1 vis *or* pussaree; 8 v. = 1 maund *or* maon, - - - -	= 30.—
20 maunds = 1 bahur *or* candy; 20 b. = 1 garce.	

Foreign.	*U. States.* Avoirdupois pounds.
Tranquebar, and Coromandel coast generally; 30 chittacks = 1 vis; $6\frac{2}{3}$ v. = 1 maund; 20 m. = 1 candy, - - - - -	= 500.—
MALAYA (Malay Peninsula, Strait of Malacca): Same as at Singapore Island; also, 1 kip *for tin* = $40\frac{1}{2}$ Av. lbs., and 20 buncals = 1 catty *for gold and silver* = $2\frac{1}{2}$ lbs., troy.	
PENANG ISLAND, *or* PRINCE OF WALES ISLAND (Areca Island, Strait of Malacca): Same as at Singapore.	
SIAM (Farther India): 2 tical = 1 tael; 20 t. = 1 catty; 100 c. = 1 pecul, - - -	= 135.2536
20 piculs = 1 cajar.	
SINGAPORE ISLAND (off S. extremity of Malay Peninsula): 16 tael = 1 catty; 100 c. = 1 pecul; 3 p. = 1 bahar, - - -	= 405.—
BANCA ISLAND (Malay Archipelago): Same as at Batavia, JAVA ISLAND.	
BORNEO ISLAND. — Same as at Batavia, JAVA.	
CELEBES ISLAND. — Same as at Batavia.	
JAVA ISLAND. — *Batavia, &c.:* 16 tael = 1 catty; 100 c. = 1 pecul; 3 p. = 1 bahur = 200 goelak, - - - - -	= 406.888
$4\frac{1}{2}$ peculs (300 goelaks) = 1 great bahar.	
MOLUCCAS *or* SPICE ISLANDS. — *Amboyna, the Banda Islands, Batshian, Booro, Ceram, Gilolo, Oby, Waigeoo:* Official, as at Batavia.	
PHILIPPINE ISLANDS (*Luzon, Mindano, Palawan, Mindoro, Panay, Marindique, Negros, Bohol, Zuba, Samar, Masbata, Leyte, &c.*): 100 catties = 1 pecul = $37\frac{1}{2}$ Castilian libra, -	= 139.4957
1 caban of rice (*usual*), - - - -	= 133.—
1 caban of cocoa, - - - -	= 83.5
SOOLOO ISLANDS (*Sooloo, Basseelan, Tawee-Tawee, Pilas, Pala, Tapul Isles, &c.*): 10 mace = 1 tael; 16 t. = 1 catty; 50 c. = 1 lachsa; 2 l. = 1 pecul, - - - - -	= $133\frac{1}{3}$.—
SUMATRA ISLAND. — 16 mace = 1 tael; 25 t. = 1 catty; 36 c. = 1 maund; $5\frac{1}{2}$ m. = 1 candil *or* bahur, - - - - -	= 423.5
4 catties = 5 goelaks. 1 tael = $133\frac{1}{3}$ lbs., Av.	
$14\frac{2}{3}$ salup = $7\frac{1}{3}$ ootan = 1 nelli, - -	= $29\frac{1}{3}$.—

Foreign.	*U. States.* Avoirdupois pounds.
ITALY. — The metric system of weights, either under the French denominations or as follows, is now the official, and may be considered the general commercial system throughout Italy, the islands of Sardinia and Sicily included.	
10,000 grani = 1000 denari = 100 grossi = 10 oncie = 1 libbra = 1 kilogram.	
1000 libbre = 100 rubbi = 10 quintali *or* centinaji = 1 migliajo *or* 1 tonnellata, - - =	2204.6214
Special and local:	
Carrara: 1 cubic palmo of marble = 884.74 cubic inches, - - - - - =	90.82
JAPAN (N. Pacific Ocean). — *Niphon I., Kioo-Sioo I., Sikokf I., the dependencies Yeso I., Bonin I., the Loo-Choo group, &c.:*	
1000 moo = 100 rin = 10 sen = 1 monme = 26.784 troy grains.	
160 monme = 1 kan, - - - - =	0.612206
350 monme ($2\frac{3}{16}$ kan) = 1 catty; 100 c. = 1 pecul, - - - - - =	133.92

NOTE. — In commercial transactions the pecul is usually reckoned at $133\frac{1}{3}$ lbs., the same as in China; but it is equal to 133.92 lbs. by the weights and analyses of the modern Japanese coins.

LIBERIA (W. Africa): Same as in the United States.	
MADEIRA ISLANDS (Atlantic Ocean, off W. coast of Morocco, N. Africa): Same as in Portugal.	
MALTA ISLAND (Mediterranean Sea, S. of Sicily): Official, same as in Great Britain.	
MAURITIUS ISLAND (Mascarene group, Indian Ocean): Official, as in Great Britain; also, 100 livres (old poids de marc of France) = 1 quintal, - - - - - =	107.9184
MEXICO. — 8 ochavas = 1 onza; 16 o. = 1 libra; 25 l. = 1 arroba; 4 a. = 1 quintal, - =	101.4232
MOROCCO (Barbary, N. Africa): 10 onza = 1 mark; 2 m. = 1 rotl; 100 r. = 1 cantaro - =	118.65664

NOTE. — The commercial rotl of Morocco, both in theory and practice, is equal to the weight of 20 old standard duros (silver dollars) of Spain; the miner's rotl is equal to 100 meticals (1 old Venetian libbra, peso grosso), or 1.054945 avoirdupois pound; and the market rotl = 160 meticals.

Foreign.	*U. States.* Avoirdupois pounds.
MOZAMBIQUE (E. Africa): Same as in Portugal.	
NORWAY. — Same as in Denmark.	
NUBIA (E. Africa): Same as at Alexandria, Egypt.	
PERSIA. — 6 dirhem = 2 mascais = 1 miscal = 73.846154 troy grains.	
100 miscals = 1 rotl *or* ratel, - - - =	1.054945
$136\frac{1}{2}$ rotl = 144 avoirdupois pounds.	
$6\frac{1}{2}$ rotl = 1 maund tabree (customs), - - =	6.85714
6 rotl = 1 " " (bazaar), - - =	6.3297
$7\frac{1}{2}$ rotl = 1 maund copra (customs), - - =	7.91209
7 rotl = 1 " " (bazaar), - - =	7.3846
Also, $116\frac{2}{3}$ miscals = 1 rotl copra (bazaar), and 6 rotl copra = 1 maund copra, bazaar.	
2 maunds tabree (of Tabreez) = 1 maund shah (of Sheeraz).	
1600 miscals = 1 reh of Teheran = 16 bazaar rotl of Tabreez = 6 okes of Turkey, - =	16.88023

NOTE. — The maund tabree is used chiefly for weighing coarse metals, coffee, sugar, drugs, &c., and the maund copra for weighing rice and provisions. The maund shah is used chiefly in Sheeraz, Bushire, and Gombroon, although the last-mentioned port now belongs to the Muscat dominion.

PORTUGAL. — 576 grao = 24 escropulo = 8 outava = 1 onça; 16 o. = 2 marco = 1 arratel, =	1.01187
32 arratel = 1 arroba; 4 arroba = 1 quintal, - =	129.5193
$13\frac{1}{2}$ quintalo = 1 tonelada.	
ROME and *Civita Vecchia:* 24 grao = 1 oncia; 12 o. = 1 libbra; 10 l. = 1 decime; 10 d. = 1 centinajo *or* cantaro, - - - =	74.7714
10 centinajo = 1 migliajo.	
RUSSIA. — 96 solotnik = 32 loth = 12 lana = 1 funt, - - - - - - =	0.902612
40 funt = 20 dowinik = $13\frac{1}{3}$ trowinik = 8 paterik = 4 desaterik = 1 pud, - - =	36.10448
10 puds = 1 berkowitz; 3 b. = 1 paken; 2 p. = 1 last.	
SENEGAMBIA (W. Africa). — *Bathurst, Sierre Leone:* Official, as in Great Britain.	
St. Louis: Official, as in France.	
SOCOTRA ISLAND (Indian Ocean, off E. coast of Africa): Same as at Muscat, Arabia.	
SPAIN. — *Standard of Castile:* 128 ochavas = 16 onzas = 2 marcos = 1 libra, - - =	1.0145143

Foreign.	*U. States.* Avoirdupois pounds.
25 libras = 1 arroba; 4 a. = 1 quintal; 20 q. = 1 tonelada, - - - -	= 2029.028
Special and local, but not official:	
VALENCIA. — *Alicante, &c.:* 12 onze = 1 libra menor (*minor*).	
18 onze = 1 libra mayor (*major*); 24 l. mayor, *or* 36 l. menor, = 1 arroba = $1\frac{1}{8}$ Castilian arroba, - - - - - -	= 27.3919
4 arrobas = 1 quintal; $2\frac{1}{2}$ q. = 1 carga; 8 c. = 1 tonelada, - - - -	= 2191.291
ASTURIAS. — *Santander, &c.:* 25 libras = 1 arroba = $1\frac{1}{2}$ arroba of Castile, - - -	= 38.04429
ARAGON. — *Saragossa, &c.:* 36 libras = 1 arroba,	= 27.3919
BISCAY. — *Bilboa, &c.:* 25 libras = 1 arroba = $1\frac{1}{16}$ Castilian arroba, - - - -	= 26.948
146 libras = 1 quintal macho (*for iron*).	
CATALONIA. — *Barcelona, &c.:* 25 libras = 1 arroba = $\frac{7}{8}$ Castilian arroba, - - -	= 22.1925
ANDALUSIA. — *Malaga:* 7 arrobas (*Castilian*) = $1\frac{3}{4}$ quintal = 1 carga of raisins, - -	= 177.54

NOTE. — The employment of the metric system of weights is sanctioned by law in Spain.

SWEDEN. — *New standard:* 100 korn = 1 ort; 100 o. = 1 skalpund; 100 s. = 1 centner, -	= 92.8583
SWITZERLAND. — *New system:* 32 lothe, *or* 8 gros, = 1 unze; 16 u. = 1 pfund *or* livre = $\frac{1}{2}$ kilogram; 100 pfunds = 1 centner, -	= 110.2311
TRIPOLI (Barbary, N. Africa.) — 16 karob = 1 drachma; 10 d. = 1 oncia *or* usano = $6\frac{1}{2}$ miscals *or* 1 troy ounce.	
16 oncia = 1 rotl; 100 r. = 1 cantaro, -	= 109.7143
400 drachma = 1 oka, - - - -	= 2.742857
TUNIS (Barbary, N. Africa). — Same as in Tripoli.	
TURKEY. — 4 grani = 1 kara, killot, *or* taim; 16 k. = 1 dirhem *or* drachmia = $\frac{2}{3}$ miscal *or* metical = $49\frac{3}{13}$ troy grains.	
100 dirhems = 1 cheki; 4 c. = 1 oka, -	= 2.813187
250 dirhems = 1 cheki for opium = $166\frac{2}{3}$ miscals, - - - - - -	= 1.75836
Constantinople, Galata: 2 cheki = 1 rotl *or* rotolo; 2 r. = 1 oka; 50 o. = 1 kantar *or* cantaro grosso, - - - - -	= 140.65934

Foreign.	*U. States.* Avoirdupois pounds.
176 dirhems = 1 rotl; 100 r. = 44 okes = 1 cantaro sottile, - - - -	= 123.78834
610 dirhems = 1 teffeh of Brusa silk, - -	= 4.29039
800 dirhems = 1 teffeh of goat's wool.	
Smyrna: 180 dirhems = 120 miscals = 1 rotolo; 100 r. = 45 okes = 1 cantaro, - -	= 126.60171
44 okes = 1 cantaro for tin.	
In Bassorah the Arabs commonly employ the light wakea of 160 krat of Mocha = 1 troy ounce, or 480 grains; and 15 wakea = 1 cheki, -	= 1.0285143
40 wakea ($2\frac{2}{3}$ cheki) = 1 oka, - - -	= 2.742857
50 okes (2,000 wakea) = 1 cuttra *or* cantaro,	= 137.142857

NOTE.—The initial for the avoirdupois values of the Turkish weights, in the absence of documentary statistics on the subject, if any exist, was derived from the Abyssinian dirhem and by comparison; and the result, I find, is almost strictly confirmed by assays carefully made at the United-States mint and elsewhere, of the modern gold and silver coins of Ottoman mintage compared with their present official standards; viz., $\frac{1}{320}$ cheki of fine silver to the piastre and $\frac{1}{4850}$ cheki of fine gold to the piastre. The *troy ounce*, it is well known, was derived from the Abyssinian dirhem (drachma) or its multiple by 10, the wakea, vakia, or wakega, and consists of 12 of the first-mentioned units, making the dirhem equivalent to 40 troy or United-States grains, while 120 of these dirhems, or 1 rotl or rotolo of Abyssinia, is equal to 65 miscals, or meticals, or $\frac{1}{10}$ maund tabree (customs) of Persia; hence, $120 \times 40 \div 65 = 73\frac{11}{13}$ troy grains, the value of the Persian miscal. But the miscal, or metical of Persia, and that of Turkey are alike: in theory it is the same specific weight everywhere; and 1 dirhem of Turkey is equal to $\frac{2}{3}$ miscal; hence, $73\frac{11}{13} \times \frac{2}{3} = 49\frac{3}{13}$ troy grains, the value of the Turkish dirhem, and 4 dirhem of Persia are equal to 1 dirhem of Turkey, and $6\frac{1}{2}$ miscals are equal to 1 troy ounce.

McCulloch (not to go farther back), in his work published in 1839, says the cantaro of Constantinople of 45 okas is equal to 127.2 avoirdupois pounds; or, in other words, that the oka of Constantinople is equal to 2.82667 pounds; and he states the oka of Smyrna to be equal to 2 lbs. 13 oz. 5 dr., or 2.83203 pounds, but, at the same time, under the last-mentioned head (Smyrna), states the weights and measures to be the same as those of Constantinople.

Alexander, in his work published in 1850, places the oka of Constantinople at 2.828571 pounds, and that of Smyrna at 2.812488 pounds, in a measure reversing the values by McCulloch; while Noback, in a work of more recent date, says the oka of Constantinople is equal to 1278.48 grammes = 2.8185644 pounds; making it nearly equal to that of Smyrna by Alexander; and that the oka of Smyrna is a little heavier, being equal to 2.83286 pounds.

From these conflicting statements no tenable idea can be gained except this; viz., that the initial and leading weights of Asia Minor (Anatolia) are probably theoretically and practically the same as those of Turkey in Europe. But this seems to admit of no question, since 1 batman of Persian silk, containing 1 reh, or 1600 miscals of Teheran, is invariably equal, both in Constantinople and Smyrna, to 6 okas of Turkey; wherefore, the oka is equal to $\frac{1600}{6} = 266\frac{2}{3}$ miscals, or $\frac{266\frac{2}{3} \times 73\frac{11}{13}}{7000} = 2\frac{74}{91}$ avoirdupois pounds, being slightly heavier than that of Smyrna by Alexander, and a trifle lighter than that of Constantinople by Noback.

Foreign.	*U. States.*
## *West Indies.*	Avoirdupois pounds.
GREAT ANTILLE ISLANDS. — CUBA: Standard of Castile. 16 onzas = 1 libra; 25 l. = 1 arroba; 4 a. = 1 quintal; 20 q. = 1 tonelada, - . - - -	= 2029.028
HAYTI: Poids du marc of France, previous to A.D. 1800.	
16 onces = 1 livre; 100 l. = 1 quintaux; 10 q. = 1 millier, - - - - -	= 1079.176
2 milliers *or* barriques = 1 tonneau.	
SAN DOMINGO, OR DOMINICA: Same as in Cuba.	
PORTO RICO: Same as in Cuba.	
JAMAICA: 16 ounces = 1 pound; 28 p. = 1 quarter; 4 q. = 1 cwt.; 20 cwt. = 1 ton, -	= 2240.—
LUCAYOS, OR BAHAMA ISLANDS. — Same as in Jamaica.	
CARIBBEE ISLANDS. *Leeward Group.* — DOMINICA, TORTOLA, VIRGIN GORDA, ST. CHRISTOPHER, ANGUILLA, BARBUDA, NEVIS, SABA: Same as in Jamaica.	
ANTIGUA, MONTSERRAT: 100 pounds = 1 cental *or* cwt., - - - - -	= 100.—
ST. EUSTACIUS: Official, as in Holland.	
GUADELOUPE, MARIE GALANTE, DESIRADE, LES SAINTES: Official, as in France; also as in Hayti.	
ST. MARTIN: Dutch, as in Holland; French, as in France; also as in Hayti.	
ST. THOMAS, SANTA CRUZ, ST. JAN: Official, as in Denmark.	
ST. BARTHOLOMEW: Official, as in Sweden.	
Windward Group. — BARBADOES, GRENADA, ST. VINCENT, TOBAGO: Same as in Jamaica.	
MARTINIQUE, ST. PIERRE, ST. LUCIA: Same as in Hayti.	
TRINIDAD: Same as in Cuba; official, as in Great Britain.	
LITTLE ANTILLES. — CURACOA, BUEN AYRE, ORUBA: Official, as in Holland.	
MARGARITA, TORTUGA, BLANQUILLA: Same as in Venezuela.	
ZANGUEBAR (E. AFRICA). — *Zanzibar* (Island and Town). Consul's report: 12 maunds = 1 frasler, - - - - -	= 35.—

FOREIGN LIQUID MEASURES,

REDUCED TO THEIR EQUIVALENT VALUES IN THE UNITED STATES.

Foreign.	*U. States.* Wine gallons.
ABYSSINIA.—By weight: see Weights.	
ALGERIA.— Official, as in France; also, $16\frac{2}{3}$ litres = 1 khoulle; 6 k. = 1 hectolitre, - - =	26.417
ARABIA.—(Generally by weight.) *Mocha:* 20 wakeas (weight) = 1 nusfiah; 8 n. = 1 cuda *or* gudda = 16 av. lbs.	
AUSTRALASIA.— Same as in Great Britain.	
AUSTRIA (*legal for the Empire*): 4 seidel = 1 maass; 10 m. = 1 viertel; 4 v. = 1 eimer *or* orna, - - - - - - =	14.9543
32 eimers = 1 fuder; $42\frac{1}{2}$ maass = 1 eimer for beer.	
AZORE ISLANDS.— Same as in Portugal.	
BALEARIC ISLANDS.—MAJORCA: 8 quartas = 1 quartera; $3\frac{1}{2}$ quarteras, *or* 4 quartinellos, = 1 quartin *or* barril; 4 quartin = 1 carga; 4 c. = 1 botta = 27 fluid arrobas, or 1 pipa of Castile, - - - - =	114.9692
MINORCA: 8 quartas = 1 quartera; 4 quarteras, *or* $2\frac{2}{3}$ gerah, = 1 quartin *or* barril; 4 barrils = 1 carga; 4 c. = 1 botta = $31\frac{1}{3}$ fluid arrobas of Castile, - - - - =	133.1635
BELGIUM.— Same as in France.	
BERBERA.— Same as at Mocha, Arabia.	
BERMUDA ISLANDS.— Official, as in Great Britain; in trade, generally as in the United States.	
BOURBON ISLAND.— Same as in France.	
CANADA, DOMINION OF.— Official, as in Great Britain; in trade, as in the United States.	
CANARY ISLANDS.— Same as in Spain, Castilian Standard.	
CANDIA ISLAND.—1 mistata = $8\frac{1}{2}$ okes weight, or, of olive oil, - - - - =	2.9397
CAPE OF GOOD HOPE.— Same as in Great Britain.	
CAPE VERDE ISLANDS.— Same as in Portugal.	

Foreign.	*U. States.*
Central and South America.	Wine gallons.
GUATEMALA, HONDURAS, SAN SALVADOR, NICARAGUA, COSTA RICA: Same as in Spain, standard of Castile; also the wine gallon of the United States is used.	
BALIZE: Official, as in Great Britain.	
BRAZIL: 24 quartilhos = 12 garrafa = 6 canada = 3 medida = 1 alqueire *or* pote = 18 arratels weight, - - - - - =	2.18418
60 potes = 1 pipa = 1080 arratels weight, - =	131.051
Bahia: 1 canada = $15\frac{1}{2}$ arratels weight = $5\frac{1}{6}$ canadas of Rio Janeiro, - - - =	1.88082
72 canadas = 1 pipa of spirits, - - - =	135.4193
100 canadas = 1 pipa of molasses.	
ARGENTINE REPUBLIC: 4 cuartos = 1 frasco; 8 f. = 1 caneca = 19 litres in theory, - - =	5.01927
3 frascos = 1 cortan; 16 c. (6 canecas) = 1 carga, - - - - - =	30.11541
4 cargas = 1 pipa catalana; also, 8 frascos = 5 U. S. gallons, nominally.	
PERU, CHILI, BOLIVIA, ECUADOR, NEW GRANADA, VENEZUELA, URUGUAY, PARAGUAY: Chiefly as in Spain, standard of Castile.	

NOTE.—In the States last mentioned, the U. S. wine gallon is more or less used in trade; and in Chili it is the customs' unit-measure for liquids. Also, in Chili, Peru, New Granada, Bolivia, and Venezuela, the use of the metric system is sanctioned by law, and may be expected to gradually come into use.

GUIANA—*Cayenne:* Same as in France.	
Paramaribo (Surinam): Same as in Holland; also as in Cayenne.	
Georgetown (Demerara): Same as in Great Britain.	
FALKLAND ISLANDS: Same as in Great Britain.	
CHINA.—By weight only; for denominations, see DRY MEASURES.	
DENMARK.—42 pagel = 4 potte = 2 kande = 1 stubchen, - - - - - - =	1.02089
40 stubchen = 20 viertel = 4 anker = 1 ohm, =	40.83522
$1\frac{1}{2}$ ohm = 1 oxehoved; 2 oxehoved = 1 piba; 2 p. = 1 fuder; $1\frac{1}{4}$ f. = 1 stykfad.	
34 stubchen = 1 toende *for beer;* 30 stubchen = 1 toende *for tar.*	
EGYPT.—By weight exclusively. See WEIGHTS.	

Foreign.	*U. States.* Wine gallons.
FRANCE. — 1,000 millilitres = 100 centilitres = 10 decilitres = 1 litre, - - - =	0.26417
100 litres = 10 decalitres = 1 hectolitre, - =	26.417029
100 hectolitres = 10 kilolitres = 1 myrialitre.	
GERMANY. — Prussian and Zollverein maasse for all the States of the tariff-alliance: 2 oessel = 1 quart; 30 q. = 1 anker; 2 a. = 1 eimer = 3840 cubic Rhine zollen; or, since $38\frac{1}{4}$ zollen = 1 metre, = $68\frac{2212764}{3581577}$ litres, - - =	18.126789
2 eimers = 1 aam, ahm, *or* ohm; 3 eimers = 1 oxhoft; 12 eimers = 1 fuder.	
$3\frac{1}{3}$ eimers = 2 barrile = 1 fass *for beer*, - =	60.42263

NOTE. — I have been thus particular in treating of the eimer, because the notion seems to be generally entertained that it is equal in theory to 68.7 litres.

Special and local, or domestic:	
BADEN: 1000 maass = 100 stubchen = 10 ohm = 1 fuder = 15 hectolitres, - - - =	396.2555
BAVARIA: 4 quartile = 1 mäss, *or* mässkanne; 60 m. = 1 eimer, - - - - =	16.9452
25 eimer = 1 fass; 64 mässe = 1 eimer *for beer*.	
BREMEN: 4 mengel = 1 quartier, *or* vierling; 4 q. = 1 stubchen, - - - - =	0.85106
$2\frac{1}{4}$ stubchen = 1 viertil; 5 v. = 1 anker; 4 a. = 1 ahm, - - - - - =	38.29781
6 ankers = 1 oxhoft; 4 o. = 1 fuder; 44 stubchen = 1 ahm *for wine*.	
6 stubchen = 1 steckannen; 6 s. = 1 tonne *for train oil* = 216 pfunde weight, or, at $7\frac{3}{4}$ av. lbs to the gallon, - - - - =	30.6073
BRUNSWICK: 10 stubchen = 1 anker; 4 a. = 1 ahm; $1\frac{1}{2}$ ahm = 1 oxhoft, - - - =	59.2803
HAMBURG: 8 oessel, plank, *or* stück = 4 quartier, *or* potts = 2 kannen = 1 stubchen, - - =	0.956404
8 stubchen = 4 viertel = 1 eimer, - - =	7.651224
32 stubchen = 1 anker; 40 stubchen = 1 ohm; 60 stubchen = 1 oxhoft; 240 stubchen = 1 fuder.	
HESSE DARMSTADT: 16 schoppen = 4 masschen = 1 viertel; 20 v. = 1 ohm = 16 decalitres, =	42.26725
LUBEC: 16 ort = 8 plank, *or* nossel = 4 quartier = 2 kanne = 1 stubchen, - - - =	0.9546
8 stubchen = 4 viertel = 1 eimer, - - =	7.6512
5 viertels = 1 anker; 6 a. = 1 fass, - - =	57.384

Foreign.	*U. States.* Wine gallons.
MECKLENBURG (*legal*): Same as in Hamburg.	
OLDENBURG: 240 quartiers, *or* 156 kannies = 1 oxhoft (*legal*) = 1 fuder of Lubec.	
SAXONY (*legal*): 144 nössel = 72 kanne = 24 viertel = 1 eimer, - - - -	= 17.8107
2 eimers = 1 aam; 3 eimers = 1 oxhoft; 5 eimers = 1 fass; 12 eimers = 1 fuder.	
WURTEMBERG. — *Helleich mäss:* 4 quartier, *or* schoppen = 1 maas; 10 m. = 1 immer; 16 i. = 1 eimer; 6 e. — 1 fuder, - - -	= 465.9036
GREAT BRITAIN. — *Imperial measure:* Denominations and relative values same as in the United States, but capacity values = $20\frac{74}{231}$ per cent greater. See LIQUID MEASURES, U. S.	
1 imperial gallon = $\frac{277274}{231000}$ or 1.200320344 wine gallons of the United States.	
GREECE. — 1 kila, *or* galloni = $2\frac{1}{2}$ okas weight.	
HOLLAND. — 10 vingerhoed = 1 maatje; 10 m. = 1 kan; 100 k. = 1 vat = 1 hectolitre, -	= 26.417
HAWAIIAN ISLANDS. — Same as in the United States.	

India and Malaysia, or East Indies.

ANNAM, BURMAH, *Calcutta*, and BENGAL generally, CEYLON I., PHILIPPINE Is., SOO-LOO Is. — By weight. See WEIGHTS.	
Bombay, Madras: By weight, chiefly; the wine gallon of the United States is sometimes used.	
Goa: Same as in Portugal.	
Pondicherry: Official, as in France.	
MALACCA. — Capacity measures, same as in the United States.	
PENANG ISLAND. — Same as in Singapore.	
SIAM. — 20 canan = 1 cohi = 80 catties weight, or 108.203 av. lbs., - - - -	= 12.9757
SINGAPORE ISLAND. — Capacity measures, same as those of the United States.	
BANCA I., BORNEO I., CELEBES I., JAVA I., MOLUCCA Is., SUMATRA I. — Official, same as in Holland.	
ITALY. — The metric measures of capacity are used here, both under the French nomencla-	

Foreign.	*U. States.* Wine gallons.
ture and as follows; viz., 10 coppa = 1 pinta; 10 p. = 1 mina; 10 m. = 1 soma = 1 hectolitre, - - . - - =	26.417
JAPAN. — 1 tsjoo = $\frac{1}{16}$ cubic kani-sasi = 106.09663 cubic inches, - - - - - =	0.459293
10 tsjoo = 1 to; 10 to = 1 kók; 10 sasi = 1 goo; 10 goo = 1 tsjoo.	
LIBERIA. — Same as in the United States.	
MADEIRA ISLANDS. — Same as in Portugal.	
MALTA ISLAND. — Official, same as in Great Britain; also, 1 caffiso of oil = 4.724 gallons, and 1 barrile of wine = 9.448 gallons.	
MAURITIUS ISLAND. — 8 pintes = 1 velt, - =	1.969
MEXICO. — Chiefly as in Spain, *Castilian standard;* but the use of the metric system is legalized, and may be expected soon to be introduced into practice.	
MOROCCO. —	
MOZAMBIQUE. — Same as in Portugal.	
NORWAY. — The Danish capacity measures are used here.	
NUBIA. — By weight, as at Alexandria.	
PERSIA. — By weight. See WEIGHTS.	
PORTUGAL. — 24 quartilhos = 6 canadas = 1 alqueire, *or* pote = 18 arratels weight, - =	2.18418
2 potes = 1 almude; 26 a. = 1 bota *or* pipa, - =	113.5775
2 botas = 1 tonelada; 18 almudes = 1 barril.	
Oporto: 1 alqueire = $27\frac{1}{4}$ arratels weight, or $\frac{100}{66}$ alqueires of Lisbon, - - - - =	3.30936
ROME, and *Civita Vecchia:* 64 cartocci = 16 quartucci = 4 foglietti = 1 boccale, - =	0.48165
32 boccali = 1 barile; 16 b. = 1 botta, - =	246.605
32 boccale *for wine* = 28 boccale *for oil.*	
RUSSIA. — 100 tscharka = 10 krushka = 1 vedro, *or* wedro, - - - - - =	3.24674
3 vedros = 1 anker; 6 a. = 1 oxhoft, - =	58.4413
40 vedros = 1 botschka.	
SPAIN (*Castilian standard*): 4 copas = 1 cuartillo; 4 c. = 1 azumbra; 8 a. = 1 arroba *or* cantaro = 35 libras weight of distilled water at maximum density, or 35.508 av. lbs., =	4.25812
16 arrobas = 1 moyo; 27 arrobas = 1 pipa; 30 arrobas = 1 bota; 60 arrobas = 1 tonelada.	
1 arroba menor *for oil* = $27\frac{1}{4}$ libras weight, - =	3.31525

Foreign.	*U. States.* Wine gallons.
Special and local:	
Alicante and Valencia: 16 cuartillos = 4 cuartos = 1 arroba = $\frac{3}{4}$ Castilian arrobas, - -	= 3.19359
40 arrobas = 1 pipa; 2 p. = 1 tonelada, -	= 255.4872
Also 100 cantaros of $\frac{3}{5}$ Castilian arroba each = 1 tonelada.	
Barcelona: 16 cortans (12 arrobas Catalan weight) = 1 carga = $7\frac{1}{2}$ fluid arrobas of Castile, - - - - -	= 31.9359
Gibraltar: 38 arrobas menor of Castile = 1 pipa,	= 125.9795
126 U. S. gallons, or 105 imp. gallons in theory = 1 pipa.	
Malaga: $33\frac{1}{3}$ fluid arrobas of Castile = 1 pipa, -	= 141.9373

NOTE.—The employment of the metric capacity measures is sanctioned by law in Spain.

SWEDEN.—4 qwarter = 2 stop = 1 kanna = $\frac{1}{10}$ cubit fot.	
48 kannas = 8 ottingar = 4 fjerding = 1 tunna, - - - - - -	= 33.184106
SWITZERLAND.—(*Official and legal for the* 22 *Cantons*): 1000 emine = 100 maass *or* potts = 10 gelt = 1 saum = 150 litres, - -	= 39.62555
TRIPOLI.—14 caraffa = 1 mataro *for oil* = $17\frac{1}{2}$ okas weight, or 48 av. lbs.	
40 caraffa = 1 barril = 50 okas weight; also 24 bozza = 1 barile = 130 rotolos weight (52 okas), or, of the standard of the United States, - - - - -	= 17.10402
TUNIS.—2 mettars for wine = 1 mettar for oil = 36 rotoli weight; $3\frac{1}{3}$ mettars = 1 millerolle = 120 rotoli weight for oil, or 231.65716 av. lbs.	
TURKEY.—By weight. See WEIGHTS. Also 1 almud of oil = 8 okas.	

West Indies.

Cuba, Porto Rico: Same as in Spain, Castilian standard; but in Cuba the U. S. gallon is also used: 36 gallons = 1 bocoy = 36 U. S. gallons.

Dominica (San Domingo, or Dominican Republic, HAYTI I.): Same as in Spain, standard of Castile.

Foreign.	*U. States.* Wine gallons.
Hayti, Empire of (HAYTI I.). — 60 gallons = 1 tierçon, - - - - - -	= 60.—

Guadeloupe, Martinique, Marie Galante, Les Saints, Desirade, northern portion of *St. Martin.* — Official, as in France; but in trade the United-States fluid gallon is chiefly used; *for molasses,* 30 gallons = 1 barāl; 65 gallons = 1 tierçon; 105 gallons = 1 baucaut: *for rum,* 114 gallons = 1 boucaut.

Jamaica, Trinidad, Bahamas, Barbadoes, St. Christopher, Dominica, Montserrat, Grenada, St. Lucia, Antigua, Tortola, Tobago, Nevis, Virgin Gorda, Grenadines: Official, as in Great Britain; in trade, mostly as in the United States.

St. Thomas, Santa Cruz, St. Jan: Official, as in Denmark.

St. Eustatius, Curacoa, Buen Ayre, Oruba, southern portion of *St. Martin:* Official, as in Holland.

Margarita, Tortuga, Blanquilla: Same as in Venezuela.

St. Bartholomew: Official, as in Sweden.

FOREIGN DRY MEASURES,

REDUCED TO THEIR EQUIVALENT VALUES IN THE UNITED STATES.

Foreign.	*U. States.* Winchester bushels.
ABYSSINIA. — 24 madega = 1 ardeb, - - =	0.33333
ALGERIA. — Official, as in France; also 16 tarrie = 8 saa *or* saha = 1 caffiso, - - =	9.—
ARABIA. — By weight: 40 kellas = 1 tomaun *for rice* = 56 maunds weight, or 168 av. lbs.	
AUSTRALIA. — Same as in Great Britain.	
AUSTRIA (*legal for the Empire*). — 4 becher = 1 mässel; 4 m. = 1 viertel; 4 v. = 1 metze, - =	1.7452
AZORE ISLANDS. — 16 quartos = 4 alqueires = 1 fanga; 15 f. = 1 moio = $\frac{7}{8}$ moio of Lisbon, - - - - - - =	20.5298
BALEARIC ISLANDS. — 6 barcella = 1 quartera = $1\frac{1}{3}$ fanega of Castile, - - - - =	2.157
BELGIUM. — 100 kop = 10 schepel = 1 mudde = 1 hectolitre, - - - - =	2.83774
BERBERA. —	
BERMUDA ISLANDS. — Official, as in Great Britain; the U. S. bushel is also used.	
BOURBON ISLAND. — Same as in France.	
CANADA, Dominion of. — Official, as in Great Britain; in trade, as in the United States, except that in Lower Canada the old French minot = 1.107436 U. S. bushels is used.	
CANARY ISLANDS. — 12 celamins = 1 fanega = 136 libras weight, - - - =	1.77737
$16\frac{7}{8}$ celamins = 1 fanega heaped.	
CANDIA ISLAND. — 1 carga, - - - =	4.3211
CAPE OF GOOD HOPE. — Same as in Great Britain; also, 4 schepels = 1 muid - - =	3.1564
CAPE VERDE ISLANDS. — Same as in Portugal.	

Central and South America.

Guatemala, Honduras, San Salvador, Nicaragua, Costa Rica: Same as in Spain, Castilian standard; but the U. S. bushel is also used.

Foreign.	*U. States.* Winchester bushels.
BALIZE. — Official, as in Great Britain.	
BRAZIL. — 16 quartas = 4 alqueirs = 1 fanga; 15 f. = 1 moio = $\frac{6}{7}$ moio of Portugal, - -	= 20.11086
Bahia: 1 alqueire = $67\frac{1}{2}$ arratels weight, or $2\frac{1}{4}$ alqueires of Portugal, - - - -	= 0.87985
Maranham: 1 alqueire = 100 arratels weight, -	= 1.30348
ARGENTINE REPUBLIC, URUGUAY. — 4 cuartillos = 1 fanega = 134 litres, - - -	= 3.80257
CHILI. — 12 celamins = 1 fanega, - - -	= 2.5753
PERU. — 1 fanega, - - - - -	= 2.31777
BOLIVIA, ECUADOR, NEW GRANADA, VENEZUELA, PARAGUAY: Chiefly as in Spain, Castilian standard.	
GUIANA. — *Cayenne:* Same as in France.	
Paramaribo: Same as in Holland, also as in Cayenne.	

NOTE. — The use of the metric system is sanctioned by law in Chili, Peru, New Granada, Bolivia, Venezuela, and French and Dutch Guiana, and, to some extent, is introduced into practice.

Georgetown: Same as in Great Britain.	
FALKLAND ISLANDS. — Same as in Great Britain.	
CHINA. — 10 hŏ = 1 shing; 10 s. = 1 tau; 10 t. = 1 hwŭh, sei, *or* tane = 120 catties weight = 160 av. lbs.	
DENMARK. — 32 sextingkar = 16 ottingkar = 2 skieppe = 1 fjerding, stubchen, *or* scheffel = 36 potte, - - - - -	= 0.98698
4 fjerding = 1 toende; 22 t. = 1 last, - -	= 86.8546
EGYPT. — *Cairo:* 24 robi = 6 usbek = 1 ardeb = 144 okas weight, - - - -	= 5.088
Alexandria, Rosetta: 1 kislos = 137 okas weight,	= 4.84065
1 rebeb = 126 okas weight, - - -	= 4.45198
1 ardeb = 230 okas weight, - - -	= 8.12663
FRANCE. — 100 litres = 10 decalitres = 1 hectolitre, - - - - - -	= 2.83774
100 hectolitres = 10 kilolitres = 1 myrialitre, -	= 283.774
GERMANY. — PRUSSIA, and Zollverein mäss of all the States of the tariff-alliance: 16 metzen (3072 cubic Rhein zollen, or 48 fluid zollverein quarts) = 1 scheffel, - - -	= 1.55776
Special and local:	
BADEN: 1000 becher = 100 mässlein = 10 sester = 1 malter = $1\frac{1}{2}$ hectolitre, - - -	= 4.25661
10 malters = 1 zober.	

Foreign.	*U. States.* Winchester bushels.
BAVARIA: 16 dreissiger = 4 mässel = 1 viertel, -	= 0.525855
12 viertels ($17\frac{1}{3}$ fluid mässkanne, or six old metzen) = 1 scheffel, - - - -	= 6.310263
BREMEN: 16 spint = 4 viertel = 1 scheffel, -	= 2.10289
10 scheffels = 1 quarter; 4 q. = 1 last, -	= 84.11572
BRUNSWICK: 4 metzen = 1 himt; 40 h. = 1 wispel, - - - - - -	= 35.3544
HAMBURG: 8 spint = 2 himten = 1 fass = 1 zollverein scheffel, - - - -	= 1.55776
10 scheffels = 1 wispel; 6 w. = 1 last, -	= 93.4656
HESSE DARMSTADT: 64 kopfchen = 32 maasschen = 8 gescheid = 2 kumpf = 1 metze, -	= 0.45404
8 metzen = 2 simmer = 1 malter = 128 litres,	= 3.632308
LUBEC: 16 fass = 4 scheffels = 1 tonne, - -	= 3.9381
3 tonnen = 1 dromt; 8 d. = 1 last, - -	= 94.5139
MECKLENBURG: 16 spint, *or* metzen = 4 fass, *or* viertel = 1 scheffel, - - - -	= 1.103628
4 scheffels = 1 wispel; 3 w. = 1 last, - -	= 105.9483
OLDENBURG: 16 kannen = 1 scheffel; 8 s. = 1 tonne, - - - . -	= 5.17536
$1\frac{1}{2}$ tonne = 1 molt; 12 m. = 1 last, - -	= 93.1565
SAXONY: 16 mässchen = 4 metzen = 1 viertel, -	= 0.73713
4 viertels = 1 scheffel; 12 s. = 1 malter, -	= 35.3823
2 malters = 1 wispel; 6 w. = 1 last, - -	= 424.5876
WURTEMBERG: 32 viertelein = 8 ecklein = 1 vierling, - - - . - -	= 0.157172
32 vierling = 8 simri = 1 scheffel, - -	= 5.0295
GREAT BRITAIN. — *Imperial measure:* Denominations and relative values same as in the United States, but capacity values = $\frac{554548}{537605}$ greater; 1 bushel = 1.0315157 U. S. bushels.	
GREECE. — 1 kila, - - - -	= 0.944
HOLLAND. — 1000 maatje = 100 kopen = 10 schepel = 1 mudde *or* zac = 1 hectolitre, -	= 2.83774
30 mudden = 1 last, - - - -	= 85.1322

India and Malaysia, or East Indies.

ANNAM, BURMAH, CEYLON ISLAND: By weight. See WEIGHTS.

HINDOSTAN. — *Bombay:* 8 tipprees = 4 seers = 1 adoulie.

Foreign.	*U. States.* Winchester bushels.
16 adoulies = 1 para = $8\frac{3}{4}$ maunds weight, or 245 av. lbs., - - - - -	= 3.15607
8 paras = 1 candy = 70 maunds weight, -	= 25.248545
Calcutta: 80 chattac = 16 koonke = 4 raik = 1 pallie = 5 seers weight, or $9\frac{1}{3}$ av. lbs.: 12 pallies = 1 morah = $1\frac{1}{2}$ factory maunds weight, or 112 av. lbs.	
20 pallies = 1 soallee = $2\frac{1}{2}$ factory maunds, or $186\frac{2}{3}$ av. lbs.	
16 soallees (40 maunds, or $2986\frac{2}{3}$ av. lbs.) = 1 kahoon, - - - - -	= 38.47397
Madras: 64 ollock = 8 puddy = 1 marcal.	
5 marcals = 1 para = $5\frac{2}{5}$ maunds weight, or 135 av. lbs.	
80 paras = 1 garce = 432 maunds, or 10800 av. lbs., - - - - - -	= 139.12464
Goa: Same as in Portugal.	
Pondicherry: Same as in France.	
MALACCA. — The Winchester bushel is used, also the coyang of Siam.	
PENANG ISLAND. — Generally as in the United States.	
SIAM. — 40 sat = 1 sesti; 40 s. = 1 cohi; 65 c. = 1 coyang = 52 peculs weight, or 7033.1872 av. lbs., - - - - -	= 90.6009
SINGAPORE ISLAND. — Generally as in the United States.	
BANCA ISLAND, BORNEO ISLAND, CELEBES ISLAND, JAVA ISLAND, MOLUCCA ISLANDS, SUMATRA ISLAND. — Official, as in Holland.	
ITALY. — See LIQUID MEASURES:	
1 soma (hectolitre), - - - -	= 2.83774
JAPAN. — See LIQUID MEASURES:	
1 kok ($6\frac{1}{4}$ cubic kani-sasi), - - -	= 4.93376
LIBERIA. — Same as in the United States.	
MADEIRA ISLANDS. — Same as in Portugal.	
MALTA ISLAND. — Official, as in Great Britain; also, 1 salma rasa ($\frac{6}{7}$ salma colma), - -	= 8.2202
MAURITIUS ISLAND. — Official, as in Great Britain.	
MEXICO. — Chiefly as in Spain, standard of Castile; but the use of the metric system is sanctioned by law.	

Foreign.	*U. States.* Winchester bushels.
MOROCCO.—	
MOZAMBIQUE.— Portuguese measures are used here.	
NORWAY.— Same as in Denmark.	
PERSIA.— 8 sextarios = 2 chenicas = 1 capicha.	
25 capichas = 8 colothuns = 1 artaba = 21 maunds tabree (customs), or 144 av. lbs., - =	1.8541
22 sextarios = 1 sabbitha, - - - =	0.20395
15 capichas = 1 legana, - - - =	1.11246
PORTUGAL.— 16 quartos = 4 alqueires = 1 fanga = 120 arratels weight, - - - =	1.56418
15 fangas = 1 moio = 1800 arratels weight, - =	23.46267
Oporto: 1 fanga = $1\frac{1}{4}$ fanga of Lisbon, or 150 arratels weight, - - - - =	1.95522
ROME.— 88 quartucci = 22 scorzi = 16 starelli = 12 staja = 8 quarterella = 4 quarte = 2 rubbiatilli = 1 rubbio, - - - - =	8.3562
RUSSIA.— 32 garnetz = 16 tschetwertka = 4 tschetwerik = 2 payak = 1 osmin, - - =	2.97607
2 osmins = 1 tschetwert; $1\frac{1}{2}$ t. = 1 kuhl.	
SANDWICH ISLANDS.— Same as in the United States.	
SPAIN (*Standard of Castile*).— 16 racion = 4 quartillos = 1 celemin, *or* almuda; 12 celamins = 4 cuartilla = 1 fanega = $4\frac{7}{8}$ arrobas weight of distilled water at maximum density, or 123.64393 av. lbs., - - - =	1.59277
12 fanegas = 1 cahiz, - - - - =	19.113241
Special and local:	
Alicante: 4 celamins = 1 barcella; 12 b. = 1 cahiz = $\frac{4}{11}$ cahiz of Castile, - - - =	6.950306
Barcelona: 48 picotin = 12 cortan = 1 quartera = $6\frac{3}{8}$ Castilian arrobas weight, - - =	2.08285
$2\frac{1}{2}$ quarteras = 1 carga; 4 quarteras = 1 salma.	
SWEDEN.— 4 quarter = 2 stop = 1 kanna = $\frac{1}{10}$ cubic fot.	
7 kanna = 4 kappe = 1 fjerding, - - =	0.519847
4 fjerding = 1 spann; 2 s. = 1 tunna, - - =	4.158777
36 kappe = 1 tunne (firm measure), - - =	4.678624
SWITZERLAND.— (Official and legal for the 22 Cantons):	
100 immi = 10 viertel, gelt, *or* quarteron = 1 malter = 150 litres, - - - - =	4.25661
Also, 16 mässli = 4 vierling = 1 viertel.	

Foreign.	*U. States.* Winchester bushels.
TRIPOLI. = 2 nufs-orbah = 1 orbah; 4 orbahs = 1 temen; 4 t. = 1 ueba = 216 rotls weight, - =	3.05279
TUNIS. — 12 zah, *or* saha = 1 quiba; 16 q. = 1 caffiso = 425 okas weight, - - - =	15.01663
TURKEY. — 4 kiloz = 1 fortin = 110 okas weight, =	3.986315

West Indies.

Cuba, Porto Rico, San Domingo: Same as in Spain, standard of Castile.

Hayti, Empire of. — 16 litrons = 1 boisseau; 12 b. = 1 setiere, - - - - - = 4.4299

Gaudeloupe, Martinique, Marie Galante, Desirade, northern portion of *St. Martin, Les Saints.* — Official, as in France; also, as in Hayti; and the U. S. bushel is often used.

Jamaica, Trinidad, Bahamas, Barbadoes, St. Christopher, Dominica, Montserrat, Grenada, St. Lucia, Antigua, Tortola, Tobago, Nevis, Virgin Gorda, Grenadines. — Official, as in Great Britain; in trade, generally as in the United States, but in *Trinidad* often as in *Hayti.*

St. Thomas, Santa Cruz, St. Jan. — Official, as in Denmark.

St. Eustatius, Curacoa, Buen Ayre, Oruba, southern portion of *St. Martin.* — Official, as in Holland.

Margarita, Tortuga, Blanquilla. — Same as in Venezuela.

St. Bartholomew. — Official, as in Sweden.

MEMORANDA AND ADDENDA,

RELATIVE TO FOREIGN MONEYS OF ACCOUNT, COINS, WEIGHTS, MEASURES, QUOTATIONS OF STOCKS, ETC.

GREAT BRITAIN. — In Great Britain, *sovereigns* weighing not less than 122¾ grains are a legal tender for a pound sterling each; which makes the minimum value of a pound sterling in gold equal to $4.8458226; and this value of the pound sterling, very nearly, is adopted by the United-States Government in assessing duties on British invoices.

Goods by weight, passing through a British custom-house, and subject to duties, are subject to an allowance, called *draft* or *tret*, for supposed waste over and above the actual *tares*, as follows; viz., on 1 cwt. (112 lbs.), 1 lb.; above 1 cwt. and under 2 cwt., 2 lbs.; on 2 cwt. and under 3 cwt., 3 lbs.; on 3 cwt. and under 10 cwt., 4 lbs; on 10 cwt. and under 18 cwt., 7 lbs; and on 18 cwt. and upwards, 9 lbs. These allowances were also made at the United-States custom-houses, until July 14, 1862, when the discontinuance of the practice was ordered by law. These are the chief reasons why goods by weight from the United States fall short of weight at the British custom-houses.

Consols, or Consolidated Annuities, represent a considerable portion of the public debt of Great Britain: they bear interest at the rate of three per cent. a year, payable semi-annually, and are transferable.

The quotations in London of the prices of United-States Bonds, and of American Stocks generally, are in cents per dollar, payable in United-States gold coins; and upon the old basis of 4\frac{4}{9}$ to the £. To these quotations, therefore, 9½ per cent. must be added to express the real price.

FRANCE. — The proposed new gold coin of France, of 25 francs, to be called an *emperor*, and to be equal in value to a British *sovereign*, must bear a premium at home of about $\frac{7}{8}$ of 1 per cent., if the present mint regulations in other respects be maintained.

A tolerance of weight in excess of the standard, and in excess only, is allowed in France; while in the United States and in Great Britain, strict conformity to the standards are required; thus it happens that the model standards of weight sent abroad from France are generally found to be slightly in excess of the true standard.

The tolerance, or remedy, spoken of is as follows: —

On *iron weights* of 50 kilogrammes, tolerance, 20 grammes; 20 kilogrammes, tolerance 10 grammes; 5 kilogrammes, tolerance 4 grammes; 1 kilogramme, tolerance 1 gramme.

On *copper* or *brass weights* of 20 kilogrammes, tolerance 1½ grammes; 5 kilogrammes, tolerance ½ gramme; 2 kilogrammes, tolerance ¼ gramme; 1 kilogramme, tolerance 1½ decigrams.

CUSTOMS TARES,

OR TARES AS ALLOWED BY THE UNITED-STATES GOVERNMENT ON DUTIABLE GOODS.

By an Act of Congress passed July 14, 1862, it is provided that when the original invoice is produced at the time of making entry thereof, and the tare shall be specified therein, it shall be lawful for the collector, if he shall see fit, with the consent of the consignees, to estimate the said tare according to such invoice; and that in all other cases the real tare shall be allowed, and may be ascertained, under such regulations as the Secretary of the Treasury may from time to time prescribe; and that hereafter there shall be no allowance for draft. And, in accordance with these provisions, the following rates of tare were adopted: —

	Per cent.
Almonds, in bags	2
" in bales	2½
" in frails	8
Alum, in casks	10
" coarse or ground, in sacks, 2 lbs. per sack.	
Barytes	3
Cheese, in casks or tubs	10
Cassia, in mats	9
Cinnamon, in bales	6
Chicory, in bags	2
Cocoa, in bags	2
" in ceroons	8
Coffee, Rio, in single bags	1
" " in double bags	2
" all others, actual tare.	
Copperas, in casks	10
Currants, in casks	10
Hemp, Manilla, in bales, 4 pounds per bale.	
Hemp, Hamburg, Leghorn, or Trieste, in bales, 5 lbs. per bale.	
Indigo, in ceroons	10
Melado	11
Nails, in bags	2
" in casks	8
Ochre, dry, in casks	8
" in oil, in casks	12
Paris white, in casks	10
Peruvian bark, in ceroons	10
Pepper, in single bags	2
" in double bags	4
Pimento, in bags	2
Raisins, in casks	12
" in boxes	25
" in half-boxes	27
" in quarter-boxes	29
" in frails	4
Rice, in bags	2
Spanish brown, dry, in casks	10
Spanish brown, in oil, in casks	12
Sugar, in mats	2½
" in bags	2
" in barrels	10
" in tierces	12
" in hhds.	12½
" in boxes	14
Salt, fine, in sacks, 3 lbs. per sack.	
Tea, China or Japan, invoice tare.	
Tea, all others, actual tare.	
Tobacco, leaf, in bales, 10 pounds per bale.	
Tobacco, leaf, in bales, extra covers, 12 pounds per bale.	
Whiting, common, in casks	10

SECTION B.

REDUCTIONS, EXCHANGE, INVESTMENTS, MIXED NEGOTIATIONS, &c., &c.

PROPOSITION 1. — When a dollar in gold is worth a dollar and thirty cents in currency, what is the value of a currency dollar?

$1 \div 1.30 = \$0.769231$, or 76 cents 9.231 mills. *Ans.*

PROP. 2. — When gold is nominally at a premium of $35\frac{3}{4}$ per cent., at what discount is currency?

$100 - \frac{100}{1.3575} = 26.335175$ per cent. *Ans.*

PROP. 3. — What is the difference between a given principal, P, and a *deduction* of r per cent. of it?

$P \sim Pr = P(1 - r)$, the present worth. *Ans.*

PROP. 4. — What is the difference between a given principal, P, and a *discount* of r per cent. of it?

$$P \sim \frac{Pr}{1+r} = \frac{P}{1+r}, \text{ the present worth. } \textit{Ans.}$$

PROP. 5. — Express the difference per dollar between *deduction* at r per cent., and *discount* at the same rate per cent.

$$(1 - r) \sim \frac{1}{1+r}. \quad \textit{Ans.}$$

PROP. 6. — Express the discount, d, on a given principal, P, for a given time in days, t, at a given rate, r, per cent. per annum.

$d = \frac{Ptr}{365 + tr} = \frac{Pi}{1+i} = P - w$, i being the interest on 1 dollar for the given time at the given rate, and w the present worth for the same time and rate; whence, $w = \frac{365P}{365 + tr} = \frac{P}{1+i}$ $= P - d$. See DISCOUNT, page 129.

PROP. 7. — The foregoing proposition (Prop. 6); except that the time, T, is in years, or years and decimal parts of a year?

$d = PTr \div (1 + Tr)$. *Ans.*

PROP. 8. — The foregoing proposition (Prop. 6), except that the time, m, is given in months, or months and decimal parts of a month?

$d = Pmr \div (12 + mr)$. *Ans.*

PROP. 9. — To convert time in days into calendar months and decimal parts of a month; $m = 12d \div 365$; but the converse of this, viz., $d = 365m \div 12$, is not liable to be true in practice, since different calendar months are made up of an unlike number of days; and, therefore, those immediately under consideration may contain a greater or less number of days.

PROP. 10. — What is the intrinsic equivalent in Federal money of £712 10*s.* 9½*d.* sterling?

$$\left(712 + \frac{12 \times s + d}{12 \times 20}\right) \times 4.86656 = \$3467.6166. \quad \textit{Ans.}$$

PROP. 11. — Reduce $3467.6166 to its intrinsic equivalent in sterling money.

$$3467.6166 \div 4.86656 = 712.53958$$

```
3467.6166 ÷ 4.86656 = 712.53958
                             20
                        -------
                        10.7916
                             12
                        -------
                         9.4992   £712 10s. 9½d.   Ans.
```

PROP. 12. — Express the equivalent in Federal money, $, of a given quantity in sterling money, £, at a given true or direct rate of exchange, *r*, that is to say, at a rate of exchange based upon the intrinsic equivalent of the two denominations of money.

$$\$ = £ \times 4.86656 \times r. \quad \textit{Ans.}$$

PROP. 13. — Express the equivalent in Federal money of a given quantity in sterling money, rated by the former intrinsic par of 4*s.* 6*d.* sterling to the dollar, the given rate of exchange, *f*, being upon the same basis.

$$\$ = \frac{£ \times 40 \times f}{9} = \frac{£ \times 4.86656 \times f}{1.09498}. \quad \textit{Ans.}$$

EXAMPLE. — What is the equivalent in Federal money of £435 7*s.* 6*d.* sterling, rated by the former intrinsic par of 4\frac{4}{9}$ to the £, the rate of exchange upon the same basis being 1.10¼?

$$\frac{435.375 \times 40 \times 1.1025}{9} = \frac{435.375 \times 4.86656 \times 1.1025}{1.09498} =$$

$2133.3375. *Ans.*

PROP. 14. — Reduce F1172.36 (1172 francs, 36 centimes of France) to its equivalent in Federal money?

$$1172.36 \times 0.19452 = 1172.36 \div 5.14086 = \$228.05. \quad \textit{Ans.}$$

PROP. 15. — Express the intrinsic or par equivalent of francs, *F*, or francs and decimal parts of a franc, in Federal money, $; and *vice versa.*

$$\$ = F \times 0.19452 = F \div 5.14086. \quad \textit{Ans.}$$
$$F = \$ \div 0.19452 = \$ \times 5.14086. \quad \textit{Ans.}$$

PROP. 16. — Express the intrinsic equivalent of marks banco, *M*, or marks and decimal parts of a mark banco of Hamburg, London rate, in Federal money; and *vice versa.*

$$\$ = M \times 0.35393 = M \div 2.8254. \quad \textit{Ans.}$$
$$M = \$ \times 2.8254 = \$ \div 0.35393. \quad \textit{Ans.}$$

EXAMPLE. — What is the equivalent in Federal money of 6472 marks, 12 schillings banco of Hamburg, London rate?

6472.75 × 0.35393 = 6472.75 ÷ 2.8254 = $2290.90. *Ans.*

PROP. 17. — Which is the most advantageous purchase, other things being equal; viz., bills of exchange on London at 1.10½, rate of 4*s.* 6*d.* sterling to the dollar; or on Paris at F5.15 to the dollar; or on Hamburg at 35⅝ cents per mark banco?

1.105 ÷ 1.09498 = 1.009151–, London.
5.15 ÷ 5.14086 = 1.00178–, Paris.
35.625 ÷ 35.393 = 1.00655+, Hamburg.

Bills on Paris. *Ans.*

NOTE. — London allows, as the intrinsic par, £1 for F25.21 = 19.304 Federal cents per franc, instead of 19.452, the prevailing commercial par. But at the same time Great Britain values the silver franc of France, measured by the silver in her own silver coinage, at £1 = $\frac{1614.545454+}{69.445575}$ = 23.249+ francs = 20.93234+ Federal cents per franc.

Of Notes for discount, and their avails at bank.

PROP. 18. — When the time of the note is expressly written in days; or when the actual number of days in the specified time are employed; that is, when the *true* time is taken, and taken in days, $P = \frac{365a}{365 - tr}$, and $a = \frac{P(365 - tr)}{365}$, P being the principal or face of the note, a the avails or sum advanced by the bank, t the time including grace, and r the rate of the interest or discount per annum. See BANK DISCOUNT, p. 127.

EXAMPLE. — What must be the principal of a note payable in 90 days, in order that the *equitable* avails at bank, for the time of the note and 3 days grace, the rate being 6 per cent., shall be $1000?

$$\frac{365 \times 1000}{365 - 93 \times .06} = \$1015.53. \quad \textit{Ans.}$$

PROP. 19. — When the time of the note is expressly written in months, and a month is taken to be the $\frac{1}{12}$ part of a calendar year,

$P = \frac{12a}{12 - r(m + 0.1)}$, and $a = \frac{P[12 - r(m \times .1)]}{12}$; m being the time in months, and .1 being $\frac{3}{30}$ of a month, or the usual 3 days grace.

EXAMPLE. — $1000 are to be obtained from a bank, on a note having three months to maturity, and three days grace; the rate being 6 per cent., for what sum must the note be drawn?

$$\frac{12 \times 100}{12 - 3.1 \times .06} = 1015.74. \quad \textit{Ans.}$$

NOTE. — 3 calendar months and 3 days, mean time, are more than 93 days, by 1¼ day.

PROP. 20. — When the true time of the note is taken in months or days, and a year is assumed to consist of 12 months of 30 days each, or of 360 days only,

$$P = \frac{360a}{360 - tr}, \text{ and } a = \frac{P(360 - tr)}{360}.$$

EXAMPLE. — What must be the principal of a note for discount, payable in 90 days after acceptance, that the proceeds at bank shall be $1000, the rate being 6 per cent., the grace 3 days, and the bank assuming that a year consists of 360 days only?

$$\frac{360 \times 1000}{360 - 93 \times .06} = \$1015.74. \quad Ans.$$

NOTE. — The Government of the United States, and the Courts, in matters of interest and discount, reckon time at 365 days to the year; and in Great Britain, France, and all Europe, a year, for like purposes, contains 365 days.

PROP. 21. — A note on time, without interest, dated Jan. 2, 1868, is to be given in exchange for the following obligations; the time of the note is required.

Note,	due March 3, 1868,	for	$370.
Bond,	" April 16,	"	830.50
Note,	" June 11,	"	1120.
Acc't,	" June 4,	"	127.50

From Jan. 2 to March 3 is 61 days	× 370	=	22570
" " 2 to April 16 is 105 "	× 831	=	87255
" " 2 to June 11 is 161 "	× 1120	=	180320
" " 2 to " 4 is 154 "	× 127	=	19558
	2448		)309703(127 days

after Jan. 2 = May 8, 1868. *Ans.*

But this is simply equivalent to finding the common time of maturity of the obligations to be transferred, which must be the answer.

EXAMPLE. — Due March 3,	$370	× 0	=	0
" April 16,	831	× 44	=	36564
" June 11,	1120	× 100	=	112000
" " 4,	127	× 93	=	11811
	2448			)160375(66 days

after March 3, = May 8, 1868. See EQUATION OF PAYMENTS, page 132.

PROP. 22. — A note dated March 1, 1869, and bearing interest at 7 per cent. from date, is to be given in exchange for the following obligations; the principal of the note is required.

1869, Feb. 16, settlement note on interest, value of, this day,	$327.36
" March 27, acceptance of this date for	1000.00
" May 4, account, averaging due this date,	658.73

1869, Feb. 16,	due	$327		
" M'ch. 27,	"	1000 × 39	= 39000	
" May 4,	"	659 × 77	= 50743	
		$1986	)89743(	45.2 days later than

Feb. 16, = April 3, 1869, the day on which the given obligations collectively become due, or average due; which is 32 days *later* or *after* March 1, 1869: then, since the adjustment involves discount instead of interest,

$$P = \frac{365a}{365 + tr} = \frac{365 \times 1986}{365 + 32 \times .07} = \$1973.89-. \quad Ans.$$

In which P represents the principal, or face of the new note, a the sum of the obligations to be transferred or cancelled, t the time in days of the adjusting interest or discount, and r the rate of the interest or discount.

Prop. 23. — The foregoing proposition (Prop. 22), except that the new note is to be dated July 1, 1869, instead of March 1, 1869.

From April 3, '69, to July 1, '69, is 59 days, the number of days that April 3d is *earlier* or *previous* to July 1st: then, since the adjustment involves interest instead of discount,

$$P = \frac{a(365 + tr)}{365} = \frac{369.13 \times 1986}{365} = \$2008.47. \quad Ans.$$

Prop. 24. — Suppose we equate the time of the following demands by the common rule, and then make up the account correctly by interest and discount to the given dates, and thence to the equated time, by way of exhibiting general principles, and the bearing of the common rule of average upon those principles.

Interest and discount at 7 per cent.

1868, note due March 10 for $500			
" " " June 8 "	750 × 90	= 67500	
" " " Sept. 6 "	1050 × 180	= 189000	
	2300	)256500(	112 days

later than March 10, = June 30, '68, the day on which the given demands ($2300) are assumed to collectively mature.

$500 due March 10, worth March 10	$500.
750 " June 8, " " 10 (90 days' disc't.)	737.28
1050 " Sept. 6, " " 10 (180 " ")	1014.96
	$2252.24;

worth, June 30 (112 days' interest), $2300.62. (True time $= 365\,(P - w) \div wr = 112 - 365\,(W - P) \div Pr.$)

$500 due March 10, worth June 8 (90 days' interest)	$508.63
750 " June 8, " " 8	750.00
1050 " Sept. 6, " " 8 (90 days' discount)	1032.18
	$2290.81;

worth, June 30 (22 days' interest) $2300.48. (True time $= 365(P - w) \div wr + 90$.)

$500 due	March 10,	worth Sept.	6	(180 days' interest)	$517.26
750 "	June 8,	" "	6	(90 " ")	762.95
1050 "	Sept. 6,	" "	6		1050.00
					$2330.21;

worth, June 30 (68 days' discount), $2300.21. (True time $= 180 - 365(w - P) \div Pr$.)

$500 due	March 10,	worth June	30	(112 days' int.)	$510.71
750 "	June 8,	" "	30	(22 " ")	753.17
1050 "	Sept. 6,	" "	30	(68 " disc't.)	1036.48 =

$2300.36. (True time $= 112 - 365(w - P) \div Pr = 111.184$ days.

NOTE. — On the assumption that simple interest is justly due and payable yearly, it is apparent that no demand should enter the account for average at a present worth more than a year due previous to the maturity of the debt latest due.

PROP. 25. — A man sold two horses for $150 apiece, one at a *profit* of 25 per cent., and the other at a *loss* of 20 per cent.; which was the greater, the profit or the loss on the two sales, and what sum of money expresses the difference?

$150 \div (1 - .20) = \$187.50$, cost of the horse sold at a loss, and $187.50 - 150 = \$37.50$ loss; $150 \div (1 + .25) = 120$, cost of the horse sold at a profit, and $150 - 120 = \$30$ profit; then $37.50 - 30 = \$7.50$ loss greater than the profit. *Ans.*

PROP. 26. — A merchant sold two packages of goods for the same sum of money each; on one of them he cleared 25 per cent., and on the other he lost 25 per cent.: did he gain or lose in the aggregate; and, if either, what per cent.?

$1 \div (1 - .25) = 1\frac{1}{3}$, cost relative to the sum received as 1 of the package sold at a loss; $1 \div (1 + .25) = \frac{8}{10}$, cost relative to the sum received as 1 of the package sold at a profit; then $\frac{(1\frac{1}{3} - 1) \sim (1 - \frac{8}{10})}{1\frac{1}{3} + \frac{8}{10}} = \frac{4}{64} = \frac{1}{16} = 6\frac{1}{4}$ per cent. loss. *Ans.*

PROP. 27. — A and B purchased a farm in company for $8000; A paid $5000 in part payment, and B paid the remainder; they then sold $\frac{1}{3}$ of the farm for $4000; and, to close the copartnership, each took $\frac{1}{2}$ of the remaining $\frac{2}{3}$ to his own private interest: how much, if any, of the undivided cash received for the land they disposed of attaches to B's private interest?

$\frac{3}{8}(4000 - \frac{1}{3}$ of $8000) = \$500$, B's share of the profit on the sale, and $3000 + 500 - \frac{1}{3}$ of $8000 = \$833\frac{1}{3}$. *Ans.*

PROP. 28. — *The sum, S, of two numbers; N and n, given, the difference, d, of their respective factors, C and c, to produce like* *products given, and the sum of the products, P, given, to find the numbers.*

Let $D =$ the difference of the assumed numbers, and let $m = P \div S$; then

1st step, $\frac{\frac{1}{2}S(m+\frac{1}{2}d)}{m} = N$ | $\frac{\frac{1}{2}P}{n} = C$

$S - N = n$ | $\frac{1}{2}P \div N = c$

$N - n = D$ | $C - c = d'$, and

$d' : d :: D : D'$.

2d step, $\frac{S + D'}{2} = N'$ | $\frac{1}{2}P \div n' = C'$

$S - N' = n'$ | $\frac{1}{2}P \div N' = c'$, $C' - c' = d''$;

$d'' : d :: D' : D''$

3d step, $\frac{S + D''}{2} = N''$ | $\frac{1}{2}P \div n'' = C''$

$S - N'' = n''$ | $\frac{1}{2}P \div N'' = c''$, and $C'' - c'' = d'''$, &c.

NOTE.—By this manner of proceeding, all the elements in propositions of this nature may be approximated to any degree of exactness desired; and the true values will be between those obtained by the first step and those obtained by the second. The greater required number (N) will be less than that obtained by the first step, and greater than that obtained by the second. Often but two steps, and seldom more than three, will be required for ordinary practical purposes. When the difference between any trial C and its corresponding c becomes equal to the given difference (d), the elements in that step will be the exact ones sought, if no error has been committed in the work. It is not necessary, however, to obtain the first trial N by the method here proposed; for any number whatever may be assumed in its stead: but, when thus obtained, it has the advantage, generally, of being near the true number sought, and of being known to be greater than that number.

EXAMPLE.—A certain farm containing 80 acres is worth in the aggregate $62.50 per acre; but one section of it, to the extent of half the gross value, is worth $11 per acre more than the other; how many acres are there in the lesser section?

$P = 80 \times 62.50 = \$5000$, the gross value of the farm; then

$\frac{(62.5 + 5.5)\ 40}{62.5} = 43.52 = N$ | $2500 \div 36.48 = 68.5307+ \quad = C$

$80 - 43.52 \quad = 36.48 = n$ | $2500 \div 43.52 = 57.444853- = c$

$7.04 = D$ | $11.085846 \quad = d'$

$11.085846 : 11 :: 7.04 : 6.985484+ = D'$

$\frac{80 + 6.985484}{2} = 43.492742 = N'$

$80 - 43.492742 = 36.507258 = n'$

$2500 \div 36.507258 = 68.479534 = C'$

$2500 \div 43.492742 = 57.480855 = c$

$10.998679 = d'' =$

11 nearly; the lesser section, therefore, contains 36.507258 acres, nearly. *Ans.*

NOTE. — All propositions of this general class having given relations of parts contain the requisite elements for direct solutions, and need not be worked, as they commonly are, by rules denominated *Double Position*. For example: "The old sea-serpent's head is 10 feet long, his tail is as long as his head and half the length of his body, and his body is as long as his head and tail; what is the whole length of the monster?"

$h = 10$, $t = h + \frac{1}{2}b$, and $b = 2h + \frac{1}{2}b$; then

$b - \frac{1}{2}b = 2h$, therefore $b = 4h = 40$

$t = h + \frac{1}{2}b$, therefore $t = 10 + 20 = 30$

$h = 10$. 80 feet. *Ans.*

PROP. 29. — Divide $1000 into four such parts that the second shall contain $10 more than the first, the third $6 more than the second, and the fourth $2\frac{1}{2}$ times as many as the first and second.

Let x represent the smallest part; then

$1000 = x + x + 10 + x + 16 + 5x + 25$, and

$1000 - (10 + 16 + 25) = 949 = 8x$, and $949 \div 8 = x =$ $118.625, 1st; $128.625, 2d; $134.625, 3d; $618.125, 4th.

PROP. 30. — A gentleman being asked his own age and the age of his wife, replied, If you subtract 5 years from my age, and divide the remainder by 8, the quotient will be $\frac{1}{3}$ of my wife's age; and if you add 2 years to her age, then multiply the sum by 3, and subtract 7 from the product, the remainder will be my age. Required the age of each.

Let $x =$ the wife's age; then $3x + 6 - 7 =$ the husband's age; but $\frac{3x + 6 - 7 - 5}{8} = \frac{x}{3}$; therefore, $9x + 18 - 36 = 8x$, and $x = 36 - 18 = 18$, the wife's age, and $18 \times 3 + 6 - 7 = 53$, the husband's age. *Ans.*

PROP. 31. — *Two relations between two unknown numbers given, to find the numbers.*

BY DOUBLE POSITION. — *Special cases.*

Let $x =$ the lesser of the correct numbers sought.

" $N =$ the greater and n the lesser of the assumed values of x, or trial numbers.

" $D =$ the greater and d the lesser of the differences between the trial numbers and those obtained in their stead; then

When both differences are in excess, or show that both the assumed values of x are too high,

$$x = \frac{(N + n)d}{D - d}.$$

When both differences are in deficiency, or show that both the assumed values of x are too low,

$$x = \frac{(N + n)D}{D + d}.$$

When one of the differences is in excess and the other in deficiency, *or when one of the assumed values of x is too high and the other too low,*

$$x = \frac{ND + nd - D - d}{D + d}.$$

Example. — Assume the wife's age (last preceding proposition) is 25, then $(25 + 2) \times 3 - 7 = 74$, the husband's age, and $\frac{(74-5) \times 3}{8} = 25.875$, instead of 25, and showing a difference of .875 in excess. Again, assume the wife's age is 20, then $(20 + 2)3 - 7 = 59$, the husband's age, and $\frac{(59-5)3}{8} = 20.25$, instead of 20, and showing a difference of .25 in excess; then

$\frac{(25 + 20) \times .25}{.875 - .25} = 18$, the wife's age, and

$(18 + 2) \times 3 - 7 = 53$, the husband's age.

Proof. $\frac{(53-5)3}{8} = 18.$

Note. — It is not necessary to assume the value of the lesser required number instead of that of the greater, for the value of either may be assumed as preferred; and when the greater is assumed, the foregoing formulas are not applicable, but a set that are can be easily made.

Prop. 32. — A said to B, Give me one of your apples and I shall then have as many as you will have left. B replied, Give me one of yours, and I shall then have twice as many as you will have left. How many apples had each?

By the first proposition, $A + 1 = B - 1$; therefore $B = A + 2$: but by the second proposition, $B + 1 = 2(A - 1)$; therefore $B = 2A - 3$: then $2A = A + 2 + 3$, and $A = 2 + 3 = 5$; $B = A + 2 = 7$. *Ans.*

Prop. 33. — The lesser and half the greater of two casks of wine = 82 gallons; and the greater and $\frac{1}{3}$ the lesser = 129 gallons: how many gallons are in each cask?

Let x = the greater and y the lesser; then
$x + \frac{1}{3}y = 129$, and $y + \frac{1}{2}x = 82$: therefore
$2y = 164 - x$, and $\frac{1}{3}y = 129 - x$; consequently
$2y - \frac{1}{3}y = 164 - 129$, and $y = 21$; also
$129 - \frac{1}{3}y = x = 129 - 7 = 122$. *Ans.*

Prop. 34. — Smith's several cows are in number to their average cost per head as 45 to 138, and they collectively cost him $690; how many cows has he?

Let n represent the number of cows, and c their average cost; then
$n : c :: 45 : 138$; but $n \times c = 690$, therefore

$$\sqrt{\frac{690 \times 45}{138}} = n = 15. \quad \textit{Ans.}$$

Solved, also, by Prob. X, page 170.

PROP. 35. — An apple-vender bought one-half of a certain lot of apples at the rate of 2 for a cent, and the other half at the rate of 3 for a cent, and concluded that they collectively cost him 2 cents for 5; being willing to dispose of them at cost, he accordingly mixed them together, and sold them out 5 for 2 cents, and lost 5 cents by so doing: how many apples had he?

Let $x =$ the whole number of apples or answer; then

$$x = \left(\frac{\frac{1}{2}x}{2} + \frac{\frac{1}{2}x}{3} - 5\right)\frac{5}{2} = \left(\frac{x}{4} + \frac{x}{6} - 5\right)\frac{5}{2} = \frac{50x - 600}{48}, \text{ therefore}$$

$48\,x + 600 = 50x$, $2x = 600$, and $x = 300$. *Ans.*

NOTE. — The sum of $\frac{1}{2}$ of a quantity and of $\frac{1}{3}$ of a like quantity, is more than $\frac{2}{5}$ of the sum of the two quantities, by $\frac{1}{30}$ of one of the quantities.

PROP. 36. — If a body were to start suddenly into motion, and move 80 miles in the first hour, and in each succeeding hour were to move through three-fourths as much space as in the hour last preceding, and were thus to continue in motion *forever*, what space would it describe?

$$80 \div \tfrac{1}{4} = 320 \text{ miles. } \textit{Ans.}$$

NOTE. — This is simply a question in Geometrical Progression descending, in which the greater extreme is 80, the ratio $\frac{1}{4}$, the less extreme 0, and the sum of the terms is required. The formula, since the ratio is less than unity, becomes $S = \frac{(E-e)(1-r)}{r-e} + E = \frac{E \times 1 - e}{r - e}$. See GEOMETRICAL PROGRESSION, p. 151.

PROP. 37. — What sum in ready money, D, may I pay for \$10,000 in stocks, P, that are redeemable at par in T, 3 years, and are bearing interest the while at 7 per cent. a year, payable half-yearly, in order that I may realize 6 per cent. simple interest a year on the investment, supposing that the payments of the interest on the stock are to be kept invested at 7 per cent. a year, from their times of maturity till the stock matures?

$$A = P + pT\left(1 + \frac{R(T-1)}{2} + \tfrac{1}{4}R\right) = P\left[1 + rn\left(1 + \frac{r(n-1)}{2}\right)\right],$$

the amount of the stock at the time of its maturity; in which p represents an interest payment, r the rate of the interest per interval between the payments, and n the whole number of the interest payments; and $D = A \div (1 + R'T) = A \div (1 + an)$, in which R' represents the rate of the discount per annum, or rate of the interest on the investment per annum, and a the rate of the discount per interval between the payments; therefore

$$\frac{10000 + 700 \times 3 \times \left(1 + \frac{.07(3-1)}{2} + \frac{.07}{4}\right)}{1.18} = \frac{10000 \times \left[1 + .035 \times 6 \times \left(1 + \frac{.035(6-1)}{2}\right)\right]}{1.18} =$$

\$10,409.96. *Ans.*

PROP. 38. — The last preceding proposition, except that my ready money is worth 6 per cent. compound interest yearly?

$D = A \div (1 + R)^3 = 12283.75 \div 1.191016 = \$10{,}313.67$. *Ans.*

See ANNUITIES, p. 156; also, p. 125.

PROP. 39. — The preceding proposition (Prop. 37), except that the payments of the interest are to be invested at 6 per cent. a year, from the times they become due, till the stock matures?

$$\frac{10000 \times (1 + .03 \times 6(1 + \frac{.03 \times 5}{2})}{1.18} = \$10{,}114.41. \quad \textit{Ans.}$$

PROP. 40. — The preceding proposition (Prop. 37), except that the interest on the stock is payable quarterly?

$$\frac{10000 + 700 \times 3 \times (1 + \frac{.07 \times (3-1)}{2} + \frac{3}{8} \times .07)}{1.18} =$$

$$\frac{10000 \times (1 + .0175 \times 12 \times (1 + \frac{.0175 \times 11}{2})}{1.18} =$$

$10,427.22. *Ans.*

PROP. 41. — The first proposition in this class (Prop. 37), except that my ready money is worth 6 per cent. interest a year, with the interest payable semi-annually?

$$D = \frac{A}{1 + T(R' + \frac{1}{4}R')} = \frac{A}{1 + T(\frac{r(n-1)}{2})} = \frac{12283.75}{1.225} =$$

$10,027.55. *Ans.*

PROP. 42. — Express the *difference*, per dollar, between the *amount* of a given algebraic principal for a given algebraic time and rate, and the *present worth* of the same principal for the same time and rate, the time being in days.

$$\frac{365 + tr}{365} \sim \frac{365}{365 + tr} = \left(1 + \frac{tr}{365}\right) \sim \frac{1}{1 + \frac{tr}{365}}. \quad \textit{Ans.}$$

EXAMPLE. — What is the difference between the amount of $1,550 for 175 days at 8 per cent. a year, and the present worth of the same sum for the same time and rate?

$$d = \frac{P(365 + tr)}{365} - \frac{365P}{365 + tr} = P\left(\frac{365 + tr}{365} - \frac{365}{365 + tr}\right) =$$

$116,708. *Ans.*

PROP. 43. — A purchased a bill of goods on six months' credit, amounting to $2,000, with the understanding that he should be allowed 5 per cent. off for ready cash, in whole or part payment: he paid $1,000 ready cash; for what sum ought he to be credited on the bill?

$1000 \div (1 - .05) = \$1052.63$. *Ans.*

PROP. 44. — Which is the lower offer, goods at $1.28\frac{3}{4}$, on 4 months' credit; or the same goods at 1.30, on 6 months' credit; allowing money to be worth 8 per cent. interest a year?

$$\frac{1.28\frac{3}{4}}{1+\frac{4\times .08}{12}}=\frac{1.2875\times 12}{12.32}=1.254+\text{, the present worth.}$$

$$\frac{1.30}{1+\frac{6\times .08}{12}}=\frac{1.30\times 12}{12.48}=1.25\text{, the present worth.}$$

The 1.30 terms, slightly. *Ans.*

Conversion of debts not yet due into others of like sums each, and having a common difference of time from maturity to maturity.

$S =$ gross sum to be converted.
$T =$ time from the present to the maturity of the gross sum.
$n =$ number of common substitutes.
$s =$ common sum of the substitutes.
$t =$ common difference of time from maturity to maturity of the substitutes.
$t' =$ assigned time from the present to the maturity of one of the substitutes.

When the common difference of time is to apply to all the common substitutes, and is to be measured from the present,

$$s = S \div n, \text{ and } t = T \div [\tfrac{1}{2}(n+1)].$$

PROP. 45. — An investment of $2,100 having 90 days to maturity is to be substituted by 3 others of like sums each, which are to become due at the close of a common difference of time from the present, and from one to another: the common sum and common difference of the times are required.

$2100 \div 3 = \$700$, the common sum of the substitutes; and
$90 \div 2 = 45$ days, the common difference, or common interval.

The three substitutes of $700 each, therefore, are to be made payable, the first at the expiration of 45 days, the second at the expiration of 90 days, and the third at the expiration of 135 days from the present time.

Proof.

$700 \times 45 = 31500$
$700 \times 90 = 63000$
$700 \times 135 = 94500$
$21 \overline{)189000} = 90$; or $700 \times (45+90+135) = 2100 \times 90$.

PROP. 46. — Five notes are to be made for like sums each, and are to become due at the close of equal intervals of time from the present, and from one to another; and these notes are to be given in exchange for the four following obligations; viz., $1600, due in 90 days, $1250.62 due in 80 days, $852.21 due in 57 days, and $1865 due in 175 days, from the present time. The common denomination of the notes, and the common interval of time are required.

$$\begin{array}{rl} 1600.00 \times 90 = & 144000 \\ 1250.62 \times 80 = & 100050 \\ 852.21 \times 57 = & 48576 \\ 1865.00 \times 175 = & 326375 \\ \hline 5567.83 \quad) & 619001 \end{array}$$

$= 111$ days, the mean or average time of maturity from the present of the obligations to be converted; and

$5567.83 \div 5 = \$1113.57$, the common denomination of the notes; and

$111 \div \frac{1}{2}(5 + 1) = 37$ days, the common difference of time, or common interval.

The special times to maturity, therefore, of the five notes of $1113.57 each, are 37, 74, 111, 148, 185 days from the present time.

When one of the common substitutes is to be treated as cash, or is to be considered as due at the present time, and the common interval is to be measured from the present,

$$s = S \div n, \text{ and } t = T \div [\tfrac{1}{2}(n - 1)].$$

PROP. 47. — Several matters of indebtedness, amounting in the aggregate to $2175.44, and which will collectively mature, or become due by average, at the close of 68 days from the present time, are to be cancelled by the payment of one-fourth of their sum down, and by passing three notes, made for one-fourth of their sum each, and payable at the close of a common difference of time from the present and from one to another. The common sum and common difference are required.

$2175.44 \div 4 = \$543.86$, the common sum; and

$68 \div \frac{1}{2}(4 - 1) = 68 \div 1.5 = 46$ days, the common interval.

The three notes, therefore, are to be made for $543.86 each, and are to be made payable, the first at 46, the second at 92, and the third at 138 days from the present time.

When the common interval is to apply between all the common substitutes, and is to be measured from an assigned time for the maturity of one of them,

$$s = S \div n, \text{ and } t = (T \sim t') \div [\tfrac{1}{2}(n - 1)].$$

PROP. 48. — A debt of $4500, due 123 days hence, without interest, is to be substituted by 4 notes, made for one-fourth of the sum each, which are to run an equal interval of time from maturity to maturity, and one of them is to be made payable at the close of an interval of 30 days from the present time. The common sum and common interval are required.

$4500 \div 4 = \$1125$, the common denomination of the notes; and $(123 - 30) \div 1.5 = 62$ days, the common difference of time.

The times to maturity of the substitutes, therefore, are 30, 92, 154, 216 days later than the present time.

PROP. 49. — The last preceding proposition (Prop. 47), except that the given debt of $4500 has but 75 days to maturity; and one of the notes is not to become due until the lapse of 105 days from the present time.

$$(75 \sim 105) \div 1.5 = 20 \text{ days, the common interval.}$$

The times from the present to the maturities of the notes, therefore, are 45, 65, 85, 105 days.

PROP. 50. — It is proposed to relinquish obligations, amounting in the aggregate to $2554.72, and which will collectively become due by equation at the close of 40 days from the present time, and to receive in their place 3 notes, made for one-fourth of the sum each, and the balance in ready cash; the said notes to run equal intervals of time from maturity to maturity, measured from the present, and one of them to run 85 days; the adjusting interest, or discount, to be at 7 per cent.

$$(85 - 40) \div \tfrac{1}{2}(4 - 1) = 30 \text{ days, the common interval.}$$

The three notes of $638.68 each, therefore, are to be made payable at 85, 55, 25 days; and the present worth of the cash payment of $638.68, due $30 - 25 = 5$ days hence, is $638.07.

To invest a given sum of money in parts, at unlike rates of interest, and the parts to gain like interest in equal intervals of time.

p, p', p'', &c. = the parts, or partial investments.

S = the sum of the investments.

r, r', r'', &c. = the given rates, or these in their relations to each other, expressed in any proportion preferred.

m = the product of r, r', r'', &c., as expressed.

$$N = \text{the sum of } \frac{m}{r} + \frac{m}{r'} + \frac{m}{r''}, \text{ \&c.}$$

$$p, p', p'', \text{\&c.} = \frac{Sm}{Nr} = \frac{Sm}{Nr'} = \frac{Sm}{Nr''}, \text{ \&c., inversely.}$$

PROP. 51. — Twenty thousand dollars ($20,000) are to be placed at interest in three such parts that the interest on them, at their respective rates of 6, 7, and 8 per cent. a year, shall be equal for all like intervals of time. The parts, or special investments, are required.

$m = 6 \times 7 \times 8 = 336$; and $336 \div 6 = 56$

$336 \div 7 = 48$

$336 \div 8 = 42$

146, the value of N; $\therefore$

$$p = \frac{20000 \times 56}{146} = \frac{20000 \times 28}{73} = \$7,671.23$$

$$p' = \frac{20000 \times 24}{73} \quad . \quad . \quad . \quad . \quad = \quad 6,575.34$$

$$p'' = \frac{20000 \times 21}{73} \quad . \quad . \quad . \quad . \quad = \quad 5,753.43$$

$$\$20,000.00$$

To invest a given sum of money in parts, at like rates of interest, and for unequal intervals of time; and the parts to gain like interest at the close of their respective times.

t, t', t'', &c. = the given times, or these in their relations to each other, expressed in whatever proportion preferred.

m = product of t, t', t'', &c., as expressed.

N = sum of $\frac{m}{t} + \frac{m}{t'} + \frac{m}{t''}$, &c.

S, and p, p', p'', &c., as in the last preceding proposition.

p, p', p'', &c., $= \frac{Sm}{Nt} = \frac{Sm}{Nt'} = \frac{Sm}{Nt''}$, &c., inversely.

PROP. 52.—It is proposed to place \$25,000 at interest in four separate sums, one of them for 60, one for 80, one for 110, and one for 150 days' time; and that these sums shall be such, that, at like rates of interest, they will gain like interest at the close of their respective times. The special sums are required.

$m = 60 \times 80 \times 110 \times 150$, or $6 \times 8 \times 11 \times 15$, or $3 \times 4 \times 5.5 \times 7.5 = 495$; and

$$495 \div 3 = 165.$$
$$495 \div 4 = 123.75$$
$$495 \div 5.5 = 90.$$
$$495 \div 7.5 = 66.$$

$444.75 = N$; therefore

$$p = \frac{25,000 \times 165}{444.75} = \frac{25,000 \times 33}{88.95} = \$9,274.87$$

$$p' = \frac{25,000 \times 12375}{44475} = \frac{25,000 \times 495}{1779} = \quad 6,956.16$$

$$p'' = \frac{25,000 \times 90}{444.75} \quad . \quad . \quad . \quad . \quad . \quad . \quad = \quad 5,059.02$$

$$p''' = \frac{25,000 \times 66}{444.75} \quad . \quad . \quad . \quad . \quad . \quad . \quad = \quad 3,709.95$$

$$\$25,000.00$$

To invest a given sum of money in parts, at unlike rates of interest, and for unequal intervals of time; and the parts to gain like interest at the close of their respective times.

$m =$ product of the given times, or of their relations to each other, by any measure whatever, that is common to them; or of the given rates, if preferred.

S, N, p, p', p'', &c., as in the preceding.

$$p, p', p'', \text{\&c.} = \frac{Sm}{Ntr} = \frac{Sm}{Nt'r'} = \frac{Sm}{Nt''r''}, \text{\&c.}$$

PROP. 53. — Ten thousand dollars ($10,000) are to be placed at interest in three separate sums, one of them at 4 per cent., for 240 days; one at 6 per cent., for 120 days; and one at 8 per cent., for 80 days; and these sums are to be such that they will gain like interest, one with another, at the close of their respective times. The special sums are required.

$240 \times 120 \times 80$, or $24 \times 12 \times 8$, or $6 \times 3 \times 2 = 36 = t \times t' \times t'' = m$.
4, 6, 8, or 2, 3, 4, $= r, r', r''$
12, 9, 8, the products of tr, $t'r'$, $t''r''$, and

$$36 \div 12 = 3$$
$$36 \div 9 = 4$$
$$36 \div 8 = 4.5$$
$$11.5 = N, \text{ therefore}$$

$$p'' = \frac{10{,}000 \times 30}{115} = \frac{10{,}000 \times 6}{23} = \$2{,}608.70$$

$$p' = \frac{10{,}000 \times 40}{115} = \frac{10{,}000 \times 8}{23} = 3{,}478.26$$

$$p = \frac{10{,}000 \times 45}{115} = \frac{10{,}000 \times 9}{23} = 3{,}913.04. \quad \$10{,}000.00.$$

To invest a given sum of money in parts, at like rates of interest, and for unequal intervals of time; and the amount (principal and interest) of the parts to be equal at the close of their respective times.

PROP. 54. — It is proposed to place $16,000 at interest in four separate sums, each at 7 per cent. a year: one of them for 80, one for 100, one for 150, and one for 200 days' time; and that these sums shall be such that their amount shall be equal, one with another, at the close of their respective times. The special sums are required.

$$N = \text{sum of } \frac{365}{365 + rt} + \frac{365}{365 + rt'} + \frac{365}{365 + rt''}, \text{\&c.}, = \frac{1}{1 + \frac{rt}{365}}, \text{\&c.}$$

$$p, p', p'', \text{\&c.} = \frac{365S}{N(365+rt)} = \frac{365S}{N(365+rt')} = \frac{365S}{N(365+rt'')},$$

&c., $= \dfrac{S}{1+\frac{rt}{365}}$, &c.; then

$365 \div 370.6 = 0.98489$
$365 \div 372. \ = 0.98118$
$365 \div 375.5 = 0.97204$
$365 \div 379. \ = 0.96306$

3.90117, the value of N; and

$$p = \frac{5,840,000}{370.6N} = \$4039.36$$
$$p' = \frac{5,840,000}{372N} = 4024.16$$
$$p'' = \frac{5,840,000}{375.5N} = 3986.65$$
$$p''' = \frac{5,840,000}{379N} = 3949.83. \quad \$16,000.00$$

Therefore, $A = S \div N = \$4101.33$, the common amount.

If the times, t, t', t'', &c., be taken in years instead of days, then $N = \dfrac{1}{1+rt} + \dfrac{1}{1+rt'} + \dfrac{1}{1+rt''}$, &c.; and $p = S \div N(1+rt)$; $p' = S \div N(1+rt')$, &c. And, if the times be taken in months, $N = \dfrac{12}{12+rt} + \dfrac{12}{12+rt'}$, &c.; and $p = 12S \div N(12+rt)$; $p' = 12S \div N(12+rt')$, &c.

To invest a given sum of money in parts, at unlike rates of interest, and for unequal intervals of time; and the amount of the parts to be equal, one with another, at the close of their respective times.

Prop. 55. — The last preceding proposition, except that the rates are to be 6 per cent. for the 80 days' term, 7 per cent. for the 100 days' term, 8 per cent. for the 150 days' term, and 9 per cent. for the 200 days' term, instead of 7 per cent. for each of the terms.

$365 \div 369.8 = 0.98702$
$365 \div 372. \ = 0.98118$
$365 \div 377. \ = 0.96817$
$365 \div 383. \ = 0.95300$

3.88937, the value of N; and

$p = 5,840,000 \div 369.8N = \$4060.38$
$p' = 5,840,000 \div 372N = 4036.37$
$p'' = 5,840,000 \div 377N = 3982.83$
$p''' = 5,840,000 \div 383N = 3920.44. \quad \$16,000.02.$

PROP. 56. — Ganson & Co. are paying $12,000 a year in monthly payments in advance for the use of the premises they occupy; and they wish to know how much less the rent would be per annum, were it fixed at the same price by the year, but payable quarterly, at the expiration of each quarter-year; allowing money to be worth 7 per cent. interest a year.

$$P + \frac{Pr(n+1)}{2n} = 12p + \frac{pr(n+1)}{2} = \$12{,}455.00,$$ the amount,

by the present manner of paying, at the close of the year; and

$$P + \frac{Prn}{2(n+1)} = 12p + \frac{pr(12-n)}{2} = \$12{,}315.00,$$ the amount,

by the proposed manner of paying, at the close of the year, in both of which cases n represents the number of payments to be made per annum; then

$$12455 - 12315 = \$140.00.$$ *Ans.*

SECTION I.

MONEYS, WEIGHTS AND MEASURES,

OF THE UNITED STATES; — THEIR DENOMINATIONS, VALUES, COMPARATIVE VALUES, MAGNITUDES, &c.

MONEYS OF ACCOUNT OF THE UNITED STATES.

These are the *mill*, the *cent*, the *dime*, and the *dollar*.

10 mills = 1 cent, 10 cents = 1 dime, 10 dimes = 1 dollar.

The *dollar* is the *unit* or ultimate money of account of the United States, or of what is sometimes called *Federal money*.

In practice, the dime, as a denomination of value, is rejected. Thus,

10 mills = 1 cent, and 100 cents = 1 dollar.

This mark, $, is equivalent to the word *dollar*, or *dollars*, in this money.

COINS OF THE UNITED STATES.

Until June, 1834, the government of the United States estimated gold in comparison with silver as 15 to 1, and in comparison with copper as 850 to 1.

From June, 1834, until February, 1853, the same government estimated gold in comparison with silver as 16 to 1, and in comparison with copper as 720 to 1.

For all time since February, 1853, this government has estimated gold in comparison with silver as $14\frac{1026}{1161}$ to 1, and in comparison with copper as 720 to 1.

The standard for mint gold with this government until 1834, was 11 parts pure gold and 1 part alloy, the alloy to consist of silver and copper mixed, not exceeding one half copper.

The gold coins, therefore, struck at the United States mint prior to 1834, are 22 carats fine.

In what, until 1834, constituted a dollar of gold coin of United States mintage, there were put 24.75 grains of pure gold; and 27 grains of the standard mint gold of that day were at that time worth \$1. Twenty-seven grains of that gold, or gold of that standard, are now, by the present government standard of valuation, worth \$1.0652.

The standard for mint silver with this government until 1834, was 1485 parts pure silver and 179 parts pure copper, $= 8\frac{59}{179}$ parts pure silver and 1 part pure copper.

The silver coins, therefore, struck at the United States mint prior to 1834, are $10\frac{295}{416}$ ounces fine.

In that which, until 1834, constituted a dollar of silver coin of this government's mintage, there were put $371\frac{1}{4}$ grains of pure silver; and 416 grains of the standard mint silver of that day were at that time of the value of \$1. Four hundred and sixteen grains of that silver, or silver of that standard, are now, by the present government standard of valuation, worth \$1.0744.

The *cent*, until 1834, was of pure copper, and weighed 208 grains; since 1834, pure copper, weight 168 grains.

The standard for mint gold with this government is now, and for all time since June, 1834, has been, 9 parts pure gold and one part alloy, the alloy to consist of silver and copper mixed, not exceeding one half silver.

The gold coins, therefore, struck at the United States mint and dated subsequent to 1834, are $21\frac{3}{5}$ carats fine.

The standard weight for these coins is $25\frac{4}{5}$ grains to the *dollar;* and in every $25\frac{4}{5}$ grains of these coins there are $23\frac{22}{100}$ grains of pure gold.

The standard for mint silver with this government is now, and for all time since June, 1834, has been, 9 parts pure silver and 1 part pure copper.

The silver coins, therefore, struck at the United States mint and dated subsequent to 1834, are $10\frac{4}{5}$ ounces fine.

In what, from June, 1834, until February, 1853, constituted a dollar of silver coin of this government's mintage, there were put $371\frac{1}{4}$ grains of pure silver; and $412\frac{1}{2}$ grains of the standard mint silver of that day (the present standard) were worth, from June, 1834, until February, 1853, \$1. Four hundred twelve and one half grains of this standard of silver are now worth, by the present standard of valuation, \$1.0742.

The standard weight for silver coins with this government at present is 384 grains to the *dollar.*

The foregoing is not applicable to the *silver three-cent pieces*, so called, authorized by the Congress of 1850–51. These pieces are composed of 3 parts silver and 1 part copper; and their standard weight is $12\frac{3}{8}$ grains each. They are worth, at full weight, measured by the present standard of 345.6 grains of fine silver to the dollar, 2.685 cents each.

The *nickel* and *bronze* tokens or coins, authorized by Congress at different times between the years 1856 and 1867, are as follows: —

Nickel 1-*cent* token: 88 parts copper and 12 parts nickel; weight, 72 grains. Nickel 3-*cent* token: copper and nickel mixed, not exceeding one-fourth nickel; weight, 32 grains. Nickel 5-*cent* token: copper and nickel mixed, not exceeding one-fourth nickel; weight, 77.16 grains, or 5 grammes; diameter, 2 centimetres.

Bronze 1-*cent* token: 95 parts copper, and 5 parts tin and zinc; weight, 48 grains. Bronze 2-*cent* token: 95 parts copper, and 5 parts tin and zinc; weight 96 grains.

NOTE. — In the preceding calculations of values, and in the following, as is now the common custom everywhere, no value has been assigned to the alloy, either in the silver coins or the gold coins. In general, it is simply copper, and will not net more, it is assumed, than the cost of recovering. In the United States, the law relating to coinage, previous to 1834, required that the alloy for gold coins should consist of silver and copper mixed, not exceeding one-half copper; and the present law provides that it shall consist of silver and copper mixed, not exceeding one-half silver.

BOSTON, July, 1869.

GOLD, — PURE.

24 carats fine	= pure gold.
1 grain	= $0.043066.
23.22 grains	= $1.00.
1 dwt.	= $1.033592.
1 ounce	= $20.671834.

MINT GOLD, — U. S.

Alloy, practically all copper.

Nine parts of pure gold and one part of alloy, or

$21\frac{3}{5}$ carats fine	= standard coin.
1 grain	= $0.03876.
$25\frac{4}{5}$ grains	= $1.00.
1 dwt.	= $0.93023.
1 ounce	= $18.60465.

GOLD COINS, — U. S.

Denominations.	Weight in grains.	Standard value.
Double Eagle, - - - - - -	516	$20.00
Eagle, - - - - - - -	258	10.00
Half-Eagle, - - - - - - -	129	5.00
Quarter-Eagle, - - - - - -	$64\frac{1}{2}$	2.50
Triple Gold Dollar, - - - - -	$77\frac{2}{5}$	3.00
Gold Dollar, - - - - - -	$25\frac{4}{5}$	1.00
Eagle, prior to 1834 (com. value = $10.62),	270	10.909
Half do. " " (com. value = $5.31),	135	5.454

Private and Uncurrent.	Weight in Grains.	Sales.
A. Bechtler, N. C., $5 piece, - -		$4.75
" " $2\frac{1}{2}$ " - - -		2.37
" " 1 " - - -		.93
T. Reed, Georgia, 5 " - - -		4.75
" " $2\frac{1}{2}$ " - - -		2.37
" " 1 " - - -		.93
Moffat, California, 5 " - - -	129	5.00

SILVER,—PURE.

12 ounces fine = Pure Silver.
1 dwt. = $0.06944.
345.6 grains = $1.
1 ounce = $1.38889.

MINT SILVER.—U. S.

Alloy, all copper.

Nine parts pure silver and one part alloy; or
10 oz. 16 dwts. fine = Standard Coin.
1 dwt. = $0.0625.
384 grains = $1.00.
1 ounce = $1.25.

SILVER COINS.—U. S.

	Weight in Grains.	Standard Value.
Dollar, - - - - - - - -	384	$1.00
Half Dollar, - - - - -	192	.50
Quarter Dollar, - - - - - -	96	.25
Dime, - - - - - -	$38\frac{2}{5}$	.10
Half Dime, - - - - - -	$19\frac{1}{5}$	.05
Three-Cent Piece, $\frac{3}{4}$ silver and $\frac{1}{4}$ copper,	$12\frac{3}{8}$	.03

The copper coins of the United States are the CENT and HALF CENT; they are of pure copper. The weight of the former is 168 grains, and that of the latter, 84 grains.

NOTE.—The silver coins of the United States, issued since February, 1853, are not legal tender in the United States in sums exceeding *five dollars.*

TABLE,

Exhibiting the standard weight and present par value of the silver coins of the United States, of dates subsequent to 1834, *and prior to* 1853.

	Weight in Grains.	Present par value.
Dollar, - - - - -	$412\frac{1}{2}$	$1.0742
Half Dollar, - . - -	$206\frac{1}{4}$	.5371
Quarter Dollar, - - - -	$103\frac{1}{8}$	.2685
Dime, - - - - -	$41\frac{1}{4}$	.1074
Half Dime, - - - -	$20\frac{5}{8}$	.0537
Three-Cent Piece, - - -	$12\frac{3}{8}$	.03

CURRENCIES OF THE DIFFERENT STATES OF THE UNION.

4 Farthings = 1 Penny, 12 Pence = 1 Shilling, 20 Shillings = 1 Pound.

In Massachusetts, Connecticut, Rhode Island, New Hampshire, Vermont, Maine, Kentucky, Indiana, Illinois, Missouri, Virginia, Tennessee, Mississippi, Texas and Florida, 6 shillings = 1 dollar; $1 = $\frac{3}{10}$ £.

In New York, Ohio and Michigan, 8 shillings = 1 dollar; $1 = $\frac{2}{5}$ £.

In New Jersey, Pennsylvania, Delaware and Maryland, 7 shillings and 6 pence = 1 dollar; 1 dollar = $\frac{3}{8}$ £.

In North Carolina, 10 shillings = 1 dollar; $1 = $\frac{1}{2}$ £.

In South Carolina and Georgia, 4 shillings and 8 pence = 1 dollar; $1 = $\frac{7}{30}$ £.

NOTE.— These *currencies*, so called, are nominal at present in a great measure. The denominations serve in the different States as verbal expressions of value. But they are neither the names of the moneys of account in any of the States, nor are they the national names of any of the real moneys in circulation. All values in money in the United States are legally expressed in *dollars*, *cents*, and *mills*.

THE METRICAL SYSTEM OF WEIGHTS AND MEASURES.

In this system, the METRE is the basis, and is one forty-millionth of the polar circumference of the earth.

The METRE is the principal unit measure of length; the ARE of *surface;* the STERE of *solidity;* the LITRE of *capacity;* and the GRAM of *weight.*

The gram is the weight, in a vacuum, of one cubic centimetre of pure water at its maximum density.

The Metre, almost exactly .	=	39.37	U. S.	inches.
The Are (100 square metres)	=	3.95367	"	square rods.
The Stere (a cubic metre) .	=	35.31445	"	cubic feet.
The Litre (a cubic decimetre)	= {	61.023	"	" inches.
		1.05668	"	wine quarts.
The Gram	=	15.43235	"	grains.

The divisions by 10, 100, 1,000, of each of these units, are expressed by the same prefixes, viz., *deci*, *centi*, *milli;* and the multiples by 10, 100, 1,000, 10,000, of each, by *deca*, *hecto*, *kilo*, *myria*. The former series were derived from the Latin language, the latter from the Greek.

To illustrate with the metre: —

10 *milli*metres = 1 *centi*metre, 10 centimetres = 1 *deci*metre, 10 decimetres = 1 METRE, 10 METRES = 1 *deca*metre, 10 decametres = 1 *hecto*metre, 10 hectometres = 1 *kilo*metre, 10 kilometres = 1 *myria*metre.

In commerce, the ordinary weight is the kilogram, and 100 kilograms (usually called kilos) = 1 quintal; 10 quintals = 1 millier, or tonneau. The kilogram = 15,432.35 ÷ 7000 = 2.20462 avoirdupois pounds.

In practice, the terms *milliare*, *deciare*, *decare*, *kiliare*, and *myriare* are usually dropped, and

100 centare = 1 are; 100 ares = 1 hectare.

Also the terms *millistere*, *hectostere*, *kilistere*, and *myriastere*, are usually rejected, and 100 centisteres = 1 decistere; 10 decisteres = 1 stere; 10 steres = 1 decastere = 353.1445 cubic feet.

1 centiare (square metre)	=	1.19598526 square yards.
1 kilometre . . .	=	0.62137 statute miles.
1 hectare . . .	=	2.471 = U. S. acres.
1 kilolitre . . .	=	1 stere = 61,023.377953 cubic in.

A hectolitre = 26.41748 wine gallons = 2.83774 Winchester bush.

NOTE.—The system is the one recommended by the Statistical Congress of 1865 as a general system of weights and measures to be adopted by all nations.

FOREIGN GOLD COINS.

NOTE.— The coins of any country, both gold and silver, circulating as foreign in any other, particularly those of the smaller denominations, are usually held at an estimate below their *standard* par value, compared with the money standard of the country in which they circulate as foreign. Many of them, more particularly the silver, having circulation in the United States, are much worn and otherwise depreciated. In some instances, owing to frequent changes made both with regard to weight and purity, certain of them, having the same name and general appearance, bear a premium at home; others, a discount. Others, again, can hardly be said to have a definable value anywhere. The par value of the old pistole of Geneva, for instance, weighing 103¼ grains, is $3.985, while that of the new, weighing 87¾ grains, would, at the same degree of purity, be worth but $3.386; whereas, owing to its higher standard of fineness, its par value is $3.443. The ducat of Austria, coined in 1831, weighs 53½ grains, — its purity is 23.64, and its par value $2.269; while the half sovereign, closely resembling the ducat, coined in 1835, and weighing 87 grains, has a purity only of 21.64, and a par value, consequently, of but $3.378. The *circulating* value of the ducat in the United States, in general, is $2.20, and that of the half sovereign of Austria, $3.25.

ARGENTINE REPUBLIC.	Standard of purity in carats.	Standard weight in grains.	Par value in Federal money.	Circulating value in Federal money.	Par value per grain. cts.
Doubloon to 1832,	19.56	418	$14.671	$	3.50
" to "	20.83	415	15.512		3.73
AUSTRIA.					
Sovereign, half in proportion, to 1785,	22.00	170	6.711	6.50	3.94
Sovereign, half in proportion, since 1785,	21.64	174	6.756	6.50	3.88
Ducat, double in proportion,	23.64	53½	2.269	2.20	4.24
BELGIUM.					
Sovereign, half in pro.,	22.00	170	6.711		3.94

	Standard of purity in carats.	Standard weight in grains.	Par value in Federal money.	Circulating value in Federal money.	Par value per grain. cts.
Twenty Franc, more in pro.	21.50	99½	$3.840	$3.83	3.85
Ducat,				2.20	
BOLIVIA, COLOMBIA, CHILI, ECUADOR, PERU, NEW GRENADA, and MEXICO.					
For the modern coins, &c., of these States, see *Foreign Moneys of Account*, SEC. A.					
Doubloon, (8 E)	20.86	417	15.620	15.60	3.74
Half do.	"	208½	7.810	7.50	"
Quarter do.	"	104¼	3.905	3.75	"
Eighth do.	"	52	1.952	1.75	"
Sixteenth do.	"	26	.976	.90	"
Pistole, half in pro.,				3.75	
BRAZIL.					
For the modern coinage of this Empire, see *Foreign Moneys of Account and Coins*, SEC. A.					
Dobraon,	22.00	828	32.719	32.00	3.95
Dobra,	"	438	17.306	17.00	"
Joannes, (*standard variable*)	"	432	17.064	$13 to $17	"
Half do. do. do.	"	216	8.532	$6 to 8.50	"
Moidore, (BBBB) half in pro., (*standard variable*)	21.79	165	6.451	6.00	3.90
Crusado, do. do.	"	16¼	.635		"
DENMARK.					
Christian d'or	21.74	103	4.018		3.90
Ducat, species,	23.48	53½	2.254	2.20	4.21
" current,	21.03	48	1.811		3.77
FRANCE.					
There are but few gold coins of France now in circulation other than multiples of the standard franc, napoleons, fractional and double, in pro.					
Chr. d'or, double in pro.,	21.60	101	3.914	3.90	3.87

	Standard of purity in carats.	Standard weight in grains.	Par value in Federal money.	Circulating value in Federal money.	Par value per grain. cts.
Franc d'or, double in pro.,	21.60	101	$3.914	$3.90	3.87
Louis d'or, " " " to 1786,	21.49	125½	4.840		3.85
Louis d'or, double in pro., since 1786,	21.68	118	4.573	4.50	3.87
Napoleon (20 F.) double &c.	21.60	99½	3.856	3.83	"
GERMANY.					
BADEN.					
Zehn Gulden, 5 in pro.,	21.60	105½	4.088	4.00	3.87
BAVARIA.					
Carolin,	18.49	149¼	4.952		3.32
Ducat, double in pro.,	23.58	53¾	2.275	2.20	4.23
Maximilian,	18.49	100	3.317		3.31
BRUNSWICK.					
Ducat,	23.22	53½	2.220		4.16
Pistole, double in pro.,	21.60	117¼	4.548		3.87
Ten Thaler, 5 in pro., to 1813,	21.55	202	7.811	7.80	3.86
Ten Thaler, less in pro., since 1813,	21.50	204	7.873	7.80	3.85
HANOVER.					
Ducat, double in pro.,	23.83	53½	2.287	2.20	4.27
George d'or, " " "	21.67	102½	3.987		3.88
Zehn Thaler, 5 " "	21.36	204½	7.838	7.80	3.83
HESSE.					
Ten Thaler, 5 in pro., to 1785,	21.36	202	7.742		"
Ten Thaler, 5 in pro., since 1785,	21.41	203	7.799		3.84
SAXONY.					
Ducat,	23.49	53½	2.256	2.20	4.21
Augustus d'or, double in pro., since 1784.	21.55	102½	3.964		3.86
WURTEMBURG.					
Carolin,	18.51	147½	4.899		3.32
Ducat,	23.28	53½	2.235		4.17

	Standard of purity in carats.	Standard weight in grains.	Par value in Federal money.	Circulating value in Federal money.	Par value per grain. *cts.*
GREAT BRITAIN.					
(*Alloy, since* 1826, *all copper.*)					
The modern gold coins of this Kingdom are the *sovereign*, fractional, double, &c.					
Guinea, half in pro., to 1785,	22.00	127	$5.016		3.95
Guinea, half in pro., since 1785,	"	129½	5.111	$5.00	"
Sovereign, half in pro.,	"	123¼	4.866	4.83	"
Five do.	"	616¼	24.332	24.20	"
Sovereign, (*dragon*) half in pro.,	"	122½	4.838	4.80	"
Double Sovereign (*dragon*)	"	246	9.717	9.67	"
GREECE.					
Twenty Drachm, more in pro.,	21.60	89	3.441	3.40	3.87
HOLLAND.					
Ducat,	23.58	53½	2.263	2.20	4.23
Ryder,	22.00	153	6.043		3.95
Double do.	"	309	12.205		"
Ten Gulden, 5 in pro.,	21.60	103	3.988	3.98	3.87
INDIA.					
Pagoda, star,	19.00	52¾	1.798		3.40
Mohur, (E. I. Co.) 1835.	22.00	180	7.106	6.75	3.95
Half Sovereign, do.				2.41	
BOMBAY.					
Rupee,	22.09	179	7.095		3.96
MADRAS.					
Rupee,	22.00	180	7.106		3.95
ITALY.					
ETURIA, Ruspone,	23.97	161¼	6.935		4.30
GENOA, Sequin,	23.86	53½	2.291		4.28
MILAN, Pistole,	21.76	97½	3.807		3.90
" Sequin,	23.76	53½	2.281		4.26

	Standard of purity in carats.	Standard weight in grains.	Par value in Federal money.	Circulating value in Federal money.	Par value per grain. *cts.*
MILAN, Twenty Lire, more in proportion,	21.58	99½	$3.853	$3.83	3.86
NAPLES, Ducat, multiples in pro.,	21.43	22½	.865		3.84
NAPLES, Oncetta,	23.88	58	2.485		4.28
PARMA, Doppia, to 1786,	21.24	110	4.192		3.81
" Pistole, since 1796,	20.95	110	4.135		3.75
" Twenty Lire,	21.60	99½	3.859	3.83	3.87
PIEDMONT, Carlino, half in pro., since 1785,	21.69	702	27.321		3.89
PIEDMONT, Pistole, half in pro., since 1785,	21.54	140	5.411		3.86
PIEDMONT, Sequin, half in pro., since 1785,	23.64	53¾	2.280		4.23
PIEDMONT, Twenty Lire, more in pro.,	20.00	99¼	3.563	3.50	3.59
ROME, Ten Scudi, 5 in pro.	21.60	267½	10.368		3.87
" Sequin, since 1760,	23.90	52½	2.251		4.28
SARDINIA, Carlino, ½ in pro.,	21.31	247½	9.465		3.82
TUSCANY, Zechino, double in pro.,	23.86	53¾	2.302		4.30
VENICE, Zechino, double in pro.,	23.84	54	2.310		
MALTA.					
Sequin,	23.70	53½	2.275		4.25
Louis d'or, double and demi in pro.,	20.25	128	4.651		3.63
NETHERLANDS.					
Ducat,	23.52	53½	2.257		4.21
Zehn Gulden, 5 in pro.,	21.55	103¾	4.013	4.00	3.86
POLAND.					
Ducat,	23.58	53½	2.264		4.23
PORTUGAL.					
The modern Portuguese gold coins are the coroa of 5000 reis, parts and multiples in proportion. See SEC. A.					

	Standard of purity in carats.	Standard weight in grains.	Par value in Federal money.	Circulating value in Federal money.	Par value per grain. *cts.*
Dobraon, 24,000 reis,	22.00	828	$32.706	$32.00	3.95
Dobra,	"	438	17.301	17.00	"
Joannes, (*standard variable*)	"	432	17.064	$13 to $17	"
Half " " "	"	216	8.532	$6 to 8.50	"
Moidore, 4000 reis, "				$4¾ to $4⅞	
Coroa, 5000 "	"	147½	5.83	5.75	"
Milrea,	22.00	19¾	.780		3.95
PRUSSIA.					
Ducat,	23.49	53½	2.255	2.20	4.21
Frederick d'or, double in pro.,	21.60	102½	3.973		3.87
RUSSIA.					
Ducat,	23.64	54	2.291		4.24
Imperial, (10 R.) half in pro., 1801,	23.55	185¼	7.828		4.22
Imperial, (10 R.) half in pro., since 1818,	22.00	201½	7.949	7.90	3.95
SICILY.					
Oncia, double in pro.,	20.39	68½	2.495		3.64
Twenty Lire, more in pro.,	21.60	99½	3.856	3.83	3.87
SPAIN.					
For the *new* standard of coinage, denominations, &c., of this Kingdom, see *Foreign Moneys of Account and Coins*, SEC. A.					
Doubloon (8 S) parts in pro.	21.45	416½	16.031	16.00	3.84
" (8 E) parts as Bolivian, &c.	20.86	417	15.620	15.60	3.74
Pistole, to 1782,	21.48	103	3.970		3.85
" since "	20.93	104	3.906		3.75
Escudo, to 1788,	20.98	52	1.957		3.76
" since "	20.42	52	1.905		3.66
Coronilla " 1800,	20.29	27	.983		3.64
SWEDEN.					
Ducat	23.45	53	2.230		4.20

	Standard of purity in carats.	Standard weight in grains.	Par value in Federal money.	Circulating value in Federal money.	Par value per grain. cts.
SWITZERLAND.					
BERNE, Ducat, double in pro.,	23.53	47	$1.984		4.22
BERNE, Pistole,	21.62	117½	4.558		3.88
GENEVA, Pistole,	21.87	87¾	3.443		3.92
" " (*old*)	21.51	103¼	3.985		3.85
ZURICH, Ducat, double in pro.,	23.50	53½	2.256		4.21
TURKEY.					
Misseir, half in pro. 1820,	15.88	36½	1.040		2.84
Sequin fonducli,	19.25	53	1.830		3.45
Yeermeeblekblek,	22.88	73¼	3.027		4.10

NOTE.—For full and particular specifications regarding modern foreign gold and silver coins, see *Foreign Moneys of Account and coins*, SEC. A.

The standard silver 5-franc piece of France is worth $1.00471 in the silver coins of the United States; but 5 francs in the standard gold coins of France are worth but $0.964726 in the gold coins of the United States.

LONG OR LINEAR MEASURE.—U. S.

STANDARD.—A brass rod, the length of which, at 62° Fahrenheit, is $\frac{36.0000}{39.1393}$ that of a pendulum beating seconds in *vacuo*, at the level of the sea, at the latitude of London, = $\frac{36.0000}{39.1013}$ at 32° Fah., at the gravitation at New York, = the Yard.

6 points = 1 line.
12 lines (72 points) = 1 inch.
12 inches = 1 foot.
3 feet (36 inches) = 1 yard.
5½ yards (16½ ft.) = 1 rod.
40 rods (220 yds.) = 1 furlong.
8 fur. (5280 feet) = 1 stat. mile.

SPECIAL, FOR CLOTH.

2¼ inches = 1 nail.
4 nails (9 inches) = 1 quarter.
4 quarters (36 inches) = 1 yard.

SPECIAL, FOR LAND.

$7\frac{92}{100}$ inches = 1 link.
25 links = 1 rod.
100 links (66 feet) = 1 chain.
80 chains (320 rods) = 1 s. mile.

ENGINEER'S CHAIN.

10 inches = 1 link.
120 links (100 feet) = 1 chain.

SHOEMAKER'S MEASURE.

No. 1 is $4\frac{1}{8}$ inches in length, and each succeeding number is an addition of $\frac{1}{3}$ of an inch. No. 1 man's size $= 8\frac{11}{24}$ inches.

MISCELLANEOUS.

Hair's breadth	$= \frac{1}{48}$ inch.	Fathom	= 6 feet.
Digit	= 10 lines.	Knot	$= 47\frac{3}{4}$ feet.
Palm	= 3 inches.	Cable's length	= 120 fathoms.
Hand	= 4 "	Geometrical pace	= 4.4 feet.
Span	= 9 "		

12 particular things	= 1 dozen.
12 dozen (144)	= 1 gross.
12 gross (1728)	= 1 great gross.
20 particular things	= 1 score.
24 sheets of paper	= 1 quire.
20 quires	= 1 ream.

SQUARE OR SUPERFICIAL MEASURE.

(*Length* × *breadth.*)

144	square inches	= 1	square foot.
9	" feet	= 1	" yard.
$30\frac{1}{4}$	" yards	= 1	" rod.
40	" rods	= 1	rood.
4	roods	= 1	acre.

SPECIAL, FOR LAND.

$62\frac{454}{625}$	square inches	= 1	square link.
10000	" links	= 1	" chain.
10	" chains	= 1	acre.

Square rod	=	$272\frac{1}{4}$ square feet.
Rood	=	1210 square yards; 10890 square feet.
Acre (160 square rods)	=	4840 square yards; 43560 square feet.
Square mile	=	640 acres; 102400 sq. rods.

220 × 198	square feet	= 1 acre.
The square of 12.649	" rods	= 1 acre.
" " of 69.5701	" yards	= 1 acre.
" " of 208.710321	" feet	= 1 acre.

CIRCULAR MEASURE.

Minute, or Geographical m. (60″)	=	1.152 s. miles. 6086 feet.
League	=	3 miles.
Degree	=	60 geo. miles. 69.158 s. ms.
Sign($\frac{1}{12}$ zod.)	=	30 degrees.
Great Circle	=	360 degrees.
Equatorial circumference of the earth	=	24897 s. m.
Equatorial diam.	=	7925 "
Polar diam.	=	7899 "
Mean radius	=	3955.92 "

NOTE. — In the expressions, *square* feet and *feet* square, there is this difference; viz., the former expresses an area in which there are as many square feet as the *number* named, and the latter an area in which there are as many square feet as the *square* of the number named. The former particularizes no *form* of area, the latter asserts a *square* form.

CUBIC OR SOLID MEASURE.—U. S.

(*Length* × *breadth* × *depth.*)

Cubic foot, 1728 cu. inches	=	1.273 cylindrical feet. 2200 " inches. 3300 spherical " 6600 conical "
Cylindrical foot 1728 " inches	=	0.785398 cubic feet. 1357.2 " inches. 2592 spherical " 5184 conical "
27 cubic feet	=	1 cubic yard.
40 " of round timber	=	1 ton.
42 " of shipping "	=	1 ton.
50 " of hewn "	=	1 ton.
128 "	=	1 cord.
Cubic foot of pure water, at the maximum density at the level of the sea, (39°.83, barometer 30 inches)	=	62½ avoirdupois pounds. 1000 " ounces.
Cylindrical foot	=	49.1 " pounds. 785.4 " ounces.
Cubic inch	=	0.036169 " pounds. 0.5787 " ounces. 253.1829 grains.
Cylindrical inch	=	0.028415 avd. pounds. 0.4546 " ounces.
Pound	=	27.648 cubic inches.
" distilled	=	27.7015 " "
Cubic inch "	=	252.6934 grains.
Pound at 62°, distilled	=	27.7274 cub. inches.
Cubic inch at 62°, "	=	252.458 grains.
" " 39°.83, in *vacuo*	=	253.0864 "

Cubic foot of salt water (sea) weighs 64.3 pounds.

GENERAL MEASURE OF WEIGHT.—U. S.

AVOIRDUPOIS.

STANDARD.—The pound is the weight, taken in air, of 27.7015 cubic inches of distilled water at its maximum density, (39°.83 F., the barometer being at 30 inches) = 27.7274 cubic inches of distilled water at 62° = 7000 Troy grains.

$27\frac{11}{32}$ grains	= 1 dram.
16 drams ($437\frac{1}{2}$ grs.)	= 1 ounce.
16 ounces (7000 grs.)	= 1 pound.

SPECIAL — GROSS.

28 pounds	=	1 quarter.
4 quarters	=	1 quintal.
112 pounds	=	1 cwt.
20 cwt.	=	1 ton.

SPECIAL — DIAMOND.

16 parts = 1 grain = 0.8 troy gr.
4 grs. = 1 carat = 3.2 " "

SPECIAL — TROY.

(Exclusively for gold and silver bullion, precious stones, and gold, silver and copper coins, and with reference to their monetary value only.)

24 grains	= 1 pennyw't.
20 dwts. (480 grs.)	= 1 ounce.
12 oz. (5760 grs.)	= 1 pound.

SPECIAL — APOTHECARIES'.

(Exclusively for compounding medicines, for recipes and prescriptions.)

20 grains	= 1 scruple, ℈.
3 scruples	= 1 dram, ʒ.
8 drams (480 g.)	= 1 ounce, ℥.
12 oz. (5760 g.)	= 1 pound, ℔.

1 lb. avoir.	= $1\frac{31}{144}$ lbs. troy.
1 lb. troy	= $\frac{144}{175}$ lbs. avoir.
1 oz. avoir.	= $\frac{175}{192}$ oz. troy.
1 oz. troy	= $1\frac{17}{175}$ oz. avoir.

NOTE.—The comparative value of diamonds of the same quality is as the square of their respective weights. A diamond of fair quality, weighing 1 carat in the rough state, is estimated worth about $\$9\frac{50}{100}$; and it will require one of twice that weight to make one when worked down equal to 1 carat in weight. Hence, to determine the value of a wrought diamond of any given number of carats:—*Rule.*—Double the weight in carats and multiply the square by 9.50. Thus, the value of a wrought diamond, weighing 2 carats, is $2+2=4\times 4=16\times 9.50=\$152$.

LIQUID MEASURE.—U. S.

The "Wine" or "Winchester" Gallon, of 231 cubic inches capacity, is the Government or Customs gallon of the United States for all liquids, and the legal gallon of each state in which no law exists fixing a state or statute gallon of its own. It contains $58372\frac{1}{4}$ grains of distilled water at 39°.83, the barometer being at 30 inches.

4 gills = 1 pint, 2 pints = 1 quart.

4 quarts, or 231 cubic in.	=	1 gallon.
0.13368 cub. ft., 294.1176 cyl. in.	=	8.355 av'd. lbs. pure water.

Liquid gallon of the State of New York,* 281.62 cylindric in. = { 0.128 cubic foot. / 221.184 " in. / 8 avoid. lbs. pure water at 39°.83, b. 30 in. }

Barrel	=	31½ gallons.	Puncheon	=	84 gallons.
Tierce	=	42 "	Pipe or Butt	=	126 "
Hogshead	=	63 "	Tun	=	252 "

Imperial gallon, 277.274 cub. in. = { 10 av'd lbs. distilled water at 62° F., b. 30 in. }

Ale gallon, 282 cub. in. = { $10\frac{1}{5}$ av'd lbs. pure water at 39°.83, b. 30 in. }

1 Wine gallon = { 0.8331 Imperial gallon. / 0.8191 Ale " / 0.10742 W. bushel. }

1 Imperial gallon = 1.2 Wine gallons.

DRY MEASURE.—U. S.

The "Winchester Bushel," so called, of $2150\frac{42}{100}$ cubic inches capacity, is the Government bushel of the United States, and the legal bushel of each state having no special or statute bushel of its own. The standard Winchester bushel measure is a cylindrical vessel having an outside diameter of 19½ inches, an inside diameter of 18½ inches, and an inside depth of 8 inches. The standard "heaped" or "coal" bushel of England was this measure heaped to a true cone 6 inches high, the base being 19½ inches, or equal to the outside diameter of the measure. Its ratio to the even bushel was, therefore, as 1.28, nearly, to 1. The present "Imperial" measure of England has the same outside diameter and the same depth as the Winchester, and an internal diameter of 18.8 inches, and the same height of cone is retained for forming the heaped bushel. Its ratio, therefore, to the even bushel is a trifle less than was that of the Winchester. In the United States the "heaped bushel" is usually estimated at 5 even pecks, or as 1.25 to 1 of the standard even bushel, which, if taken as

* By enactment of the Legislature of the State of New York, this gallon ceased to be the legal gallon of that State, April 11, 1852; and the United States Government gallon, of 231 cubic inches capacity, was adopted in its stead.

the rule, requires a cone on the Winchester measure of 5.4 inches to equal the heaped Winchester bushel.

4 gills . . .	=	1 pint.
2 pints . . .	=	1 quart.
4 quarts . .	=	1 gallon or half peck.
8 quarts . .	=	1 peck.
4 pecks 2150.42 cubic in. 1.244456 " ft. 1.5844 cyl. "	=	1 bushel. 2738 cyl. in. 77.7785 av'd lbs. pure water.
Bushel of the State of New York,* 2816.1955 cyl. in.	=	1.28 cubic feet. 2211.84 " in. 80 av'd lbs. pure water.
Bushel of Connecticut,†	=	1.272 cubic feet. 2198 " in. 79.50 av'd lbs. pure water.
Heaped Win. bushel 1.28—even " "	=	2747.7 cubic in. 1.59 cubic ft.
Imperial bushel	=	2218.192 " in.
Chaldron	=	36 Winch. heaped bushels.
1 Winchester bushel	=	0.9694 Imperial bushel. 9.3092 Wine gallons.
1 Imperial bushel	=	1.0315 Winchester bushels.

NOTE.—The Imperial bushel, mentioned above, is the present legal bushel of Great Britain; and the Imperial gallon, mentioned on the preceding page, is the present legal gallon of Great Britain, for all liquids. The gallon for liquids is the same as the gallon for dry measure. Eight Imperial gallons make one bushel. The subdivisions of the gallon and the bushel, and their denominations, are the same as in the Winchester measures. In Great Britain, in addition to the denominations of dry measure used in the United States, the

Strike, ... = 2 bushels.		Last, ... = 80 bushels.
Coomb, ... = 4 "		Sack of corn, ... = 3 "
Quarter, ... = 8 "		Bole of corn, ... = 6 "
Wey or load, ... = 40 "		Last of gunpowder, . . = 42 barrels.

* This bushel ceased to be the legal bushel of this State April 11, 1852, and the United States Government bushel, of $2150\frac{42}{100}$ cubic inches capacity, was adopted as the legal bushel in its stead.

† This bushel is now, January, 1852, no longer the legal bushel of this State, and the standard Winchester bushel is adopted in its stead.

SECTION II.

MISCELLANEOUS FACTS, CALCULATIONS, AND PRACTICAL MATHEMATICAL DATA.

SPECIFIC GRAVITIES.

The specific gravity of a body is its weight relative to the weight of an equal bulk of pure water at the maximum density, (39°.83, b. 30 in.) the water being taken as 1., a cubic foot of which weighs 1000 avoirdupois ounces, or 62½ lbs. The specific gravity, therefore, of any body multiplied by 1000, or, which is the same thing, the decimal being carried to three places of figures, or thousands, as in the following TABLES, the whole taken as an integer equals the number of ounces in a cubic foot of the material: multiplied by 62.5, or considered an integer and divided by 16, it equals the number of pounds in a cubic foot; and multiplied by .036169, or taken as an integer and divided by 27648, it equals the decimal fraction of a pound per cubic inch; by which, it is readily seen, the specific gravity of a commodity being known, its weight per any given bulk is easily and accurately ascertained; as, also, its specific gravity, the weight and bulk being known. The weight of any one article relative to that of any other, is as its respective specific gravity to the specific gravity of the other.

METALS.	Specific gravity.		Specific gravity.
Antimony,	6.712	Gold, pure, hammered,	19.546
Arsenic,	5.810	Iridium,	15.363
Bismuth,	9.823	Iron, cast,	7.209
Bronze,	8.700	" wrought,	7.787
Brass, best,	8.504	Lead,	11.352
Copper, cast,	8.788	Mercury, 32°,	13.598
" wire-drawn,	8.878	" 60°,	13.580
Cadmium,	8.604	" —39°,	15.000
Cobalt,	7.700	Manganese,	8.013
Chromium,	5.900	Molybdenum,	8.611
Glucinium,	3.000	Nickel,	8.280
Gold, pure, cast,	19.258	Osmium,	10.000

	Specific gravity.
Platinum, cast,	19.500
" hammered,	20.337
" rolled,	22.069
Potassium, 60°,	0.865
Palladium,	11.870
Rhodium,	11.000
Silver, pure, cast,	10.474
" hammered,	10.511
Sodium,	0.970
Steel, soft,	7.836
" tempered,	7.818
Tin, cast,	7.291
Tellurium,	6.115
Tungsten,	17.600
Titanium,	4.200
Uranium,	9.000
Zinc, cast,	6.861

STONES AND EARTHS.

Alabaster, white,	2.730
" yellow,	2.699
Amber,	1.078
Asbestos, starry,	3.073
Borax,	1.714
Bone, ox,	1.656
Brick,	1.900
Chalk, white,	2.782
Charcoal,	.441
" triturated,	1.380
Cinnabar,	7.786
Clay,	1.934
Coal, bitum. avg.,	1.270
" anth. "	1.520
Coral, red,	2.700
Earth, loose,	1.500
Emery,	4.000
Feldspar,	2.500
Flint, white,	2.594
" black,	2.582
Garnet,	4.085
Glass, flint,	2.933
" white,	2.892
" plate,	2.710
" green,	2.642

	Specific gravity.
Granite, red,	2.625
" Lockport,	2.655
" Quincy,	2.652
" Susquehanna,	2.704
Grindstone,	2.143
Gypsum, opaque,	2.168
Hone, white,	2.876
Hornblende,	3.600
Ivory,	1.822
Jasper,	2.690
Limestone, green,	3.180
" white,	3.156
Lime, compact,	2.720
" foliated,	2.837
" quick,	0.804
Loadstone,	4.930
Magnesia, hyd.,	2.333
Marble, common,	2.686
" white Ital.	2.708
" Rutland, Vt.,	2.708
" Parian,	2.838
Nitre, crude,	1.900
Pearl, oriental,	2.650
Peat, hard,	1.329
Porcelain, China,	2.385
Porphyra, red,	2.766
" green,	2.675
Quartz,	2.647
Rock Crystal,	2.654
Ruby,	4.283
Stone, common,	2.520
" paving,	2.416
" pumice,	0.915
" rotten,	1.981
Salt, common, solid,	2.130
Saltpetre, refined,	2.090
Sand, dry,	1.800
Serpentine,	2.430
Shale,	2.600
Slate,	2.672
Spar, fluor,	3.156
Stalactite,	2.324
Talc, black,	2.900
Topaz,	4.011

SIMPLE SUBSTANCES, *neither metallic nor gaseous.*	Specific gravity.
Boron,	1.968
Bromine,	2.970
Carbon,	3.521
Iodine,	4.943
Phosphorus,	1.770
Selenium,	4.320
Silicon,	1.184
Sulphur,	1.990
WOODS, (*dry.*)	
Apple,	0.793
Alder,	.800
Ash,	.760
Beech,	.696
Birch,	.720
Box, French,	1.328
" Dutch,	.912
Cedar,	.561
Cherry,	.715
Chestnut,	.610
Cocoa,	1.040
Cork,	.240
Cypress,	.644
Ebony, American,	1.331
" foreign,	1.290
Elm,	.671
Fir, yellow,	.657
" white,	.569
Hacmetac,	.592
Hickory, red,	.900
Lignum vitæ,	1.333
Larch,	.544
Logwood,	.913
Mahogany, Spanish, best,	1.065
" " com.,	.800
" St. Domingo,	.720
Maple, red,	.750
Mulberry,	.897
Oak, live,	1.120
" white,	.785
Orange,	.705
Pear,	.661
Pine, white,	.554
Pine, yellow,	.568
Poplar, white,	.383
Plum,	.785
Quince,	.705
Spruce, white,	.551
Sassafras,	.482
Sycamore,	.604
Walnut,	.671
Willow,	.585
Yew, Spanish,	.807
" Dutch,	.788
Highly seasoned Am.	
Ash, white,	.722
Beech,	.624
Birch,	.526
Cedar,	.452
Cherry,	.606
Cypress,	.441
Elm,	.600
Fir,	.491
Hickory, red,	.838
Maple, hard,	.560
Oak, white, upland,	.687
" James River,	.759
Pine, yellow,	.541
" pitch,	.536
" white,	.473
Poplar, (tulip,)	.587
Spruce, white,	.465
GUMS, FATS, &c.	
Asphaltum,	{ .905 1.650
Beeswax,	.965
Butter,	.942
Camphor,	.988
Gamboge,	1.222
Gunpowder,	.900
" shaken,	1.000
" solid,	{ 1.550 1.800
Gum, Arabic,	1.454
" Caoutchouc,	.933
" Mastic,	1.074

	Specific gravity.
Honey, ...	1.450
Ice, ...	.930
Indigo, ...	1.009
Lard, ...	.941
Pitch, ...	1.253
Rosin, ...	1.100
Spermaceti, . . .	.943
Starch, ...	1.530
Sugar, dry, . . .	1.606
Tallow, ...	.938
Tar, ...	1.257

LIQUIDS.

Acid, acetic, . .	1.062
" citric, . .	1.034
" fluoric, . .	1.060
" nitric, . .	1.485
" nitrous, . .	1.420
" sulphuric, . .	1.846
" muriatic, . .	1.200
" silicic, . .	2.660
Alcohol, anhyd. . .	.794
" 90 % . .	.834
Beer, ...	1.034
Blood, human, . .	1.054
Camphene, pure, . .	.863
Cider, whole, . .	1.018
Ether, sulph., . .	.715
" nitric, . .	.908
Milk, cow's, . .	1.032
Molasses, 75 % . .	1.400
Oils, linseed, . .	.934
" olive, . .	.917
" rapeseed, . .	.927
" sassafras, . .	1.090
" turpentine, com., .	.875
" sperm, pure, .	.874
" whale, p'f'd, .	.923
Proof spirits, . .	.925
Vinegar, . . .	1.025
Water, pure, . .	1.000
" sea, . .	1.026
" Dead sea, .	1.240

	Specific gravity.
Wine, champagne, .	.997
" claret, . .	.994
" port, . .	.997
" sherry, . .	.992

ELASTIC FLUIDS:

The measure of which is atmospheric air, at 60°, b. 30 in., its assumed gravity 1; one cubic foot of which weighs 527.04 grains, = .305 of a grain per cubic inch. It is, at this temperature and density, to pure water at the maximum density, as .0012046 to 1, or as 1 to 830.1.

SIMPLE OR ELEMENTARY GASES.

Hydrogen, . . .	.0689
Oxygen, . . .	1.1025
Nitrogen, . . .	.9720
Fluorine, . . .	
Chlorine, . .	2.470
Carbon, vapor of, (*theoretically*,) .	.422

COMPOUND GASES.

Ammoniacal, . .	.591
Carbonic acid, . . .	1.525
" oxide, . .	.973
Carbureted hydrogen, .	.559
Chloro-carbonic, .	3.389
Cyanogen, . . .	1.818
Muriatic acid gas, .	1.247
Nitrous acid gas, .	3.176
Nitrous oxide gas, .	1.040
Olefiant, gas . .	.978
Phosphureted hydrogen,	1.185
Sulphureted " .	1.177
Steam, 212° . .	.484
Smoke, of wood, .	.900
" of coal, .	.102
Vapor, of water, .	.623
" of alcohol, .	1.613
" of spirits turpentine,	5.013

Weight per Bushel (corn Ex. tariff) *of different Grains, Seeds, &c.*

Articles.	lbs.	Articles.	lbs.
Barley, (N. E. 47 lbs.) .	48	Hemp seed, . . .	40
Beans, ...	62	Oats, ...	32
Buckwheat, . . .	46	Peas, ...	64
Blue-grass seed . .	14	Rye, ...	56
Corn, ...	56	Salt, T. I., . . .	80
Cranberries, . . .		" boiled, . . .	56
Clover seed, . . .	60	Timothy seed, . .	46
Dried Apples, . .	22	Wheat, ...	60
" Peaches, . .	33	Potatoes, h'p'd, . .	60
Flax seed, (N. E. 52 lbs.)	56	Malt, ...	38

Weight per Barrel (*Legal or by Usage*) *of different Articles.*

Flour, . . .	196 lbs.	Cider, in Mass., .	32 gals.
Boiled Salt, . .	280 "	Soap, . . .	256 lbs.
Beef, . . .	200 "	Raisins, . . .	112 "
Pork, . . .	200 "	Anchovies, . .	30 "
Pickled Fish, . .	200 "	Lime, . . .	220 "
" " in Massachusetts,	30 gls.	Ground Plaster, . Hydraulic Cement, .	300 "

A Gallon of Oil weighs	$7\frac{3}{4}$ lbs.
A " " Molasses, standard, (75 per cent.,)	$11\frac{2}{3}$ "
A " " Linseed Oil (usage, $7\frac{1}{2}$ lbs.) .	7.788 "
A Firkin of Butter, (legal,) . . .	56 "
A Keg of powder, ...	25 "
A Hogshead of Salt is	8 bush.
A Perch of Stone = $24\frac{3}{4}$ cubic feet.	
A Gallon of Alcohol, 90 per cent., weighs .	6.965 lbs.
A " " Proof Spirits, " .	7.732 "
A " " Wine, (average,) " .	8.3 "
A " " Sperm Oil, " .	7.33 "
A " " Whale " p'f'd, " .	7.71 "
A " " Olive " " .	7.66 "
A " " Spirits Turpentine, " .	7.31 "
A " " Camphene, pure, " .	7.21 "

Weight of Coals, &c., broken to the medium size, per Measure of Capacity.

The average weight of Bituminous Coals, broken as above, is about 62 per cent. that of a bulk of equal dimensions in the solid mass, or of the specific gravity of the article; that of Anthracite is about 57 per cent.

Average weight per cubic foot.	lbs.	Average weight per W. Coal bushel.	lbs.
Anthracite, . . .	54	Anthracite, . . .	86
Bituminous, . . .	50	Bituminous, . . .	80
Charcoal, of pine, . .	18.6	Charcoal, hard wood, .	30
" of hard wood, .	19.02	Coke, best, . . .	32

Practical Approximate Weight in Pounds of Various Articles.

Sand, dry, per cubic foot, ...	95
Clay, compact, per cubic foot, . . .	135
Granite, " " " . . .	165
Lime, quick, " " " . . .	50
Marble, " " " . . .	169
Slate, " " " . . .	167
Peat, hard, " " " . . .	83
Seasoned Beech Wood, per cord, . . .	5616
" Yellow Birch Wood, per cord, . .	4736
" Red Maple Wood, " " . .	5040
" " Oak Wood, " " . .	6200
" White Pine Wood, " " . .	4264
" Hickory Wood, " " . .	6960
" Chestnut Wood, " " . .	4880
Meadow Hay, well settled, per cubic foot, $8\frac{1}{3}$ lbs., or 240 cubic feet = 2000 lbs., or $268\frac{8}{10}$ cubic feet = 1 long ton	
Meadow Hay, in large old stacks, per cubic foot,	$9\frac{8}{10}$
Clover Hay, in settled bulk, " " "	$7\frac{3}{4}$
Corn on Cob, in crib, " " "	22
" shelled, in bin, " " "	45
Wheat, in bin, " " "	48
Oats, in bin, " " "	$25\frac{1}{2}$
Potatoes, in bin, " " "	$38\frac{1}{2}$
Common Brick, $7\frac{3}{4} \times 3\frac{3}{4} \times 2\frac{1}{4}$ in. " M, . .	4500
Front " $8 \times 4\frac{1}{2} \times 2\frac{1}{2}$ in. " " . .	6185

ROPES AND CABLES.

The STRENGTH of cords depends somewhat upon the fineness of the strands; — damp cordage is stronger than dry, and untarred stonger than tarred; but the latter is impervious to water and less elastic.

SILK cords have three times the strength of those of flax of equal circumference, and MANILLA has about half that of hemp.

Ropes made of IRON WIRE are full three times stronger than those of hemp of equal circumference.

White ropes are found to be most durable. The best qualities of hemp are — 1. *pearl gray;* 2. *greenish;* 3. *yellow.* A brown color has less strength.

THE BREAKING WEIGHT of a good hemp rope is 6400 lbs. per square inch, but no cordage may be counted on with safety as capable of sustaining a weight or strain above half that required to break it, and the weight of the rope itself should be included in the estimate.

THE RELIABLE STRENGTH of a good hemp cable, in pounds, is usually estimated as equal to the square of its circumference in inches $\times$ by 120. That of rope $\times$ 200. Thus, a cable of 9 inches in circumference may be relied on as having a sustaining power $= 9 \times 9 \times 120 = 9720$ lbs.

THE WEIGHT, in pounds, of a cable laid rope, per linear foot = the square of its circumference in inches $\times$ .036, very nearly.

The weight, in pounds, of a linear foot of manilla rope = the square of its circumference in inches $\times$ .03, very nearly. Thus, a manilla rope of three inches circumference weighs per linear foot $3 \times 3 \times .03 = \frac{27}{100}$ lbs., $= 3\frac{7}{10}$ feet per lb.

A good hemp rope stretches about $\frac{1}{6}$, and its diameter is diminished about $\frac{1}{5}$ before breaking.

WEIGHT AND STRENGTH OF IRON CHAINS.

Diameter of Wire in Inches.	Weight of 1 Foot of Chain. *lbs.*	Breaking Weight of Chain. *lbs.*	Diameter of Wire in Inches.	Weight of 1 Foot of Chain. *lbs.*	Breaking Weight of Chain. *lbs.*
$\frac{3}{16}$	0.325	2240	$\frac{5}{8}$	4.217	26880
$\frac{1}{4}$	0.65	4256	$\frac{11}{16}$	4.833	32704
$\frac{5}{16}$	0.967	6720	$\frac{3}{4}$	5.75	38752
$\frac{3}{8}$	1.383	9634	$\frac{13}{16}$	6.667	45696
$\frac{7}{16}$	1.767	13216	$\frac{7}{8}$	7.5	51744
$\frac{1}{2}$	2.633	17248	$\frac{15}{16}$	9.333	58464
$\frac{9}{16}$	3.333	21728	1	10.817	65632

Comparative Weight of Metals, Weight per Measure of Solidity, &c

	Specific Gravity.	Ratio of Comparison	Pounds in a Cubic Foot.	Pounds in a Cubic Inch.
Iron, wrought or rolled .	7.787	1.	486.65	.28163
Cast Iron, . . .	7.209	.9258	450.55	.26073
Steel, soft, rolled, . .	7.836	1.0064	489.75	.28342
Copper, pure, " . .	8.878	1.1401	554.83	.32110
Brass, best, " . .	8.604	1.1050	537.75	.3112
Bronze, gun metal, . .	8.700	1.1173	543.75	.31464
Lead, ...	11.352	1.4579	709.50	.4106

TABLE,

Exhibiting the Weight in pounds of One Foot in Length of Wrought or Rolled Iron of any size, (cross section,) from ⅛ inch to 12 inches.

SQUARE BAR.

Size in Inches.	Weight in Pounds.	Size in Inches.	Weight in Pounds.	Size in Inches.	Weight in Pounds.	Size in Inches.	Weight in Pounds.
⅛	.053	2⅜	19.066	4⅝	72.305	7¾	203.024
¼	.211	2½	21.120	4¾	76.264	8	216.336
⅜	.475	2⅝	23.292	4⅞	80.333	8¼	230.068
½	.845	2¾	25.560	5	84.480	8½	244.220
⅝	1.320	2⅞	27.939	5⅛	88.784	8¾	258.800
¾	1.901	3	30.416	5¼	93.168	9	273.792
⅞	2.588	3⅛	33.010	5⅜	97.657	9¼	289.220
1	3.380	3¼	35.704	5½	102.240	9½	305.056
1⅛	4.278	3⅜	38.503	5⅝	106.953	9¾	321.332
1¼	5.280	3½	41.408	5¾	111.756	10	337.920
1⅜	6.390	3⅝	44.418	5⅞	116.671	10¼	355.136
1½	7.604	3¾	47.534	6	121.664	10½	372.672
1⅝	8.926	3⅞	50.756	6¼	132.040	10¾	390.628
1¾	10.352	4	54.084	6½	142.816	11	408.960
1⅞	11.883	4⅛	57.517	6¾	154.012	11¼	427.812
2	13.520	4¼	61.055	7	165.632	11½	447.024
2⅛	15.263	4⅜	64.700	7¼	177.672	11¾	466.684
2¼	17.112	4½	68.448	7½	190.136	12	486.656

To determine the weight, in pounds, of one foot in length, or of any length, of a bar of any of the following metals of form prescribed, of any size, multiply the weight in pounds, of an equal length of square rolled iron of the same size, (see table of square rolled iron,) if the weight be sought of

Iron,	Round rolled, by		.7854
Steel,	Square " "		1.0064
"	Round " "		.7904
Cast Iron,	Square bar, "		.9258
" "	Round " "		.7271
Copper,	Square rolled, "		1.1401
"	Round " "		.8954
Brass,	Square " "		1.105
"	Round " "		.8679
Bronze,	Square bar, "		1.1173
"	Round " "		.8775
Lead,	Square " "		1.4579
"	Round " "		1.145

The weight of a bar of any metal, or other substance, of any given length, of a *flat form*, (and any other form may be included in the rule,) is readily obtained by multiplying its cubic contents (feet or inches) by the weight (pounds, ounces, or grains) of a cubic foot or inch of the article sought to be weighed; that is—

Length × *breadth* × *thickness* × *weight per unit of measure.*

For the weight in pounds of a cubic foot or inch of different metals, see "TABLE of weights of metals per measure of solidity, &c."

OR, FOR FLAT OR SQUARE BARS,

Multiply the sectional area in inches by the length in feet, and that product, if the metal be

Wrought Iron,	by	3.3795
Cast "	"	3.1287
Steel,	"	3.4

EXAMPLE.—Required the weight of a bar of steel, whose length is 7 feet, breadth $2\frac{1}{2}$ inches, and thickness $\frac{3}{4}$ of an inch.

$2.5 \times .75 \times 7 \times 3.4 = 44.625$ lbs. *Ans.*

EXAMPLE.—Required the weight of a cast iron beam, whose length is 14 feet, breadth 9 inches, and thickness $1\frac{1}{2}$ inch.

$14 \times 9 \times 1.5 \times 3.1287 = 591.32$ lbs. *Ans.*

TABLE,

Exhibiting the weight in pounds of One Foot in Length of Round Rolled Iron of any diameter, from 1/8 inch to 12 inches.

Diameter in inches.	Weight in lbs.	Diam. in inches.	Weight in lbs.	Diam. in inches.	Weight in lbs.	Diam. in inches.	Weight in lbs.
1/8	.041	2 3/8	14.975	4 5/8	56.788	7 3/4	159.456
1/4	.165	2 1/2	16.688	4 3/4	59.900	8	169.856
3/8	.373	2 5/8	18.293	4 7/8	63.094	8 1/4	180.696
1/2	.663	2 3/4	20.076	5	66.752	8 1/2	191.808
5/8	1.043	2 7/8	21.944	5 1/8	69.731	8 3/4	203.260
3/4	1.493	3	23.888	5 1/4	73.172	9	215.040
7/8	2.032	3 1/8	25.926	5 3/8	76.700	9 1/4	227.152
1	2.654	3 1/4	28.040	5 1/2	80.304	9 1/2	239.600
1 1/8	3.360	3 3/8	30.240	5 5/8	84.001	9 3/4	252.376
1 1/4	4.172	3 1/2	32.512	5 3/4	87.776	10	266.288
1 3/8	5.019	3 5/8	34.886	5 7/8	91.634	10 1/4	278.924
1 1/2	5.972	3 3/4	37.332	6	95.552	10 1/2	292.688
1 5/8	7.010	3 7/8	39.864	6 1/4	103.704	10 3/4	306.800
1 3/4	8.128	4	42.464	6 1/2	112.160	11	321.216
1 7/8	9.333	4 1/8	45.174	6 3/4	120.960	11 1/4	336.004
2	10.616	4 1/4	47.952	7	130.048	11 1/2	351.104
2 1/8	11.988	4 3/8	50.815	7 1/4	139.544	11 3/4	366.536
2 1/4	13.440	4 1/2	53.760	7 1/2	149.328	12	382.208

To find the weight of an equilateral three-sided cast iron prism.

width of side in inches^2 × 1.354 × length in feet = weight in lbs.

Example. — A three-sided cast iron prism is 14 feet in length, and the width of each side is 6 inches; required the weight of the prism.

$$6^2 \times 1.354 \times 14 = 682.4 \text{ lbs.} \quad \textit{Ans.}$$

To find the weight of an equilateral rectangular cast iron prism.

width of side in inches^2 × 3.128 × length in feet = weight in lbs.

To find the weight of an equilateral five-sided cast iron prism.

width of side in inches^2 × 5.381 × length in feet = weight in lbs.

To find the weight of an equilateral six-sided cast iron prism.

width of side in inches^2 × 8.128 × length in feet = weight in lbs.

To find the weight of an equilateral eight-sided cast iron prism

width of side in inches^2 × 15.1 × length in feet = weight in lbs.

To find the weight of a cast iron cylinder.

diameter in inches^2 × 2.457 × length in feet = weight in lbs.

In a quantity of cast iron weighing 125 lbs., how many cubic inches?

By tabular weight per cubic inch —

$$125 \div .26073 = 479.4 \text{ cubic inches.} \quad \textit{Ans.}$$

Or, by tabular weight per cubic foot —

450.55 : 1728 : : 125 : 479.4 cubic inches. *Ans.*

How many cubic inches of copper will weigh as much as 479.4 cubic inches of cast iron?

By tabular weight per cubic inch —

.3211 : .26073 : : 479.4 : 389.27 cubic inches. *Ans.*

Or, by specific gravities —

8.878 : 7.209 : : 479.4 : 389.27 cubic inches. *Ans.*

Or, by tabular ratio of weight —

$$479.4 \times \frac{.9258}{1.1401} = 389.28.$$

A cast iron rectangular weight is to be constructed having a breadth of 4 inches and a thickness of 2 inches, and its weight is to be 18 lbs.; what must be its length?

$$\frac{18}{4 \times 2 \times .26073} = 8.63 \text{ inches. } \textit{Ans.}$$

A cast iron cylinder is to be 2 inches in diameter, and is to weigh 6 lbs.; what must be its length?

$.26073 \times .7854 = .2047$ lb. = weight of 1 cyl. inch, then

$$\frac{6}{2^2 \times .2047} = 7.327 \text{ inches. } \textit{Ans.}$$

A cast iron cylinder is to weigh 6 lbs., and its length is to be 7.327 inches; what must be its diameter?

$$\sqrt{\left(\frac{6}{7.327 \times .2047}\right)} = 2 \text{ inches. } \textit{Ans.}$$

A cast iron weight, in the form of a *prismoid*, or the *frustrum of a pyramid*, or the *frustrum of a cone*, is to be constructed that will weigh 14 lbs., and the area of one of the bases is to be 16 inches, and that of the other 4 inches; what must be the length of the weight?

$\sqrt{16 \times 4} = 8$ and $\overline{8 + 16 + 4} \div 3 = 9.33$, and $\frac{14}{9.33 \times .26073}$ = 5.75 inches. *Ans.*

NOTE. — For Rules in detail pertaining to the foregoing, see GEOMETRY, *Mensuration of superficies — of solids.*

A model for a piece of casting, made of dry white pine, weighs 7 lbs.; what will the casting weigh, if made of common brass?

By specific gravities —

.554 : 8.604 : : 7 : 108.71 lbs. *Ans.*

NOTE. — As the specific gravity of the substance of which the model is composed must generally remain to some extent uncertain, calculations of this kind can only be relied on as approximate.

4*

TABLE

Exhibiting the Weight of One Foot in Length of Flat, Rolled Iron; Breadth and Thickness in Inches, Weight in Pounds.

Br. and Th. inch.	Wei't. lbs.	Br. and Th. inch.	Wei't. lbs.	Br. and Th. inch.	Wei't. lbs.	Br. and Th. inch.	Wei't. lbs.
½ by ⅛	.211	1¼ by ⅞	3.696	1¾ by ½	2.957	2⅛ by ½	3.591
¼	.422	1	4.224	⅝	3.696	⅝	4.488
⅜	.634	1⅛	4.752	¾	4.435	¾	5.386
⅝ by ⅛	.264	1⅜ by ⅛	.581	⅞	5.175	⅞	6.284
¼	.528	¼	1.161	1	5.914	1	7.181
⅜	.792	⅜	1.742	1⅛	6.653	1⅛	8.079
½	1.056	½	2.323	1¼	7.393	1¼	8.977
¾ by ⅛	.316	⅝	2.904	1⅜	8.132	1⅜	9.874
¼	.633	¾	3.485	1½	8.871	1½	10.772
⅜	.950	⅞	4.066	1⅝	9.610	2¼ by ⅛	.950
½	1.267	1	4.647	1⅞ by ⅛	.792	¼	1.901
⅝	1.584	1⅛	5.228	¼	1.584	⅜	2.851
⅞ by ⅛	.369	1¼	5.808	⅜	2.376	½	3.802
¼	.739	1½ by ⅛	.634	½	3.168	⅝	4.752
⅜	1.108	¼	1.267	⅝	3.960	¾	5.703
½	1.478	⅜	1.901	¾	4.752	⅞	6.653
⅝	1.848	½	2.534	⅞	5.544	1	7.604
¾	2.218	⅝	3.168	1	6.336	1⅛	8.554
1 by ⅛	.422	¾	3.802	1⅛	7.129	1¼	9.505
¼	.845	⅞	4.435	1¼	7.921	1⅜	10.455
⅜	1.267	1	5.069	1⅜	8.713	1½	11.406
½	1.690	1⅛	5.703	1½	9.505	1⅝	12.356
⅝	2.112	1¼	6.337	1⅝	10.297	1¾	13.307
¾	2.534	1⅜	6.970	1¾	11.089	2⅜ by ⅛	1.003
⅞	2.957	1⅝ by ⅛	.686	2 by ⅛	.845	¼	2.006
1⅛ by ⅛	.475	¼	1.373	¼	1.690	⅜	3.010
¼	.950	⅜	2.059	⅜	2.534	½	4.013
⅜	1.425	½	2.746	½	3.379	⅝	5.016
½	1.901	⅝	3.432	⅝	4.224	¾	6.019
⅝	2.376	¾	4.119	¾	5.069	⅞	7.023
¾	2.851	⅞	4.805	⅞	5.914	1	8.026
⅞	3.326	1	5.492	1	6.759	1⅛	9.029
1	3.802	1⅛	6.178	1⅛	7.604	1¼	10.032
1¼ by ⅛	.528	1¼	6.864	1¼	8.449	1⅜	11.036
¼	1.056	1⅜	7.551	1⅜	9.294	1½	12.039
⅜	1.584	1½	8.237	1½	10.138	1⅝	13.042
½	2.112	1¾ by ⅛	.739	2⅛ by ⅛	.898	1¾	14.046
⅝	2.640	¼	1.478	¼	1.795	2	16.052
¾	3.168	⅜	2.218	⅜	2.693	2½ by ⅛	1.056

TABLE. — *Continued.*

Br. and Th. *inch.*	Weight. *lbs.*	Br. and Th. *inch.*	Weight. *lbs.*	Br. and Th. *inch.*	Weight. *lbs.*	Br. and Th. *inch.*	Weight. *lbs.*
2½ by ¼	2.112	2¾ by 1¾	16.264	3¼ by ⅝	6.865	3¾ by 1⅝	20.594
⅜	3.168	1⅞	17.426	¾	8.238	1¾	22.178
½	4.224	2	18.587	⅞	9.610	1⅞	23.762
⅝	5.280	2⅛	19.749	1	10.983	2	25.347
¾	6.336	2¼	20.911	1⅛	12.356	2¼	28.515
⅞	7.393	2⅞ by ⅛	1.214	1¼	13.729	2½	31.683
1	8.449	¼	2.429	1⅜	15.102	2¾	34.851
1⅛	9.505	⅜	3.644	1½	16.475	4 by ⅛	1.690
1¼	10.561	½	4.858	1⅝	17.848	¼	3.379
1⅜	11.617	⅝	6.073	1¾	19.221	½	6.759
1½	12.673	¾	7.287	1⅞	20.594	¾	10.139
1⅝	13.729	⅞	8.502	2	21.967	1	13.518
1¾	14.785	1	9.716	2¼	24.713	1¼	16.898
1⅞	15.841	1⅛	10.931	2½	27.459	1½	20.277
2	16.898	1¼	12.145	3½ by ⅛	1.478	1¾	23.657
2⅝ by ⅛	1.109	1⅜	13.360	¼	2.957	2	27.036
¼	2.218	1½	14.574	⅜	4.436	2¼	30.416
⅜	3.327	1⅝	15.789	½	5.914	2½	33.795
½	4.436	1¾	17.003	⅝	7.393	2¾	37.175
⅝	5.545	1⅞	18.218	¾	8.871	3	40.555
¾	6.653	2	19.432	⅞	10.350	3¼	43.934
⅞	7.762	2⅛	20.647	1	11.828	4¼ by ⅛	1.795
1	8.871	2¼	21.861	1⅛	13.307	¼	3.591
1⅛	9.980	3 by ⅛	1.267	1¼	14.785	½	7.181
1¼	11.089	¼	2.535	1⅜	16.264	¾	10.772
1⅜	12.198	⅜	3.802	1½	17.743	1	14.363
1½	13.307	½	5.069	1⅝	19.221	1¼	17.954
1⅝	14.416	⅝	6.337	1¾	20.700	1½	21.544
1¾	15.525	¾	7.604	1⅞	22.178	1¾	25.135
1⅞	16.634	⅞	8.871	2	23.657	2	28.726
2	17.742	1	10.139	2¼	26.614	2¼	32.317
2⅛	18.851	1⅛	11.406	2½	29.571	2½	35.908
2¾ by ⅛	1.162	1¼	12.673	2¾	32.528	2¾	39.498
¼	2.323	1⅜	13.941	3¾ by ⅛	1.584	3	43.089
⅜	3.485	1½	15.208	¼	3.168	3¼	46.680
½	4.647	1⅝	16.475	⅜	4.752	3½	50.271
⅝	5.808	1¾	17.743	½	6.337	4½ by ¼	3.802
¾	6.970	1⅞	19.010	⅝	7.921	½	7.604
⅞	8.132	2¼	20.277	¾	9.505	¾	11.406
1	9.294	2	22.812	⅞	11.089	1	15.208
1⅛	10.455	2½	25.345	1	12.673	1¼	19.010
1¼	11.617	3¼ by ⅛	1.373	1⅛	14.257	1½	22.812
1⅜	12.779	¼	2.746	1¼	15.842	1¾	26.614
1½	13.940	⅜	4.119	1⅜	17.426	2	30.416
1⅝	15.102	½	5.492	1½	19.010	2¼	34.218

TABLE. — *Continued.*

Br. and Th. *inch.*	Weight. *lbs.*	Br. and Th. *inch.*	Weight. *lbs.*	Br. and Th. *inch.*	Weight. *lbs.*	Br. and Th. *inch.*	Weight. *lbs.*
4½ by 2½	38.020	4¾ by 3	48.158	5¼ by ¾	13.307	5½ by 2	37.175
2¾	41.822	3¼	52.172	1	17.743	2½	46.469
3	45.624	3½	56.185	1¼	22.178	3	55.762
3¼	49.426	5 by ¼	4.224	1½	26.614	5¾ by ¼	4.858
3½	53.228	½	8.449	1¾	31.049	½	9.716
4¾ by ¼	4.013	¾	12.673	2	35.485	¾	14.574
½	8.026	1	16.898	2¼	39.921	1	19.432
¾	12.040	1¼	21.122	2½	44.356	1¼	24.290
1	16.053	1½	25.347	3	53.228	1½	29.146
1¼	20.066	1¾	29.571	5½ by ¼	4.647	1¾	34.007
1½	24.079	2	33.795	½	9.294	2	38.865
1¾	28.092	2¼	38.020	¾	13.941	2¼	43.723
2	32.106	2½	42.244	1	18.587	2½	48.581
2¼	36.119	3	46.469	1¼	23.234	3	58.297
2½	40.132	5¼ by ¼	4.436	1½	27.881	6 by ¼	5.069
2¾	44.145	½	8.871	1¾	32.528		

WEIGHT OF METALS IN PLATE.

The weight of a SQUARE FOOT *one* inch thick of

Malleable Iron . . .	= 40.554 lbs.
Com. plate " . .	= 37.761 "
Cast Iron . . .	= 37.546 "
Copper, wrought . .	= 46.240 "
" com. plate . .	= 45.312 "
Brass, plate, com. . .	= 42.812 "
Zinc, cast, pure . .	= 35.734 "
" sheet . . .	= 37.448 "
Lead, cast . . .	= 59.125 "

And for any other thickness, greater or less, it is the same in proportion; thus, a square foot of sheet copper $\frac{1}{16}$ of an inch thick $= 46.24 \div 16 = 2.89$ lbs. And 5 square feet at that thickness $= 2.89 \times 5 = 14.45$ lbs., &c. So, too, 5 square feet at 2½ inches thickness $= 46.24 \times 2.5 \times 5 = 578$ lbs.

THE AMERICAN WIRE GAUGE.

The American Wire Gauge was prepared by Messrs. Brown and Sharp, manufacturers of machinists' tools, Providence, R. I. It is graded upon geometrical principles, is rapidly becoming the standard gauge with manufacturers of wire and plate in the United States, and cannot fail to supersede the use of the Birmingham Gauge in this country.

TABLE

Showing the Linear Measures represented by Nos. American Wire Gauge and Birmingham Wire Gauge, or the values of the Nos. in the United-States Standard Inch.

No.	American Gauge. Inch.	Birm. Gauge. Inch.	No.	American Gauge. Inch.	Birm. Gauge Inch.	No.	American Gauge. Inch.	Birm. Gauge. Inch.	No.	American Gauge. Inch.	Birm. Gauge. Inch.
0000	.46000	.454	8	.12849	.165	19	.03589	.042	30	.01003	.012
000	.40964	.425	9	.11443	.148	20	.03196	.035	31	.00893	.010
00	.36480	.380	10	.10189	.134	21	.02846	.032	32	.00795	.009
0	.32486	.340	11	.09074	.120	22	.02535	.028	33	.00708	.008
1	.28930	.300	12	.08081	.109	23	.02257	.025	34	.00630	.007
2	.25763	.284	13	.07196	.095	24	.02010	.022	35	.00561	.005
3	.22942	.259	14	.06408	.083	25	.01790	.020	36	.00500	.004
4	.20431	.238	15	.05707	.072	26	.01594	.018	37	.00445	
5	.18194	.220	16	.05082	.065	27	.01419	.016	38	.00396	
6	.16202	.203	17	.04526	.058	28	.01264	.014	39	.00353	
7	.14428	.180	18	.04030	.049	29	.01126	.013	40	.00314	

Thus the DIAMETER or size of No. 4 *wire*, American gauge, is 0.20431 of an inch; Birmingham gauge, 0.238 of an inch: so the THICKNESS of No. 4 *plate*, American gauge, is 0.20431 of an inch; Birmingham gauge, 0.238 of an inch; and so for the other Nos. on the gauges respectively.

TABLE

Showing the Number of Linear Feet in One Pound, Avoirdupois, of Different Kinds of Wire; Sizes or Diameters corresponding to Nos. American Wire-gauge.

No.	Iron. *Feet.*	Copper. *Feet.*	Brass. *Feet.*	No.	Iron. *Feet.*	Copper. *Feet.*	Brass. *Feet.*
0000	1.7834	1.5616	1.6552	19	293.00	256.57	271.94
000	2.2488	1.9692	2.0872	20	396.41	347.12	367.92
00	2.8356	2.4830	2.6318	21	465.83	407.91	432.35
0	3.5757	3.1311	3.3187	22	587.35	514.32	545.13
1	4.5088	3.9482	4.1847	23	740.74	648.63	687.50
2	5.6854	4.9785	5.2768	24	934.03	817.89	866.90
3	7.1695	6.2780	6.6542	25	1177.7	1031.3	1093.0
4	9.0403	7.9162	8.3906	26	1485.0	1300.4	1378.3
5	11.400	9.9825	10.581	27	1872.7	1639.8	1738.1
6	14.375	12.588	13.342	28	2361.4	2067.8	2191.7
7	18.127	15.873	16.824	29	2977.9	2607.6	2763.8
8	22.857	20.015	21.214	30	3754.8	3287.9	3484.9
9	28.819	25.235	26.748	31	4734.2	4145.5	4394.0
10	36.348	31.828	33.735	32	5970.6	5221.2	5541.4
11	45.829	40.131	42.535	33	7528.1	6592.0	6987.0
12	57.790	50.604	53.636	34	9495.6	8314.9	8813.1
13	72.949	63.878	67.706	35	11972	10483	11111
14	91.861	80.439	85.258	36	15094	13217	14009
15	115.86	100.75	107.53	37	19030	16664	17662
16	146.10	127.94	135.60	38	24003	21018	22278
17	184.26	168.35	171.02	39	30266	26503	28091
18	232.34	203.45	215.64	40	38176	33342	35432

NOTE.—In this TABLE the iron and copper employed are supposed to be nearly pure. The specific gravity of the former was taken at 7.774; that of the latter, at 8.878. The specific gravity of the brass was taken at 8.376.

To find the number of feet in a pound of wire of any material not given in the TABLE, *of any size, American gauge, its specific gravity being known.*

RULE. — Multiply the number of feet in a pound of iron wire of the same size by 7.774, and divide the product by the specific gravity of the wire whose length is sought; or ordinarily, for steel wire, multiply the number of feet in a pound of iron wire of the same size by 0.991.

To find the number of feet in a pound of wire of any given No., Birmingham gauge.

RULE. — Multiply the number of feet in a pound of the same kind of wire, same No., American gauge, by the size, American gauge, and divide the product by the size, Birmingham gauge.

EXAMPLE. — In a pound of copper wire No. 16, American gauge, there are 127.94 feet: how many feet are there of the same kind of wire, same No., Birmingham gauge?

$$(127.94 \times .05082) \div .065 = 100.03. \quad \textit{Ans.}$$

To find the weight of any given length of wire of any given No. or size, American gauge, or the length in any given weight, by help of the foregoing TABLE.

EXAMPLE. — Required the weight of 600 feet of No. 18 iron wire.

$$600 \div 232.34 = 2.5822 \text{ lbs.} = 2 \text{ lbs. } 9\tfrac{1}{3} \text{ oz., nearly.} \quad \textit{Ans.}$$

EXAMPLE. — Required the length in feet of $2\frac{1}{2}$ lbs. of No. 31 brass wire.

$$4394 \times 2.5 = 10985. \quad \textit{Ans.}$$

Characteristics of Alloys of Copper and Zinc — Brass.

Parts by Weight.		Specific Gravity.	Color.	Denomination.
Copper.	Zinc.			
83	17	8.415	Yellowish Red.	Bath Metal.
80	20	8.448	" "	Dutch Brass.
$74\frac{1}{2}$	$25\frac{1}{2}$	8.397	Pale yellow.	Rolled Sheet Brass.
66	34	8.299	Full "	English Sheet Brass.
$49\frac{1}{2}$	$50\frac{1}{2}$	8.230	" "	German Sheet Brass.
33	67	8.284	Deep "	Watchmaker's Brass.

NOTE. — To alloys of copper and zinc, generally, there is added a small quantity of lead, which renders them the better adapted for turning, planing, or filing; and, for the same reason, to alloys of copper and tin, there is usually added a small quantity of zinc (see ALLOYS AND COMPOSITIONS).

TABLE

Showing the Weight of One Square Foot of Rolled Metals, thickness corresponding to Nos., American Wire-gauge.

Thickness. *No.*	Iron. *Pounds.*	Steel. *Pounds.*	Copper. *Pounds.*	Brass. *Pounds.*	Lead. *Pounds.*	Zinc. *Pounds.*
1	10.849	10.999	13.109	12.401	17.102	10.833
2	9.6611	9.7953	11.674	11.043	15.228	9.6466
3	8.6032	8.7227	10.396	9.8340	13.562	8.5903
4	7.6616	7.7680	9.2578	8.7576	12.078	7.6501
5	6.8228	6.9175	8.2442	7.7988	10.755	6.8126
6	6.0758	6.1601	7.3416	6.9450	9.5779	6.0667
7	5.4105	5.4856	6.5377	6.1845	8.5291	5.4024
8	4.8184	4.8853	5.8222	5.5077	7.5957	4.8112
9	4.2911	4.3507	5.1851	4.9050	6.7645	4.2847
10	3.8209	3.8740	4.6169	4.3675	6.0233	3.8151
11	3.4028	3.4501	4.1117	3.8896	5.3642	3.3977
12	3.0303	3.0720	3.6616	3.4638	4.7770	3.0257
13	2.6985	2.7360	3.2607	3.0845	4.2539	2.6934
14	2.4035	2.4365	2.9042	2.7473	3.7889	2.3999
15	2.1401	2.1698	2.5829	2.4463	3.3737	2.1369
16	1.9058	1.9322	2.3028	2.1784	3.0043	1.9029
17	1.6971	1.7207	2.0506	1.9399	2.6753	1.6945
18	1.5114	1.5324	1.8263	1.7276	2.3826	1.5091
19	1.3459	1.3646	1.6263	1.5384	2.1217	1.3439
20	1.1985	1.2152	1.4482	1.3700	1.8893	1.1967
21	1.0673	1.0821	1.2897	1.2300	1.6768	1.0657
22	.95051	.96371	1.1485	1.0865	1.4984	.94908
23	.84641	.85815	1.0227	.96749	1.3343	.84514
24	.75375	.76422	.91078	.86158	1.1882	.75262
25	.67125	.68057	.81109	.76728	1.0582	.67024
26	.59775	.60605	.72228	.68326	.94229	.59685
27	.53231	.53970	.64345	.60846	.83913	.53151
28	.47404	.48062	.57280	.54185	.74728	.47333
29	.42214	.42800	.51009	.48242	.66546	.42151
30	.37594	.38116	.45426	.42972	.59263	.37538

NOTE.—In calculating the foregoing TABLE, the specific gravities were taken as follows: viz., iron, 7.200; steel, 7.300; copper, 8.700; brass, 8.230; lead, 11.350; Zinc, 7.189.

TIN PLATES.

Brand Marks.	Size of Sheets in Inches.	No. of Sheets in Box.	Net Weight in lbs.	Brand Marks.	Size of Sheets in Inches.	No. of Sheets in Box.	Net Weight in lbs.
IC	14 × 14	200	140	SDXX	15 × 11	200	210
IC	14 × 10	225	112	SDXXX	15 × 11	200	231
HC	14 × 10	225	119	SDXXXX	15 × 11	200	252
HX	14 × 10	225	147	TT	14 × 10	225	112
IX	14 × 10	225	140	" IC	12 × 12	225	119
IXX	14 × 10	225	161	" IX	12 × 12	225	147
IXXX	14 × 10	225	182	" IXX	12 × 12	225	168
IXXXX	14 × 10	225	203	" IXXX	12 × 12	225	189
IX	14 × 14	200	174	" IXXXX	12 × 12	225	210
IXX	14 × 14	200	200	" IC	20 × 14	112	112
DC	17 × 12½	100	105	" IX	20 × 14	112	140
DX	17 × 12½	100	126	" IXX	20 × 14	112	161
DXX	17 × 12½	100	147	" IXXX	20 × 14	112	182
DXXX	17 × 12½	100	168	" IXXXX	20 × 14	112	203
DXXXX	17 × 12½	100	189	Ternes IC	20 × 14	112	112
SDC	15 × 11	200	168	" IX	20 × 14	112	140
SDX	15 × 11	200	189				

NOTE.—The above TABLE includes all the regular sizes and qualities of tin plates, except "*wasters*." Other sizes, such as 10 × 10, 11 × 11, 13 × 13, &c., of the different brands, are often imported into the United States to order.

Common English Sheet Iron, Nos. 10 to 28, Birmingham gauge, widths from 24 to 36 inches.

R. G. Sheet Iron, Nos. 10 to 30, Birmingham gauge, widths from 24 to 36 inches.

American Puddled Sheet Iron, Nos. 22 to 28, Birmingham gauge, widths from 24 to 36 inches.

Russia Sheet Iron, Nos. 16 to 8 inclusive, Russia gauge, sheets 28 × 56 inches.

Sheet Zinc, Nos. 16 to 8, Liege gauge, widths from 24 to 40 inches; length 84 inches.

Copper Sheathing, 14 × 48 inches, 14 to 32 oz. (even numbers), per square foot.

Yellow Metal, in sheets, 48 × 14 inches, 14 to 32 oz. (even numbers), per square foot.

TABLE

Showing the Capacity, in Wine Gallons, of Cylindrical Cans, of different diameters, at One Inch depth. Diameter in Inches.

Diam'r. *inches.*	Gallons.	Diam'r. *inches.*	Gallons.	Diam'r. *inches.*	Gallons.	Diam'r. *inches.*	Gallons.
6	.1224	12¼	.5102	18½	1.164	24¾	2.083
6¼	.1328	12½	.5313	18¾	1.195	25	2.125
6½	.1437	12¾	.5527	19	1.227	25¼	2.167
6¾	.1549	13	.5746	19¼	1.260	25½	2.211
7	.1666	13¼	.5969	19½	1.293	25¾	2.254
7¼	.1787	13½	.6197	19¾	1.326	26	2.298
7½	.1913	13¾	.6428	20	1.360	26¼	2.343
7¾	.2042	14	.6664	20¼	1.394	26½	2.388
8	.2176	14¼	.6904	20½	1.429	26¾	2.433
8¼	.2314	14½	.7149	20¾	1.464	27	2.479
8½	.2457	14¾	.7397	21	1.499	27¼	2.524
8¾	.2603	15	.7650	21¼	1.535	27½	2.571
9	.2754	15¼	.7907	21½	1.572	27¾	2.518
9¼	.2909	15½	.8169	21¾	1.608	28	2.666
9½	.3069	15¾	.8434	22	1.646	28¼	2.713
9¾	.3233	16	.8704	22¼	1.683	28½	2.762
10	.3400	16¼	.8978	22½	1.721	28¾	2.810
10¼	.3572	16½	.9257	22¾	1.760	29	2.859
10½	.3749	16¾	.9539	23	1.799	29¼	2.909
10¾	.3929	17	.9826	23¼	1.837	29¾	3.009
11	.4114	17¼	1.0120	23½	1.877	30	3.060
11¼	.4303	17½	1.0410	23¾	1.918	30½	3.163
11½	.4497	17¾	1.0710	24	1.958	31	3.264
11¾	.4694	18	1.1020	24¼	1.999	31½	3.374
12	.4896	18¼	1.1320	24½	2.041	32	3.482

Applications of the foregoing TABLE.

EXAMPLE. — A cylindrical can is 11¼ inches in diameter, and its depth is 18⅗ inches; required its capacity.

$.4303 \times 18\frac{3}{5} = 8$ gallons. *Ans.*

EXAMPLE. — The diameter of a can containing oil is 26½ inches, and the oil is 14½ inches in depth. How many gallons are there of the oil?

$2.388 \times 14\frac{1}{2} = 34.6$ gallons. *Ans.*

EXAMPLE. — A can is to be constructed that will hold just 36 gallons, and its diameter is to be 18 inches; what must be its depth?

$36 \div 1.102 = 32\frac{2}{3}$ inches. *Ans.*

EXAMPLE. — A cylindrical can is to be constructed that shall have a depth of 15 inches and a capacity of just 5 gallons; what must be its diameter?

$5 \div 15 = .3333$ = capacity of can in gallons for each inch of depth; and against .3333 gallon in the table, or the quantity in gallons nearest thereto, is 10 inches, the required, or nearest tabular diameter. *Ans.*

NOTE. — The table is not intended to meet demands of the nature of the one contained in the last example, with accuracy, unless the fractional part of the diameter, if there be a fractional part, is $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ inch. As, however, the diameter opposite the tabular gallon nearest the one sought, even at its greatest possible remove, can be but about $\frac{1}{8}$ inch from the diameter required, we can, by inspection, determine the diameter to be taken, or true answer to the inquiry, sufficiently near for practical purposes, be the fraction what it may. Or, to throw the demand into a mathematical formula: As the tabular gallon nearest the one sought is to the diameter opposite, so is the tabular gallon required to the required diameter, nearly. Thus, in answer to the last query,

.3400 : 10 :: 3333 : 9.8 inches, the required or true diameter, nearly.

For a mathematical formula strictly applicable to this question, see GAUGING

Or, for a formula more strictly geometrical, we have

$$\sqrt{\frac{\text{Capacity} \times 231}{\text{Depth} \times .7854}} = \text{diameter.}$$

The true diameter, therefore, for the supposed can, is

$$\sqrt{\frac{231 \times 5}{15 \times .7854}} = 9.9\text{— inches.}$$

WEIGHT OF PIPES.

The weight of ONE FOOT IN LENGTH of a pipe, of any diameter and thickness, may be ascertained by multiplying the square of its exterior diameter, in inches, by the weight of 12 *cylindrical* inches of the material of which the pipe is composed, and by multiplying the square of its interior diameter, in inches, by the same factor and subtracting the product of the latter from that of the former, — the remainder or difference will be the weight. This is evident from the fact that the process obtains the weight of two solid cylinders of equal length, (one foot,) the diameter of one being that of the pipe, and the other that of the vacancy, or bore. For very large pipes, the dimensions may be taken in feet, and the weight of a cylindrical foot of the material used as the factor, or multiplier, if desired.

The weight of 12 *cylindrical* inches (length 1 foot, diameter 1 inch) of

Malleable Iron	= 2.6543 lbs.
Cast Iron	= 2.4573 "
Copper, wrought,	= 3.0317 "
Lead "	= 3.8697 "
Cast Iron — 1 *cyl.* foot —	= 353.86 "

Therefore — EXAMPLE. — Required the weight of a copper pipe whose length is 5 feet, exterior diameter $3\frac{1}{4}$ inches, and interior diameter 3 inches.

$$3\tfrac{1}{4} = \tfrac{13}{4} \times \tfrac{13}{4} = 10.5625 \times 3.0317 = 32.022 +$$
$$3 \times 3 = 9 \times 3.0317 = 27.285 +$$
$$\textit{Ans.}\quad 4.737 \times 5 = 23.685 \text{ lbs.}$$

EXAMPLE. — Required the weight of a cast iron pipe, whose length is 10 feet, exterior diameter 38 inches, and interior diameter 3 feet.

$38^2 \times 2.4573 - 36^2 \times 2.4573 = 363.68 \times 10 = 3636.8$ lbs. *Ans.*

Or, $\overline{38^2 - 36^2} = 148 \times 2.4573 = 363.68 \times 10 = 3636.8$ lbs. *Ans.*

EXAMPLE. — Required the weight of a lead pipe, whose length is 1200 feet, exterior diameter $\frac{7}{8}$ of an inch, and interior diameter $\frac{9}{16}$ of an inch.

$\frac{7}{8} \times \frac{7}{8} = \frac{49}{64} = .765625$, and $\frac{9}{16} \times \frac{9}{16} = \frac{81}{256} = .316406$, and $.765625 - .316406 = .449219 \times 3.8697 \times 1200 = 2086$ lbs. *Ans.*

EXAMPLE. — The length of a cast-iron cylinder is 1 foot, its exterior diameter is 12 inches, and its interior diameter 10 inches; required its weight.

$$12^2 - 10^2 = 44 \times 2.4573 = 108.12 \text{ lbs.} \quad \textit{Ans.}$$
$$\text{Or, } 144 : 353.86 :: 44 : 108.12 \text{ lbs.} \quad \textit{Ans.}$$

The following TABLE *exhibits the coefficients of weight, in pounds, of one foot in length, of various thicknesses, of different kinds of pipe, of any diameter whatever.*

Thickness in Inches.	Wrought Iron.	Copper.	Lead.
$\frac{1}{32}$	.332	.379	.484
$\frac{1}{16}$	.664	.758	.9675
$\frac{3}{32}$	.995	1.137	1.451
$\frac{1}{8}$	1.327	1.516	1.935
$\frac{5}{32}$	1.658	1.894	2.417
$\frac{3}{16}$	1.99	2.274	2.901
$\frac{7}{32}$	2.323	2.653	3.386
$\frac{1}{4}$	2.654	3.032	3.87
$\frac{5}{16}$	3.318	3.79	4.837
$\frac{3}{8}$	3.981	4.548	5.805

CAST IRON.

Thickness.	Factor.	Thickness.	Factor.	Thickness.	Factor.
$\frac{3}{16}$	1.842	$\frac{5}{8}$	6.143	$1\frac{1}{4}$	12.287
$\frac{1}{4}$	2.457	$\frac{3}{4}$	7.372	$1\frac{1}{2}$	14.744
$\frac{3}{8}$	3.686	$\frac{7}{8}$	8.6	$1\frac{3}{4}$	17.201
$\frac{1}{2}$	4.901	1	9.829	2	19.659

To obtain the weight of pipes by means of the above TABLE—

RULE.—Multiply the diameter of the pipe, taken from the interior surface of the metal on the one side to the exterior surface on the opposite, (interior diameter + thickness,) in inches, by the number in the table under the respective metal's name, and opposite the thickness corresponding to that of the pipe—the product will be the weight, in pounds, of ONE foot in length of the pipe, and that product multiplied by the length of the pipe, in feet, will give the weight for any length required.

EXAMPLE.—Required the weight of a copper pipe whose length is 5 feet, interior diameter and thickness $3\frac{1}{8}$ inches, and thickness $\frac{1}{8}$ of an inch.

$$3\tfrac{1}{8} = \tfrac{25}{8} = 3.125 \times 1.516 \times 5 = 23.687 \text{ lbs.} \quad \textit{Ans.}$$

EXAMPLE.—Required the weight of a cast iron pipe, 10 feet in length, whose interior diameter is 3 feet, and whose thickness is 1 inch.

$$36 + 1 = 37 \times 9.829 \times 10 = 3636.73 \text{ lbs.} \quad \textit{Ans.}$$

WEIGHT OF CAST IRON AND LEAD BALLS.

To find the weight of a sphere or globe of any material—

RULE.—Multiply the cube of the diameter, in inches, or feet, by the weight of a *spherical* inch or foot of the material.

The weight of a spherical inch of

Cast Iron . = .1365 lbs.
Lead . . = .215 "

Therefore—EXAMPLE.—Required the weight of a leaden ball whose diameter is $\frac{1}{4}$ of an inch.

$\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64} = .015625 \times .215 = .00336$ lb. *Ans.*

EXAMPLE.—Required the weight of a cast iron ball whose diameter is 8 inches.

$8^3 \times .1365 = 69.888$ lbs. *Ans.*

EXAMPLE.—How many leaden balls, having a diameter $\frac{1}{4}$ of an inch each, are there in a pound?

$1 \div .00336 = \frac{100000}{336} = 298.$ *Ans.*

EXAMPLE.—What must be the diameter of a cast iron ball, to weigh 69.888 lbs?

$69.888 \div .1365 = \sqrt[3]{512} = 8$ inches. *Ans.*

EXAMPLE.—What must be the diameter of a leaden ball to equal in weight that of a cast iron ball, whose diameter is 8 inches?

[Lead is to cast iron as .215 to .1365, as 1.575 to 1.]

$8^3 = 512 \div 1.575 = \sqrt[3]{325} = 6.875$ inches. *Ans.*

WEIGHT OF HOLLOW BALLS OR SHELLS.

The weight of a hollow ball is the weight of a solid ball of the same diameter, *less* the weight of a solid ball whose diameter is that of the interior diameter of the shell.

EXAMPLE.—Required the weight of a cast iron shell whose exterior diameter is $6\frac{1}{4}$ inches, and interior diameter $4\frac{1}{4}$ inches.

$6\frac{1}{4} = \frac{25}{4} \times \frac{25}{4} \times \frac{25}{4} = 244.14 \times .1365 = 33.33$
$4\frac{1}{4} = 4.25^3 \times .1365 = 10.48$

22.85 lbs. *Ans.*

Or, If we multiply the difference of the cubes, in inches, of the two diameters—the exterior and interior—by the weight of a spherical inch, we shall obtain the same result.

EXAMPLE.—Required the weight of a cast iron shell whose exterior diameter is 10 inches and interior diameter 8 inches.

$\overline{10^3 - 8^3} \times .1365 = 66.612$ lbs. *Ans.*

ANALYSIS OF COALS.

Description.	Volatile Matter.	Carbon.	Ash.
Breckinridge, Ky.,	62.25	29.10	8.65
"Albert," N. B.,	61.74	32.14	6.12
Chippenville, Pa.,	49.80		
Kanawha, "	41.85		
Pittsburg, "	32.95		
Cannel, sp. gr. 1.4	35.28	64.72	
Newcastle,	24.72	75.28	
Cumberland,	18.40	80.	1.60
Anthracite, a'v'g.,	3.43	89.46	7.11

Woods of most descriptions vary little from 80 per cent. volatile matter, and 20 per cent. charcoal.

TABLE—*Exhibiting the Weights, Evaporative Powers, &c., of Fuels, from Report of Professor Walter R. Johnson.*

Designation of Fuel.	Specific Gravity.	Weight per Cubic Foot.	Lbs. of Water at 212 degrees converted into Steam by 1 Cubic Foot of Fuel.	Lbs. of Water at 212 degrees converted into Steam by 1 lb. of Fuel.	Weight of Clinkers from 100 lbs. of Coal.
ANTHRACITE COALS.					
Beaver Meadow, No. 3	1.610	54.93	526.5	9.21	1.01
Beaver Meadow, No. 5	1.554	56.19	572.9	9.88	.60
Forest Improvement	1.477	53.66	577.3	10.06	.81
Lackawanna	1.421	48.89	493.0	9.79	1.24
Lehigh	1.590	55.32	515.4	8.93	1.08
Peach Mountain	1.464	53.79	581.3	10.11	3.03
BITUMINOUS COALS.					
Blossburgh	1.324	53.05	522.6	9.72	3.40
Cannelton, Ia.	1.273	47.65	360.0	7.34	1.64
Clover Hill	1.285	45.49	359.3	7.67	3.86
Cumberland, *average,*	1.325	53.60	552.8	10.07	3.33
Liverpool	1.262	47.88	411.2	7.84	1.86
Midlothian	1.294	54.04	461.6	8.29	8.82
Newcastle	1.257	50.82	453.9	8.66	3.14
Pictou	1.318	49.25	478.7	8.41	6.13
Pittsburgh	1.252	46.81	384.1	8.20	.94
Scotch	1.519	51.09	369.1	6.95	5.63
Sydney	1.338	47.44	386.1	7.99	2.25
COKE.					
Cumberland		31.57	284.0	8.99	3.55
Midlothian		32.70	282.5	8.63	10.51
Natural Virginia	1.323	46.64	407.9	8.47	5.31
WOOD.					
Dry Pine Wood		21.01	98.6	4.69	

MENSURATION OF LUMBER.

To find the contents of a board.

RULE. — Multiply the length in feet by the width in inches, and divide the product by 12; the quotient will be the contents in square feet.

EXAMPLE. — A board is 16 feet long and 10 inches wide; how many square feet does it contain?

$$16 \times 10 = 160 \div 12 = 13\tfrac{4}{12}. \quad \textit{Ans.}$$

To find the contents of a plank, joist, or stick of square timber.

RULE. — Multiply the product of the depth and width in inches by the length in feet, and divide the last product by 12; the quotient is the contents in feet, *board measure.*

EXAMPLE. — A joist is 16 feet long, 5 inches wide, and 2½ inches thick; how many feet does it contain, board measure?

$$5 \times 2.5 \times 16 \div 12 = 16\tfrac{8}{12}. \quad \textit{Ans.}$$

To find the solidity of a plank, joist, or stick of square timber.

RULE. — Multiply the product of the depth and width in inches by the length in feet, and divide the last product by 144; the quotient will be the contents in cubic feet.

EXAMPLE. — A stick of timber is 10 by 6 inches, and 14 feet in length; what is its solidity?

$$10 \times 6 = 60 \times 14 = 840 \div 144 = 5\tfrac{5}{6} \text{ feet.} \quad \textit{Ans.}$$

NOTE. — If a board, plank, or joist is narrower at one end than the other, add the two ends together and divide the sum by 2; the quotient will be the mean width. And if a stick of squared timber, whose solidity is required, is narrower at one end than the other $(A + a + \sqrt{Aa}) \div 3 =$ mean area. A and a being the areas of the ends.

To measure round timber.

RULE (IN GENERAL PRACTICE.) — Multiply the length, in feet, by the square of ¼ the girt, in inches, taken about ⅓ the distance from the larger end, and divide the product by 144; the quotient is considered the contents in cubic feet. For a strictly correct rule for measuring round timber, see MENSURATION OF SOLIDS — *Frustum of a Cone.*

EXAMPLE. — A stick of round timber is 40 feet in length, and girts 88 inches; what is its solidity?

$$88 \div 4 = 22 \times 22 = 484 \times 40 = 19360 \div 144 = 134.44 \text{ cub. ft.} \quad \textit{Ans.}$$

The following TABLE *is intended to facilitate the measuring of Round Timber, and is predicated upon the foregoing* RULE.

¼ Girt in Inches.	Area in Feet.	¼ Girt in Inches.	Area in Feet.	¼ Girt in Inches.	Area in Feet.	¼ Girt in Inches.	Area in Feet.
6	.25	12	1.	18	2.25	24	4.
6¼	.272	12¼	1.042	18¼	2.313	24¼	4.084
6½	.294	12½	1.085	18½	2.376	24½	4.168
6¾	.317	12¾	1.129	18¾	2.442	24¾	4.254
7	.34	13	1.174	19	2.506	25	4.34
7¼	.364	13¼	1.219	19¼	2.574	25¼	4.428
7½	.39	13½	1.265	19½	2.64	25½	4.516
7¾	.417	13¾	1.313	19¾	2.709	25¾	4.605
8	.444	14	1.361	20	2.777	26	4.694
8¼	.472	14¼	1.41	20¼	2.898	26¼	4.785
8½	.501	14½	1.46	20½	2.917	26½	4.876
8¾	.531	14¾	1.511	20¾	2.99	26¾	4.969
9	.562	15	1.562	21	3.062	27	5.062
9¼	.594	15¼	1.615	21¼	3.136	27¼	5.158
9½	.626	15½	1.668	21½	3.209	27½	5.252
9¾	659	15¾	1.722	21¾	3.285	27¾	5.348
10	.694	16	1.777	22	3.362	28	5.444
10¼	.73	16¼	1.833	22¼	3.438	28¼	5.542
10½	.766	16½	1.89	22½	3.516	28½	5.64
10¾	.803	16¾	1.948	22¾	3.598	28¾	5.74
11	.84	17	2.006	23	3.673	29	5.84
11¼	.878	17¼	2.066	23¼	3.754	29¼	5.941
11½	.918	17½	2.126	23½	3.835	29½	6.044
11¾	.959	17¾	2.187	23¾	3.917	30	6.25

To find the solidity of a log by help of the preceding TABLE.

RULE. — Multiply the tabular area opposite the corresponding ¼ girt, by the length of the log in feet, and the product will be the solidity in feet.

EXAMPLE. — The ¼ girt of a log is 22 inches, and the length of the log is 40 feet; required the solidity of the log.

$$3.362 \times 40 = 134.48 \text{ cubic feet.} \quad \textit{Ans.}$$

NOTE. — Though custom has established, in a very general way, the preceding method as that whereby to measure round timber, and holds, in most instances, the solidity to be that which the method will give, there seems, if the object sought be the real solidity of the stick, neither accuracy, justice, nor certainty, in the practice.

Thus, in the preceding example, the stick was supposed to be 40 feet in length, and 88 inches in circumference at ⅓ the distance from the larger end, and was found, by the method, to contain 134.44 cubic feet: now 88 ÷ 3.1416 = 28 inches, = the diameter at ⅓ the distance from the greater base, and retaining this diameter and the length, we may suppose, with sufficient liberality, and without being far from the general run of such sticks, the diameter at the greater base to be 30 inches, and that of the less to be 24 inches, and —

By a correct rule the stick contains —

$30 \times 24 = 720 + 12 = 732 \times .7854 \times 40 = 22996 \div 144 = 159.7$ cubic feet, or 19 per cent. more than given by the method under consideration; and we need hardly add that the nearer the stick approaches to the figure of a cylinder, the wider will be the difference between the truth and the result obtained by the method referred to. Thus, suppose the stick a cylinder, 28 inches in diameter, and 40 feet in length; and we have, by the fallacious rule, as above, 134.44 cubic feet; and —

By a correct method, we have —

$28^2 \times .7854 \times 40 = 24630 \div 144 = 171$ cubic feet, or over 27 per cent. more than furnished by the erroneous mode of practice.

Again: suppose the stick in the form of a cone, 30 inches at the base, and tapering to a point at 150 feet in length; and we have, by a correct rule —

$30^2 \div 3 = 300 \times .7854 \times 150 = 35343 \div 144 = 245.44$ cubic feet; and by the ordinary method of gauging, or the aforementioned practice, we have —

$20 \times 3.1416 = 62.832 \div 4 = 15.708^2 \times 150 = 37011.19 \div 144 = 257$ cubic feet, or nearly $4\frac{3}{4}$ per cent. more than the stick actually contains.

In short, without taking into account anything for the thickness of the bark, that may be supposed to be on the stick, the method is correct only when the stick tapers at the rate of $5\frac{1}{4}$ inches diameter per each 10 feet in length, or over $\frac{1}{2}$ inch diameter to each foot in length of the stick.

If, however, we suppose the stick as before, (30 inches at the greater base, 24 inches at the smaller, and 40 feet in length,) and suppose the bark upon it to be 1 inch thick, we shall have, by the usual method, 134.44 cubic feet, as before. And, exclusive of the bark, by a correct method, we shall have.

$\overline{30 - 2} \times \overline{24 - 2} = 616 + 12 = 628 \times .7854 \times 40 = 19729 \div 144 = 137$ cubic feet, or only about 2 per cent. more than that furnished us by the usual practice.

The following simple rule for measuring round timber is sufficiently correct for most practical purposes: —

Rule. — Multiply the square of one-fifth of the mean girt, (exclusive of bark,) in inches, by twice the length of the stick in feet, and divide the product by 144; the quotient will be the solidity in feet.

To find the solidity of the greatest rectangular stick that may be cut from a given log, or from a stick of round timber of given dimensions.

Rule. — Multiply the square of the mean diameter of the log, in inches, by half the length of the log, in feet, and divide the product by 144.

Example. — The diameter (exclusive of bark) of the greater base of a stick of round timber is 30 inches, and that of the less base is 24 inches, and the stick is 40 feet in length; required the solidity of the greatest rectangular stick that may be cut from it.

$30 \times 24 + \frac{1}{3}(30 - 24)^2 = 732 =$ square of mean diameter,* and
$732 \times 20 = 14640 \div 144 = 101\frac{2}{3}$ cubic feet. *Ans.*

* Except in the case of a cylinder, there is a difference betwixt the *mean* diameter of a solid having circular bases, and the *middle* diameter of that solid. The mean diameter reduces the solid to a cylinder; the middle diameter is the diameter midway between the two bases.

NOTE. — The foregoing stick will make —
$14640 \div 16 = 915$ feet of square-edged boards 1 inch thick;
Or, $101\frac{2}{3} \times 9 = 915$.

To find the solidity of the greatest square stick that may be cut from a given log, or from a stick of round timber of given dimensions.

RULE. — Multiply the square of the diameter of the less end of the log, in inches, by half the length of the log, in feet, and divide the product by 144.

EXAMPLE. — The preceding supposed log will make a square stick containing —

$$24^2 \times \frac{40}{2} = 1152 \div 144 = 80 \text{ cubic feet.}$$

Diameter multiplied by $.7071 =$ side of inscribed square.

To find the contents, in Board Measure, of a log, no allowance being made for wane or saw-chip.

RULE. — Multiply the square of the mean diameter, in inches, by the length in feet, and divide the product by 15.28.

Or, Multiply the square of the mean diameter in inches, by the length in feet, and that product by .7854, and divide the last product by 12.

The cubic contents of a log multiplied by 12, equal the contents of the log, board measure.

The convex surface of a Frustum of a Cone $= (C + c) \times \frac{1}{2}$ slant length; C being the circumference of the greater base, and c the circumference of the less.

GAUGING.

Rules *for finding the capacity in gallons or bushels of different shaped Cisterns, Bins, Casks, &c., and also, by way of examples, for constructing them to given capacities.*

Rule — 1. *When the vessel is rectangular.* Multiply the interior length, breadth, and depth, in feet together, and the product by the capacity of a cubic foot, in gallons or bushels, as desired for its capacity.

Rule — 2. *When the vessel is cylindrical.* Multiply the square of its interior diameter in feet, by its interior depth in feet, and the product by the capacity of a *cylindrical* foot in gallons or bushels, as desired for its capacity.

Rule — 3. *When the vessel is a rhombus or rhomboid.* Multiply its interior length, in feet, its right-angular breath in feet, and its depth in feet together, and the product by the capacity of a cubic foot in the special measure desired for its capacity.

Rule — 4. *When the vessel is a frustum of a cone* — a round vessel larger at one end than the other, whose bases are planes. Multiply the interior diameter of the two ends together, in feet, add ⅓ the square of their difference in feet to the product, multiply the sum by the perpendicular depth of the vessel in feet, and that product by the capacity of a *cylindrical* foot in the unit of measure desired for its capacity.

Rule — 5. *When the vessel is a prismoid or the frustum of any regular pyramid.* To the square root of the product of the areas of its ends in feet, add the areas of its ends in feet, multiply the sum by ⅓ its perpendicular depth in feet, and that product by the capacity of a cubic foot in gallons or bushels, as desired for its capacity.

If it is found more convenient to take the dimensions in inches, do so; proceed as directed for feet, divide the product by 1728, and multiply the quotient by the capacity of the respective foot as directed. Or, multiply the capacity in inches by the capacity of the respective inch in gallons or bushels; — by the quotient obtained by dividing the capacity of the respective foot in gallons or bushels by 1728 — for the contents.

Rule — 6. *When the vessel is a barrel, hogshead, pipe, &c.* Multiply the difference in inches between the bung diameter and head diameter, (interior,) if the staves be

much curved, . .	by .7	*See* page 63.
medium curved, .	by .65	
straighter than medium,	by .6	
nearly straight, .	by .55	

and add the product to the head diameter, taken in inches; then multiply the square of the sum by the length of the cask in inches, and divide the product by the capacity in *cylindrical* inches of a gallon or bushel as desired for the contents. Or, divide the contents in *cylindrical* inches, as above found, by 1728, and multiply the quotient by the capacity of a cylindrical foot in gallons or bushels as desired for its contents. Or, multiply the capacity in cylindrical inches by the capacity of a cylindrical inch, in gallons or bushels, as desired,— that is, by the quotient obtained by dividing the capacity of a cylindrical foot in gallons or bushels, by 1728, for the contents.

The capacity of a

CUBIC FOOT =			CYLINDRICAL FOOT =		
7.4805	Winchester wine	gallons.	5.8751	Winchester wine	gallons.
6.1276	Ale	"	4.8126	Ale	"
6.2321	Imperial	"	4.8947	Imperial	"
.80356	Winchester	bushel.	.63111	Winchester	bushel.
.62888	" heaped	"	.49391	" heaped	"
.64285	" 1¼ even	"	.50489	" 1¼ even	"
.779	Imperial "	"	.61183	Imperial	"

EXAMPLE.—Required the capacity in Winchester bushels of a rectangular bin, whose interior length is 12 feet, breadth 6 feet, and depth 5 feet.

$$12 \times 6 \times 5 \times .8035 = 289.26 \text{ bushels.} \quad \textit{Ans.}$$

EXAMPLE.—Required the capacity in Winchester wine gallons of a cylindrical can, whose interior diameter is 18 inches, and depth 3 feet.

$$18 \times 18 \times 36 \times 5.875 \div 1728 = 39.66 \text{ gallons.} \quad \textit{Ans.}$$

Or, $1.5 \times 1.5 \times 3 \times 5.875 = 39.66$ gallons. *Ans.*

Or, $18 \times 18 \times 36 \times .0034 = 39.66$ gallons. *Ans.*

EXAMPLE.—How many Winchester bushels in 39.66 wine gallons?

$$39.66 \times .10742 = 4.26 \text{ bushels.} \quad \textit{Ans.}$$

EXAMPLE.—How many wine gallons in 4.26 Winchester bushels?

$$4.26 \times 9.3092 = 39.66 \text{ gallons.} \quad \textit{Ans.}$$

EXAMPLE.—How many wine gallons will a cistern in the form of a frustum of a cone hold, having the interior diameter of one of its ends 6 feet, and that at the other 8 feet, and its perpendicular depth 9 feet?

$8 - 6 = 2$, and $2^2 \div 3 = 1.333 = \frac{1}{3}$ square of dif. of diameters, and
$6 \times 8 + 1.333 = 49.333 \times 9 \times 5.8751 = 2608.55$ gals. *Ans.*

Or, $6 \times 8 + 8^2 + 6^2 = 148 \times \frac{9}{3} \times 5.8751 = 2608.55$ gals. *Ans.*

Or, $(8^3 - 6^3) \div (8 - 6) = 148 \times \frac{9}{3} \times 5.8751 = 2608.55$ gals. *Ans.*

Or, $96 - 72 = 24$ and $(24^2 \div 3) = 192$, and
$96 \times 72 + 192 = 7104 \times 108 \times .0034 = 2608.55$ gals. *Ans.*

Example. — What is the capacity in Winchester bushels of a cistern whose form is prismoid, the dimensions (interior) of one end being 8 by 6 feet, of the other 4 by 3 feet, and its perpendicular depth 12 feet?

$8 \times 6 = 48 =$ area of one end, and $4 \times 3 = 12 =$ area of the other end; then —

$48 \times 12 = \sqrt{576} = 24 + 48 + 12 = 84 \times \frac{12}{3} \times .80356 = 270$ bushels. *Ans.*

Or, $(8 + 4) \div 2 = 6$, and $(6 + 3) \div 2 = 4.5 =$ mean sectional areas of ends, and

$6 \times 4.5 \times 4 = 4$ area of mean perimeter, then

$8 \times 6 + 4 \times 3 + 6 \times 4.5 \times 4 = 168 \times \frac{12}{6} \times .80356 = 270$ bus. *Ans.*

Example. — What must be the depth of a rectangular bin whose length is 12 feet, and breadth 6 feet, to hold 289.26 bushels?

$289.26 \div (12 \times 6 \times .80356) = 5$ feet. *Ans.*

Example. — A cylindrical can, whose depth is to be 36 inches, is required to be made that will hold 40 gallons; what must be the diameter of the can?

$40 \div (3 \times 5.8751) = \sqrt{2.27} = 1.506$ feet. *Ans.*

Or, $40 \div (36 \times .0034) = \sqrt{326.8} = 18.07$ inches. *Ans.*

Example. — A cylindrical can, whose interior diameter is to be 18 inches, is required that will hold 40 gallons; what must be the interior depth of the can?

$40 \div (18^2 \times .0034) = 36.31$ inches. *Ans.*

Or, $40 \div (1.5^2 \times 5.8751) = 3.026$ feet. *Ans.*

Example. — A cistern is to be built in the form of a frustum of a cone, that will hold 1800 gallons, and the diameter of one of its ends is to be 5 feet, and that of the other $7\frac{1}{2}$ feet; what must be the depth?

$7.5 - 5 = 2.5$, and $2.5^2 \div 3 = 2.0833 = \frac{1}{3}$ square of difference of diameter, and

$1800 \div (7.5 \times 5 + 2.0833) \times 5.8751 = 7.74$ feet. *Ans.*

$$\text{Or, } 1800 \div \left(\frac{7.5 \times 5 + 7.5^2 + 5^2}{3} \times 5.8751\right) = 7.74 \text{ feet. } \textit{Ans.}$$

Example. — The form, capacity, depth, and diameter of one end being determined on, and being as above, what must be the diameter of the other end?

$\frac{c}{\frac{1}{3}h} - \frac{3}{4}d^2 = y$, c being the solidity in cylindrical measurement, h

the depth, d the diameter of the given end or base, and y a quantity the square root of which is the sum of the required base and half the given base; then

$1800 \div 5.8751 = 306.378 =$ solidity in cylindrical feet, and

$306.378 \div \frac{7.74}{3} = 118.75 - (5^2 \div \frac{4}{3}) = \sqrt{100} = 10 - \frac{5}{2} = 7.5$ feet. *Ans.*

EXAMPLE. — A measure is to be built in the form of a frustum of a cone, that will hold exactly 1 wine gallon, and the diameter of one of its ends is to be 4 inches, and that of the other 6 inches; what must be its depth?

$1 \div (6 \times 4 + 1\frac{1}{3}) \times .0034 = 11.61$ inches. *Ans.*

$$\text{Or, } \frac{231}{.7854} \div \frac{6 \times 4 + 6^2 + 4^2}{3} = 11.61 \text{ inches. } Ans.$$

EXAMPLE. — A measure in the form of a frustum of a cone holds 1 wine gallon; the diameter of one of its ends is 6 inches, and its depth is 11.61 inches; what is the diameter of the other end?

$\frac{231}{.7854} = 294.1176 \div \frac{11.61}{3} = 76 - (6^2 \div \frac{4}{3}) = \sqrt{49} = 7 - \frac{6}{2} = 4$ inches. *Ans.*

CASK GAUGING.

CASK-GAUGING, in a general sense, is a practical art, rather than a scientific achievement or problem, and makes no pretensions to strict accuracy with regard to the conclusions arrived at. The aim is, by means of a few satisfactory measurements taken of the outside, and an estimate of the probable mean thickness of the material of which the cask is composed (of which there must always remain some doubt), or by means of a few measurements taken of the inside, to determine, 1st, the capacity of the cask, and, 2d, the ullage, or capacity of the occupied or unoccupied space in a cask but partly full. And the Rule (RULE 6, page 60), which reduces the supposed cask, or cask of supposed curvature, to a cylinder, is as practically correct for the capacity of ordinary casks, as any rule, or set of rules, that can be offered for general purposes.

Casks have no fixed form of their own, to which they severally and collectively correspond, nor are they in any considerable degree in conformity with any regular geometrical figure.

Some casks — a few — those having their staves much curved throughout their entire length, are nearest in keeping with the *middle frustum of a spheroid;* others, slightly less curved than the preceding, correspond in a considerable degree to the *middle* *frustum of a parabolic spindle;* others, again — those having very little longitudinal curvature of stave to their semi-lengths — are nearly in keeping with the *equal frustums of a paraboloid;* and others — a very few — those whose staves are straight from the bung diameter to the heads, or equal to that form, are in accordance with the *equal frustums of a cone.*

The *gauging rod*, which is intended to be correct for casks of the most common form, gives for all casks, as may be seen in one of the following EXAMPLES, a solidity slightly greater (about $2\frac{1}{2}$ per cent.) than would be obtained by supposing the cask in conformity with the third figure above alluded to.

The RULE for finding the contents of a cask, *by four dimensions*, hereafter to be given, is intended as a general Rule for all casks, and, when the diameter midway between the bung and head can be accurately ascertained, will lead to a very close approach to the truth.

From the length of a cask, taken from outside to outside of the heads, with callipers, it is usual to deduct from 1 to 2 inches, to correspond with the thickness of the heads, according to the size of the cask, and the remainder is taken as the length of the interior.

To the diameter of each head, taken externally, from $\frac{1}{4}$ inch to $\frac{3}{10}$ inch should be added for common-sized barrels, $\frac{4}{10}$ inch for 40 gallon casks, and from $\frac{1}{2}$ inch to $\frac{6}{10}$ inch for larger casks, to correspond with the interior diameters of the heads.

If the staves are of uniform thickness, any sectional diameter of a cask may be nearly or quite ascertained, by dividing the circumference at that place by 3.1416, and subtracting twice the thickness of the stave from the quotient.

For obtaining the diagonal of a cask by mathematical process, — the interior length, &c. &c. — see *Rules*, below.

In the following formulas D denotes the bung diameter, d the head diameter, and l the length of the cask.

The solidity of any cask is equal to its length multiplied by the square of its mean diameter multiplied by .7854.

To calculate the contents of a cask from four dimensions.

RULE. — To the square of the bung diameter add the square of the head diameter, and the square of double the diameter midway between the bung and head, and multiply the sum by $\frac{1}{6}$ the length of the cask, for its *cylindrical* contents; the product multiplied by .0034 expresses the contents in wine gallons.

EXAMPLE. — The length of the cask is 40 inches, its bung diameter 28 inches, head diameter 20 inches, and the diameter midway between the bung and head is 25.6 inches; how many gallons' capacity has the cask?

$20^2 + 28^2 + \overline{25.6 \times 2}^2 = 3805.44 \times \frac{40}{6} \times .0034 = 86.26$ gals. *Ans.*

$$(D^2 + d^2 + \overline{2\,m}^2) \times \tfrac{1}{6}\, l \times .7854 = \text{cubic contents.}$$

$$\frac{D^2 + d^2 + \overline{2\,m}^2}{6} = \text{square of mean diameter.}$$

By Rule 6, p. 68, this cask will hold —

$28 - 20 = 8 \times .65 = 5.2 + 20 = 25.2 \times 25.2 \times 40 \times .0034 = 86.36$ gallons.

When the cask is in the form of the middle frustum of a spheroid.

$$\tfrac{2}{3}\, D^2 + \tfrac{1}{3}\, d^2 = \text{square of mean diameter.}$$

And a cask of this form, having the same head diameter, bung diameter, and length as the preceding, will hold —

$$\frac{2 \times 28^2 + 20^2}{3} \times 40 \times .0034 = 89.216 \text{ gallons.}$$

When the cask is in the form of the middle frustum of a parabolic spindle.

$$\overline{\tfrac{2}{3} D^2 + \tfrac{1}{3}\, d^2} - \tfrac{2}{15}\,(D \backsim d)^2 = \text{square of mean diameter.}$$

And a cask of this form, having the same head diameter, bung diameter, and length as the preceding, will hold —

$522\tfrac{2}{3} + 133\tfrac{1}{3} = 656 - 8.533 = 647.467 \times 40 \times .0034 = 88.055$ gals.

When the cask is in the form of two equal frustums of a paraboloid.

$$\tfrac{1}{2}\, D^2 + \tfrac{1}{2}\, d^2 = \text{square of mean diameter.}$$

And a cask of this form, having the same head diameter, bung diameter, and length as the preceding, will hold —

$$\frac{28^2 + 20^2}{2} \times 40 \times .0034 = 80.51 \text{ gallons.}$$

When the cask is in the form of the equal frustums of a cone.

$\overline{\tfrac{1}{2}\, D^2 + \tfrac{1}{2}\, d^2} - \tfrac{1}{6}\,(D \backsim d)^2 =$ square of mean diameter.

Or, $\tfrac{1}{3} D^2 + \tfrac{1}{3} d^2 + \tfrac{1}{3} Dd =$ " " " "

Or, $D \times d + \tfrac{1}{3}(D \backsim d)^2 =$ " " " "

And a cask of this form, having the same head diameter, bung diameter, and length as the preceding, will hold —

$$\overline{28 \times 20 + 21\tfrac{1}{3}} \times 40 \times .0034 = 79.06 \text{ gals.}$$

To find the contents of a cask the same as would be given by the gauging rod.

The *gauging rod* is constructed upon the principle that the cube of the diagonal of a cask, in inches, multiplied by $\frac{144}{64000}$, equals the contents of the cask, in Imperial gallons.

The contents in wine gallons of either of the aforementioned casks, therefore, by the gauging rod, would be —

$$\overline{31.241}^3 \times .0027 = 82\tfrac{1}{3} \text{ gals.}$$

The decimal coëfficient to take the place of .0027, for finding the contents of a cask in the form of the middle frustum of a spheroid = .002926; and for finding the contents of a cask in the form of the equal frustums of a cone = .002593. And between these extremes lies the decimal for other casks, or casks of intervening figures.

To find the diagonal of a cask, when the interior is inaccessible.

Rule. — From the bung diameter subtract half the difference of the bung and head diameters, and to the square of the remainder add the square of half the length of the cask, and the square root of the sum will be the diagonal.

Example. — What is the diagonal of a cask whose bung diameter is 28 inches, head diameter 20 inches, and length 40 inches?

$$28 - 20 = 8 \div 2 = 4, \text{ and } 28 - 4 = 24, \text{ then}$$
$$\sqrt{(24^2 + 20^2)} = 31.241 \text{ inches. } \textit{Ans.}$$

To find the length of a cask, the head diameter, bung diameter and diagonal being given.

$$\sqrt{\left(\text{diagonal}^2 - \overline{D - \frac{D \backsim d}{2}}^2\right)} = \tfrac{1}{2}\, l.$$

And the interior length of a cask, whose interior head diameter, bung diameter and diagonal, are as the preceding, will be

$$\sqrt{(31.241^2 - 24^2)} = 20 \times 2 = 40 \text{ inches.}$$

To find the solidity of a sphere.

$$D^2 \times \tfrac{2}{3} D \times .7854 = \text{cubic contents, D being the diameter.}$$

To find the solidity of a spherical frustum.

$$\left(\tfrac{2}{3}\, h^2 + \frac{b^2 + d^2}{2}\right) \times h \times .7854 = \text{cubic contents, } b \text{ and } d \text{ being the bases, and } h \text{ the height.}$$

Note. — For Rules in detail pertaining to the foregoing figures, and for other figures, see Mensuration of Solids.

ULLAGE.

The *ullage* or *wantage* of a cask is the quantity the cask lacks of being full.

To find the ullage of a standing cask, when the cask is half full or more.

RULE. — To the square of the head diameter, add the square of the diameter at the surface of the liquor, and the square of twice the diameter midway between the surface of the liquor and the upper head, and divide the sum by 6; the quotient, multiplied by the distance from the surface of the liquor to the upper head, multiplied by .0034, will give the ullage in wine gallons.

EXAMPLE. — The diameters are as follows — at the upper head, 20 inches; at the surface of the liquor, 22 inches; and at a point midway between these, $21\frac{1}{4}$ inches; and the distance from the upper head to the surface of the liquor is 5 inches; required the ullage.

$(20^2 + 22^2 + \overline{21.25 \times 2}^2) \div 6 = 448.37 \times 5 \times .0034 = 7.62$ gallons. *Ans.*

When the cask is standing, and less than half full, to find the ullage.

RULE. — Make use of the bung diameter in place of the head diameter, and proceed in all respects as directed in the last *Rule*, and add the quantity found to half the capacity of the cask; the sum will be the ullage.

EXAMPLE. — The bung diameter is 28 inches; the diameter at the surface of the liquor, below the bung, is 26 inches; the diameter midway between the bung and the surface of the liquor is 27.3 inches; and the distance from the surface of the liquor to the bung diameter is 5 inches; required the quantity the cask lacks of being half full; and also the ullage of the cask, its capacity being 86.26 gallons.

$(28^2 + 26^2 + \overline{27.3 \times 2}^2) \div 6 = 740.2 \times 5 \times .0034 = 12.58$ gallons less than $\frac{1}{2}$ full. *Ans.*

And, $86.26 \div 2 = 43.13 + 12.58 = 55.73$ gallons ullage. *Ans.*

When the cask is upon its bilge, and half full or more, to find the ullage.

RULE. — Divide the distance from the bung to the surface of the liquor — (the height of the empty segment) — by the whole bung diameter, and take the quotient as the height of the segment of a circle whose diameter is 1, and find the area of the segment; multiply the area by the capacity of the cask, in gallons, and that product by 1.25; the last product will be the ullage, in gallons, as found by the aid of the *wantage-rod;* and will be correct for casks of the most common form.

NOTE. — The area of the segment of a circle =

(ch'd $\frac{1}{2}$ arc + $\frac{1}{3}$ ch'd $\frac{1}{2}$ arc + ch'd seg.) × height seg. × $\frac{4}{10}$*, very nearly.

And, having the diameter of the circle and the height of the segment given, the chord of half the arc, and the chord of the segment may be found, thus —

radius — height = cosine; radius^2 *— cosine*^{2} *= sine*^{2}*;* $\surd$ *(sine*^{2}*)* × 2 = *ch'd of seg.*

sine^2 + *height seg.*^{2} = *ch'd* $\frac{1}{2}$ *arc*^{2}, and $\surd$ *(ch'd* $\frac{1}{2}$ *arc2)* = *ch'd* $\frac{1}{2}$ *arc.*

EXAMPLE. — The bung diameter is 28 inches, the height of the empty segment 5.6 inches, and the capacity of the cask 86.26 gallons; required the ullage of the cask, in gallons.

$5.6 \div 28 = .2$ = height of seg., diameter as 1.

$1 \div 2 = .5$ = radius.

$.5 - .2 = .3$ = cosine.

$.5^2 - .3^2 = .16$ = sine^2, or square of half the base of the segment.

$\surd .16 = .4 \times 2 = .8$ = chord of segment, or base of segment.

$.4^2 + .2^2 = .2$ = square of chord of half the arc.

$\surd .2 = .4472$ = chord of half the arc, then —

$.4472 \div 3 = .1491$, and $\overline{.1491 + .4472 + .8} \times .2 \times \frac{4}{10} = .1117$, area of segment, and

$.1117 \times 86.26 \times 1.25 = 12$ gallons. *Ans.*

When the cask is upon its bilge, and less than half full, to find the ullage.

RULE. — Divide the depth of the liquor by the bung diameter, and proceed in all respects as directed in the last Rule; then subtract the quantity found from the capacity of the cask, and the difference will be the ullage of the cask.

To find the quantity of liquor in a cask by its weight.

EXAMPLE. — The weight of a cask of proof spirits is 300 lbs., and the weight of the empty cask (*tare*) is 32 lbs. How many gallons are there of the liquor?

$300 - 32 = 268 \div 7.732 = 34\frac{2}{3}$ gallons. *Ans.*

Customary Rule by Freighting Merchants, for finding the cubic measurement of casks.

Bung diameter^2 × $\frac{4}{5}$ length of cask = cubic measurement.

NOTE. — One cubic foot contains 7.4805 wine gallons.

* For several Rules in detail, for finding the area of the segment of a circle, see GEOMETRY — *Mensuration of Superficies.*

TONNAGE.

GOVERNMENT MEASUREMENT.*

$$\frac{\overline{\text{length} - \frac{3}{5}\text{ breadth}} \times \text{breadth} \times \text{depth}}{95} = \text{tonnage}.$$

In a double-decked vessel, the length is reckoned from the fore part of the main stem to the after side of the sternpost above the upper deck; the breadth is taken at the broadest part above the main wales, and half this breadth is taken for the depth.

In a single-decked vessel the length and breadth are taken as for a double-decked vessel, and the distance between the ceiling of the hold and the under side of the deck plank is taken as the depth.

Example. — The length of a double-decked vessel is 260 feet, and the breadth is 60 feet; required the tonnage.

$260 - \frac{60 \times 3}{5} = 224 \times 60 \times \frac{60}{2} = 403200 \div 95 = 4244.2$ tons. *Ans.*

Example. — The length of a single-decked vessel is 180 feet, the breadth 34 feet, and depth 18 feet; required the tonnage.

$180 - \frac{3}{5}$ of $34 = 159.6 \times 34 \times 18 \div 95 = 1028.16$ tons. *Ans.*

CARPENTER'S MEASUREMENT.

For a double-decked —

$$\frac{\text{length of keel} \times \text{breadth main beam} \times \frac{1}{2}\text{ breadth}}{95} = \text{tonnage}.$$

For a single-decked —

$$\frac{\text{length of keel} \times \text{breadth main beam} \times \text{depth of hold}}{95} = \text{tonnage}$$

* The set of rules legalized by the Congress of 1865 for ascertaining the nominal or Government tonnage of vessels are not inserted in this work, partly because of their great lengths in detail, and the multiplicity of the mechanical admeasurements required, and partly because they can be of use to but a few individuals; and to those they are furnished by the Government.

OF CONDUITS OR PIPES.

Pressure of Water in Vertical Pipes, &c.

h = height of column in inches ; o = circumference of column in inches; t = thickness of pipe in inches equal in strength to lateral pressure at base of column ; w = weight of a cubic inch of water in pounds ; C = cohesive strength in pounds per inch area of transverse section of the material of which the pipe is composed — TABLE, p. 74.

ho = area of interior of pipe in inches ; hw = pressure in pounds per square inch at the base of the column, or maximum lateral pressure in pounds per square inch on the pipe tending to burst it ; how = maximum lateral pressure in pounds on the pipe, tending to burst it at the bottom ; and $how \div 2$ = mean lateral pressure in pounds on the pipe, or pressure in pounds on the pipe tending to burst it at half the height of the column.

$$how \div C = t;\ how \div t = C;\ Ct \div ow = h;\ Ct \div hw = o.$$

NOTE. — The *reliable* cohesion of a material is not above ⅓ its ultimate force, as given in the Table of Cohesive Forces. By experiment, it has been found that a cast iron pipe 15 inches in diameter and ¾ of an inch thick, will support a head of water of 600 feet; and that one of the same diameter made of oak, and two inches thick, will support a head of 180 feet: 12000 lbs. per square inch for cast iron, 1200 for oak, 750 for lead, are counted safe estimates. The ultimate cohesion of an alloy, composed of lead 8 parts and zinc 1 part, is 3000 pounds per square inch.

Concerning the Discharge of Pipes, &c.

Small pipes, whether vertical, horizontal, or inclined, under equal heads, discharge proportionally less water than large ones. That form of pipe, therefore, which presents the least perimeter to its area, other things being equal, will give the greatest discharge. A round pipe, consequently, will discharge more water in a given time than a pipe of any other form, of equal area.

The greater the length of a pipe discharging vertically, the greater the discharge. Because the friction of the particles against its sides, and consequent retardation, is more than overcome by the gravity of the fluid.

The greater the length of a pipe discharging horizontally, the less proportionally will be the discharge. The proportion compared with a less length is in the inverse ratio of the square root of the two lengths, nearly.

Other things being equal, rectilinear pipes give a greater discharge than curvilinear, and curvilinear greater than angular. The head, the diameters and the lengths being the same, the time occupied in passing an equal quantity of water through a straight pipe is 9, through one curved to a semicircle 10, and through one having one right angle, otherwise straight, 14. All interior inequalities and roughness should be avoided.

It has been ascertained that a velocity of 60 feet a minute (1 foot a second) through a horizontal pipe, 4 inches in diameter and 100 feet in length, is produced by a head $2\frac{1}{7}$ inches, only $\frac{1}{7}$ of an inch above the upper surface of the orifice; and that, to maintain an equal velocity through a pipe similarly situated, of equal length, having a diameter of $\frac{1}{2}$ inch only, a head of $1\frac{5}{12}$ feet is required. To increase the velocity through the last mentioned pipe to 2 feet a second, requires a head $4\frac{10}{12}$ feet; to 3 feet, a head of $10\frac{1}{12}$; to 4 feet, a head of $17\frac{10}{12}$, &c.

From the foregoing, the following, it is believed, reliable rules, are deduced.

To find the velocity of water passing through a straight horizontal pipe of any length and diameter, the head, or height of the fluid above the centre of the orifice, being known.

RULE. — Multiply the head, in feet, by 2500, and divide the product by the length of the pipe, in feet, multiplied by 13.9, divided by the interior diameter of the pipe in inches; the square root of the quotient will be the velocity in feet per second.

EXAMPLE. — The head is 6 feet, the length of the pipe 1340 feet, and its diameter 5 inches; required the velocity of the water passing through it.

$2500 \times 6 = 15000 \div (\frac{1340 \times 13.9}{5}) = \sqrt{4.03} - = 2$ feet per second. *Ans.*

To find the head necessary to produce a required velocity through a pipe of given length and diameter.

RULE. — Multiply the square of the required velocity, in feet, per second, by the length of the pipe multiplied by the quotient obtained by dividing 13.9 by the diameter of the pipe in inches, and divide the product thus obtained by 2500; the quotient will be the head in feet.

EXAMPLE. — The length of a pipe lying horizontal and straight is 1340 feet, and its diameter is 5 inches; what head is necessary to cause the water to flow through it at the rate of 2 feet a second?

$$2^2 \times 1340 \times \frac{13.9}{5} \div 2500 = 6 \text{ feet.} \quad \textit{Ans.}$$

To find the quantity of water flowing through a pipe of any length and diameter.

RULE. — Multiply the velocity in feet per second by the area of the discharging orifice, in feet, and the product is the quantity in cubic feet discharged per second.

EXAMPLE. — The velocity is 2 feet a second, and the diameter of the pipe 5 inches; what quantity of water is discharged in each second of time?

$5 \div 12 = .4166$, and $.4166^2 \times .7854 \times 2 = .273$ cubic foot. *Ans.*

MISCELLANEOUS PROBLEMS.

To find the specific gravity of a body heavier than water.

RULE. — Weigh the body in water and out of water, and divide the weight out of water by the difference of the two weights.

EXAMPLE. — A piece of metal weighs 10 lbs. in atmosphere, and but $8\frac{1}{4}$ in water; required its specific gravity.

$$10 - 8.25 = 1.75, \text{ and } 10 \div 1.75 = 5.714. \quad \textit{Ans.}$$

To find the specific gravity of a body lighter than water.

RULE. — Weigh the body in air; then connect it with a piece of metal whose weight, both in and out of water, is known, and of sufficient weight that the two will sink in water; and find their combined weight in water; then divide the weight of the body in air by the weight of the two substances in air, less the sum of the difference of the weight of the metal in air and water and the combined weight of the two substances in water, and the quotient will be the specific gravity sought.

EXAMPLE. — The combined weight, in water, of a piece of wood, and piece of metal, is 4 lbs.; the wood weighs in atmosphere 10 lbs.; and the metal in atmosphere 12, and in water 11 lbs.; required the specific gravity of the wood.

$$10 \div (10 + 12 - \overline{12 \backsim 11 + 4}) = .588. \quad \textit{Ans.}$$

To find the specific gravity of a fluid.

RULE. — Multiply the known specific gravity of a body by the difference of its weight in and out of the fluid, and divide the product by its weight out of the fluid; the quotient will be the specific gravity of the fluid in which the body is weighed.

EXAMPLE. — The specific gravity of a brass ball is 8.6; its weight in atmosphere is 8 oz., and in a certain fluid $7\frac{1}{4}$ oz.; required the specific gravity of the fluid.

$$8 - 7.25 = .75, \text{ and } 8.6 \times .75 = 6.45, \text{ and } 6.45 \div 8 = .806. \quad \textit{Ans.}$$

To find the proportion of one to the other of two simples forming a compound, or the extent to which a metal is debased, (the metal and the alloy used being known.)

The Rule strictly bears upon that of *Alligation Alternate*, which see.

EXAMPLE. — The specific gravity of gold is 19.258, and that of copper, 8.788; an article composed of the two metals, has a specific gravity of 18; in what proportion are the metals mixed?

$$18 \backsim 19.258 \times 8.788 = 11.055$$
$$18 \backsim 8.788 \times 19.258 = 177.4, \text{ then}$$

$\overline{11.055 + 177.4} : 11.055 :: 18 = 1.056$ copper, } *Ans.*
$\overline{11.055 + 177.4} : 177.4 :: 18 = 16.944$ gold. }

Or, $18 - 1.056 = 16.944$ gold. Copper to gold as 1 to 16.04 +

To find the lifting power of a balloon.

RULE. — Multiply the capacity of the balloon, in feet, by the difference of weight between a cubic foot of atmosphere and a cubic foot of the gas used to inflate the balloon, and the product is the weight the balloon will raise.

EXAMPLE. — A balloon, whose diameter is 24 feet, is inflated with hydrogen; what weight will it raise?

Specific gravity of air is 1, weight of a cubic foot 527.04 grains; specific gravity of hydrogen is .0689.

$527.04 \times .0689 = 36.31$ grains = weight of 1 cubic foot of hydrogen.
$527.04 - 36.31 = 490.73$ grs. = dif. of weight of air and hydrogen.
$24^3 \times .5236 = 7238.24$ = capacity in cubic feet of balloon.
Then, $7238.24 \times 490.73 = 3552021$ grs. $= \frac{3552021}{7000} = 507\frac{4}{10}$ lbs. *Ans.*

To find the diameter of a balloon that shall be equal to the raising of a given weight.

The weight to be raised is $507\frac{4}{10}$ lbs.

$\overline{507.4 \times 7000} \div 490.73 = 7238.24$, and $7238.24 \div .5236 = \sqrt[3]{13824} = 24$ feet. *Ans.*

To find the thickness of a concave or hollow metallic ball or globe, that shall have a given buoyancy in a given liquid.

EXAMPLE. — A concave globe is to be made of brass, specific gravity 8.6, and its diameter is to be 12 inches; what must be its thickness that it may sink exactly to its centre in pure water?

Weight of a cubic inch of water .036169 lb.; of the brass .3112 lb.
Then, $\overline{12^3 \times .5236 \times .036169} \div 2 = 16.3625$ cubic inches of water to be displaced.

$16.3625 \div .3112 = 52.5787$ cubic inches of metal in the ball.

$\overline{12^2 \times 3.1416} = 452.39$ square inches of surface of the ball.

And, $52.5787 \div 452.39 = .1162 + = \frac{1}{9}$ inch thick, full. *Ans.*

To cut a square sheet of copper, tin, etc., so as to form a vessel of the greatest cubical capacity the sheet admits of.

RULE. — From each corner of the sheet, at right angles to the side, cut $\frac{1}{6}$ part of the length of the side, and turn up the sides till the corners meet.

Comparative Cohesive Force of Metals, Woods, and other substances, Wrought Iron (medium quality) being the unit of comparison, or 1; the cohesive force of which is 60000 lbs. per inch. transverse area.

Wrought iron,	1.00	Ash, white,	.23
" " wire,	1.71	" red,	.30
Copper, cast,	.40	Beech,	.19
" wire,	.76	Birch,	.25
Gold, cast,	.34	Box,	.33
" wire,	.51	Cedar,	.19
Iron, cast, (average).	.38	Chestnut, sweet,	.17
Lead, "	.015	Cypress,	.10
" milled,	.055	Elm,	.22
Platinum, wire,	.88	Locust,	.34
Silver, cast,	.66	Mahogany, best,	.36
" wire,	.68	Maple,	.18
Steel, soft,	2.00	Oak, Amer., white,	.19
" fine,	2.25	Pine, pitch,	.20
Tin, cast block,	.083	Sycamore,	.22
Zinc, "	.043	Walnut,	.30
" sheet,	.27	Willow,	.22
Brass, cast,	.75	Ivory,	.27
Gun metal,	.50	Whalebone,	.13
Gold 5, copper 1,	.83	Marble,	.15
Silver 5, " 1,	.80	Glass, plate,	.16
Brick,	.05	Hemp fibres, glued,	1.53
Slate,	.20		

The *strength* of white oak to cast iron, is as 2 to 9.
The *stiffness* of " " " " is as 1 to 13.

To determine the weight, or force, in pounds, necessary to tear asunder a bar, rod, or piece of any of the above named substances, of any given transverse area:

Rule. — Multiply the comparative cohesive force of the substance, as given in the table, by the cohesive force per square inch, area of cross section (60000 lbs.) of wrought iron, which gives the cohesive force of 1 square inch area of cross section of the substance whose power is sought to be ascertained, and the product of 1 square inch thus found, multiplied by area of cross section, in inches, of the rod, piece, or bar itself, gives the cohesive force thereof.

Alloys having a tenacity greater than the sum of their constituents.

Swedish copper 6 pts., Malacca tin 1; tenacity per sq. inch, 64000 lbs.
Chili copper 6 pts., Malacca tin 1; " " " 60000 "
Japan copper 5 pts., Banca tin 1; " " " 57000 "
Anglesea copper 6 pts., Cornish tin 1; " " " 41000 "

Common block-tin 4 pts., lead 1, zinc 1;	tenacity per sq.	in.,	13000	lbs.
Malacca tin 4 pts., regulus of antimony 1;	" "	"	12000	"
Block-tin 3 pts., lead 1 part;	" "	"	10000	"
Block-tin 8 pts., zinc 1 part;	" "	"	10200	"
Zinc 1 part, lead 1 part;	" "	"	4500	"

Alloys having a density greater than the mean of their constituents.

GOLD with *antimony*, *bismuth*, *cobalt*, *tin*, or *zinc*.
SILVER with *antimony*, *bismuth*, *lead*, *tin*, or *zinc*.
COPPER with *bismuth*, *palladium*, *tin*, or *zinc*.
LEAD with *antimony*.
PLATINUM with *molybdinum*.
PALLADIUM with *bismuth*.

Alloys having a density less than the mean of their constituents.

GOLD with *copper*, *iron*, *iridium*, *lead*, *nickel*, or *silver*.
SILVER with *copper* or *cobalt*.
IRON with *antimony*, *bismuth*, or *lead*.
TIN with *antimony*, *lead*, or *palladium*.
NICKEL with *arsenic*.
ZINC with *antimony*.

RELATIVE POWER OF DIFFERENT METALS TO CONDUCT ELECTRICITY,

(*the mass of each being equal.*)

Copper,	1000	Platinum,	188
Gold,	936	Iron,	158
Silver,	736	Tin,	155
Zinc,	285	Lead,	83

LINEAR DILATION OF SOLIDS BY HEAT.

Length which a bar heated to 212° has greater than when at the temperature of 32°.

Brass, cast,	.0018671	Iron, wrought,	.0012575
Copper,	.0017674	Lead,	.0028568
Fire brick,	.0004928	Marble,	.0011016
Glass,	.0008545	Platinum,	.0009342
Gold,	.0014880	Silver,	.0020205
Granite,	.0007894	Steel,	.0011898
Iron, cast,	.0011111	Zinc,	.0029420

NOTE.—To find the *surface* dilation of any particular article, double its linear dilation, and to find the dilation in *volume*, triple it. To find the elongation in linear inches per linear foot, of any particular article, multiply its respective linear dilation, as given in the TABLE, by 12.

MELTING POINT OF METALS AND OTHER BODIES.

Lime, *palladium*, *platinum*, *porcelain*, *rhodium*, *silex*, may be melted by means of strong lenses, or by the hydro-oxygen blowpipe. *Cobalt*, *manganese*, *plaster of Paris*, *pottery*, *iron*, *nickel*, &c., at from 2700° to 3250° Fahrenheit; others as follows: —

	Degrees Fah.		Degrees Fah.
Antimony,	809	Nitre,	660
Beeswax, bleached,	155	Silver,	1873
Bismuth,	506	Solder, common,	475
Brass,	1900	" plumbers',	360
Copper,	1996	Sugar,	400
Glass, flint,	1178	Sulphur,	226
Gold,	2016	Tallow,	127
Lead,	612	Tin,	442
Mercury,	—39	Zinc,	680

Cast iron thoroughly melts at	2786
Greatest heat of a smith's forge, (com.)	2346
Welding heat of iron,	1892
Iron red hot in twilight,	884
Lead 1, tin 1, bismuth 4, melts at	201
Lead 2, tin 3, bismuth 5, " "	212

RELATIVE POWER OF DIFFERENT BODIES TO RADIATE HEAT.

Water,	100	Lead, bright,	19
Copper,	12	Mercury,	20
Glass,	90	Paper, white,	100
Ice,	85	Silver,	12
India ink,	88	Tin, blackened,	100
Iron, polished,	15	" clean,	12
Lampblack,	100	" scraped,	16

NOTE. — The power of a body to *reflect* heat is inverse to its power of radiation.

BOILING POINT OF LIQUIDS.

Barometer at 30 in.

Acid, nitric,	253°	Oils, essential, avg.,	318°
" sulphuric,	600°	" turpentine,	316°
Alcohol, *anhyd.*,	168.5°	" linseed,	640°
" 90 per cent.,	174°	Phosphorus,	554°
Ether, sulph.,	97°	Sulphur,	560°
Mercury,	656°	Water,	212°

NOTE. — Barometer at 31 inches, water boils at 213°.57; at 29, it boils at 210°.38; at 28, it boils at 208°.69; at 27, it boils at 206°.85, and in *vacuo* it boils at 88°. No liquid, under pressure of the atmosphere alone, can be heated above its boiling point. At that point the steam emitted sustains the weight of the atmosphere.

FREEZING POINT OF LIQUIDS.

Acid, nitric,	—55°	Oil, linseed, avg.,	—11°
" sulphuric,	1°	Proof spirits,	—7°
Ether,	—47°	Spirits turpentine,	16°
Mercury,	—39°	Vinegar,	28°
Milk,	30°	Water,	32°
Oil, cinnamon,	30°	Wine, strong,	20°
" fennel,	14°	Rapeseed Oil,	25°
" olive,	36°		

NOTE. — Water expands in freezing .11, or $\frac{1}{9}$ its bulk.

EXPANSION OF FLUIDS BY BEING HEATED FROM 32° TO 212°, F.

Atmospheric air, $\frac{1}{480}$ per each degree, =	.375
Gases, all kinds, $\frac{1}{480}$ " " "	
Mercury, exposed,	.018
Muriatic acid, (sp. gr. 1.137,)	.060
Nitric acid, (sp. gr. 1.40,)	.110
Sulphuric acid, (sp. gr. 1.85,)	.060
" ether, — to its boiling point,	.070
Alcohol, (90 per cent.,) " "	.110
Oils, fixed,	.080
" turpentine,	.070
Water,	.046

RELATIVE POWER OF SUBSTANCES TO CONDUCT HEAT.

Gold,	1000	Zinc,	363
Silver,	973	Tin,	304
Copper,	898	Lead,	180
Platinum,	381	Porcelain,	12
Iron,	374	Fire brick,	11

NOTE. — Different woods have a conducting power in ratio to each other, as is their respective specific gravities, the more dense having the greater.

METALS IN ORDER OF DUCTILITY AND MALLEABILITY.

Ductility.	*Malleability.*
1. Platinum.	1. Gold.
2. Gold.	2. Silver.
3. Silver.	3. Copper.
4. Iron.	4. Tin.
5. Copper.	5. Platinum.
6. Zinc.	6. Lead.
7. Tin.	7. Zinc.
8. Lead.	8. Iron.

Quantity per cent. by weight of Nutritious Matter contained in different articles of Food.

Articles.	per ct.	Articles.	per ct.
Lentils,	94	Oats,	74
Peas,	93	Meats, avg.,	35
Beans,	92	Potatoes,	25
Corn, (maize,)	89	Beets,	14
Wheat,	85	Carrots,	10
Barley,	83	Cabbage,	7
Rice,	88	Greens,	6
Rye,	79	Turnips, white,	4

Specific gravity, and quantity per cent., by volume, of Absolute Alcohol contained, necessary to constitute the following named unadulterated articles.

Articles.	Sp. grav. 60°, b. 30.	Per cent. of Alcohol.
Absolute Alcohol, (*anhydrous*,)	.7939	100
Alcohol, highest by distillation,	.825	92.6
" commercial standard,	.8335	90
Proof Spirits, — standard,	.9254	54

Quantity per cent., by volume, (general average) of Absolute Alcohol contained in different pure or unadulterated Liquors, Wines, &c.

Liquors, &c.	per cent.	Wines.	per cent.
Rum,	50	Port,	22
Brandy,	50	Madeira,	20
Gin, Holland,	48	Sherry,	18
Whiskey, Scotch,	50	Lisbon,	17
" Irish,	50	Claret,	10
Cider, whole,	9	Malaga,	16
Ale,	8	Champagne,	14
Porter,	7	Burgundy,	12
Brown Stout,	6	Muscat,	17
Perry,	9	Currant,	19

Proof of Spirituous Liquors.

The weight, in air, of a cubic inch of *Proof Spirits*, at 60° F., is 233 grains; therefore, an inch cube of any heavy body, at that temperature, weighing 233 grains less in spirits than in air, shows the spirits in which it is weighed to be *proof*. If the body lose less of its weight, the spirit is above proof, — if more, it is below.

Comparative Weight of different kinds of Timber in a green and perfectly seasoned state.

Assuming the weight of each kind destitute of water to be 100, that of the same kind green is as follows: —

Ash,	153	Cedar,	148	Maple, red,	149
Beech,	174	Elm, swamp,	198	Oak, Am.,	151
Birch,	169	Fir, Amer.,	171	Pine, white,	152

NOTE. — Woods which have been felled, cleft and housed for 12 months, still retain from 20 to 25 per cent. of water. They therefore contain but from 75 to 80 per cent. of heating matter: and it will require from 25 to 29 per cent. the weight of such woods to dispel the water they contain. They are, therefore, less valuable by weight, as fuel, by this per cent., than woods perfectly free from moisture. They never, however, contain, exposed to an ordinary atmosphere, less than 10 per cent. of water, however long kept; and even though rendered anhydrous by a strong heat, they again imbibe, on exposure to the atmosphere, from 10 to 12 per cent. of dampness.

Relative power of different seasoned Woods, Coals, &c., as fuel, to produce heat, — the Woods supposed to be seasoned to mean dryness, (77½ per cent.,) and the other articles to contain but their usual quantity of moisture.

	Ratio of Heating Power per equal Bulk.	Weight.
Hickory, shell-bark,	1.00	1.00
" red-heart,	.81	.99
Walnut, com.	.95	.98
Beech, red,	.74	.99
Chestnut,	.49	.98
Elm, white,	.58	.98
Maple, hard,	.66	.98
Oak, white,	.81	.99
" red,	.69	.99
Pine, white,	.42	1.01
" yellow,	.48	1.03
Birch, black,	.63	.99
" white,	.48	.99
Coal, Cumberland, (bit.)	2.56	2.28
" Lackawanna (anth.)	2.28	2.22
" Lehigh, "	2.39	2.03
" Newcastle, (bit.)	2.10	1.96
" Pictou, (bit.)	2.21	1.91
" Pittsburgh, (bit.)	1.78	1.82
" Peach Mountain, (anth.)	2.69	2.29
Charcoal,	1.14	2.53
Coke, Virginia, natural,	1.89	2.12
" Cumberland,	1.31	2.25
Peat, ordinary,		62
Alcohol, common,		2.02
Beeswax, yellow,		2.90
Tallow,		3.10

NOTE. — By help of the preceding table, the price of either one article being known, the relative or par value of either other, as fuel, may be readily ascertained: — EXAMPLE:

Maple (66) : $5.00 : : Pine (42) : $3.18.

ILLUMINATION — ARTIFICIAL.

The following TABLE shows: —

1. The materials and methods of using — column *Materials.*

2. The comparative maximum intensity of light afforded by each material, used or consumed as indicated, — column *Intensities.*

3. The weight, in grains, of material consumed per hour, by each method respectively, in producing its respective light, or light of intensity ascribed — column *Weight.*

4. The *ratio* of weight required of each material, under each special method of consumption, for the production of equal lights in equal times — column *Ratios.*

Materials.		Intens.	Weight.	Ratios.
Camphene,	Paragon Lamp,	16.	853	1.
Sperm Oil,	Parker's heating Lamp,	11.	696	1.19
" "	Mech. or Carcel "	10.	815	1.53
" "	French annular "	5.	543	2.04
" "	Common hand "	1.	112	2.10
Whale "	p'f'd., P's heating "	9.	780	1.63
Wax Candles,	3's or 4's, 15 in. or 12 in.,	1.	125	2.35
" "	6's, 9 in.,	.92	122	2.50
Sperm "	4's, 13½ in.,	1.	142	2.66
Stearine "	4's, 13½ in.,	1.	168	3.15
Tallow, "	dipped, 10's,	.70	150	4.02
" "	mould, 10's,	.66	132	3.75
" "	" 8's,	.57	132	4.35
" "	" 6's,	.79	163	3.87
" "	" 4's, 13¾ in.,	1.	186	3.49
" *Coal Gas,*" intensity being		1.	411	

NOTE. — The consumption of 1.43 cubic feet of gas per hour, gives a light equal to one wax candle, — the consumption of 1.96 cubic feet per hour, a light equal to four wax candles, and the consumption of 3 cubic feet per hour, a light equal to ten wax candles. A cubic foot of gas weighs 286 grains.

The average yield of bi-carbureted hydrogen — Olefiant gas — Coal Gas, obtained from the following articles, is as annexed.

1 lb. Bituminous Coal,	4½ cubic feet.
1 lb. Oil, or Oleine,	15 " "
1 lb. Tar,	12 " "
1 lb. Rosin, or Pitch,	10 " "

A pound of good Lancashire cannel coal, or of good Scotch cannel, will yield, on an average, 5.95 cubic feet of good illuminating gas.

From the English Boghead cannel, by White's hydro-carbon process, 17 cubic feet to the pound are commonly obtained.

A pipe whose interior diameter is ½ inch, will supply gas equal in illuminating power to 20 wax candles.

The sp. gr. of coal-gas from cannel coal ranges from 0.640 to 0.370, while that from good working bituminous coal in general ranges from 0.420 to 0.370.

Results of Experiments by Mr. Clegg of London, relative to the conveyance of coal-gas through pipes of different lengths, diameters, &c.

Diameter of pipe in inches.	Length of pipe in yards.	Pressure in inches of water.	Specific gravity, air as 1.	Quantity in cubic feet discharged per hour.
0.5	10	1.25	0.4	120
0.5	59	1.25	0.4	60
0.62	41	1.38	0.559	99
0.62	62	1.34	0.559	83
0.62	93	1.34	0.559	74
0.62	119	1.34	0.559	57
0.62	138	1.34	0.559	53
2.	25	0.5	0.528	1630
10.	100	3.	0.4	120,000
10.	1760	3.	0.4	30,000
18.	1760	1.	0.4	66,000
26.	4300	0.475	0.42	80,000
26.	3130	0.8	0.42	103,000
26.	4300	2.25	0.42	175,000

From these experiments and others, Mr. Clegg derives the data for the following general approximate rules, viz.:

$Q = 1350d^2\sqrt{\frac{hd}{s(l+d)}}$; or, when the length of the pipe is above 400 or 500 times its diameter; or, in any case, if the pressure be measured in the pipe instead of in the gas-holder, $Q = 1350d^2\sqrt{\frac{hd}{ls}}$; in which Q represents the quantity of gas in cubic feet passed per hour, l the length of the pipe in yards, d the diameter of the pipe in inches, h the pressure in inches of water, and s the specific gravity of the gas.

Therefore, ordinarily for short mains of large diameters, or when the quotient of $36l \div d$ is less than 400,

$$l = \frac{1350^2 d^5 h}{Q^2 s} - d;\ h = \frac{Q^2 s(l+d)}{1350^2 d^5};\ \text{and } l + d = \frac{1350^2 d^5 h}{Q^2 s}.$$

And for long pipes of small diameters, or when l and $l + d$ are practically equal.

$$l = \frac{1350^2 d^5 h}{Q^2 s};\ h = \frac{Q^2 l s}{1350^2 d^5};\ \text{and } d = \sqrt[5]{\frac{Q^2 l s}{1350^2 h}}.$$

Now, in these formulas for Q, by Mr. Clegg, the probable retardation of the flow of the gas due to the friction of its particles in the pipes has been taken into account; and the results by the formulas agree as closely as could be expected with the given experiments, although the former average about 7 per cent. more than the latter.

The pipes are here supposed to be straight, and to lie horizontal, or equal in effect to that condition, and the gas is supposed to be delivered without force at the discharging end.

Example. — What pressure in inches of water is required to convey 852 cubic feet of gas per hour, of specific gravity 0.398, through a pipe 4 inches in diameter and 6 miles in length?

$$\frac{852^2 \times 10560 \times .398}{1350^2 \times 4^5} = 1.6348 \text{ inches. } \textit{Ans.}$$

Note. — The illuminating power of coal-gas is nearly as its specific gravity, the more dense being the better.

THERMOMETERS.

	Boiling point.	Freezing point.
Fahrenheit's, ...	212°	32°
Reaumur's, ...	80°	0°
Centigrade, ...	100°	0°

To reduce Reaumur to Fahrenheit.

When it is desired to reduce the +°, (degrees above the zero) : —

Rule. — Multiply the degrees Reaumur, by 2.25, and add 32° to the product; the sum will be the degrees Fahrenheit.

When it is desired to reduce the —°, (degrees below the zero) : —

Rule. — Multiply the —° Reaumur by 2.25, and subtract the product from 32°; the difference will be the degrees Fahrenheit.

Example. — The degrees R. are 40; required the equivalent degrees F.

$$40 \times 2.25 = 90 + 32 = 122°. \quad \textit{Ans.}$$

Example. — The degrees below 0, R., are 10; what are the corresponding degrees F.?

$$10 \times 2.25 = 22.5, \text{ and } 32 - 22.5 = 9\tfrac{1}{2}°. \quad \textit{Ans.}$$

Example. — The degrees below 0, R., are 16; what point on the scale F. corresponds thereto?

$$16 \times 2.25 = 36, \text{ and } 32 - 36 = -4: 4° \text{ below } 0. \quad \textit{Ans.}$$

To reduce the Centigrade to Fahrenheit.

Rule. — Multiply the degrees C. by 1.8, and in all other respects proceed as directed for Reaumur, above.

Note. — The *zero* of Wedgewood's pyrometer is fixed at the temperature of iron red-hot in daylight, = 1077° F., and each degree W. equals 130° F. The instrument is not considered reliable, and is but little used

HORSE POWER.

A HORSE-POWER, in machinery, as a measure of force, is estimated equal to the raising of 33000 lbs. over a single pulley one foot a minute, = 550 lbs. raised one foot a second, = 1000 lbs. raised 33 feet a minute.

ANIMAL POWER.

A man of ordinary strength is supposed capable of exerting a force of 30 lbs. for 10 hours in a day, at a velocity of 2½ feet a second, = 75 lbs. raised 1 foot a second.

The ordinary working power of a horse is calculated at 750 lbs. for 8 hours in a day, at a velocity of 2 feet a second, = 375 lbs. raised 1 foot a second, = 5 times the effective power of a man during associated labor, and 4 times his power per day; and as machinery may be supposed to work continually, = a trifle less than 23 per cent. per day of a machine horse-power.

STEAM.

Table exhibiting the expansive force and various conditions of steam under different degrees of temperature.

Degrees of heat.	Pressure in atmospheres.	Density. Water as 1.	Volume. Water as 1.	Spec. gravity. Air as 1.	Weight of a cubic foot in grains.
212	1	.00059	1694	.484	254
250.5	2	.00110	909	.915	483
276	3	.00160	625	1.330	700
293.8	4	.00210	476	1.728	910
308	5	.00258	387	2.120	1110
359	10	.00492	203	3.970	2100
418.5	20	.00973	106	7.440	3940

[An atmosphere is $14\frac{7}{10}$ lbs. to the square inch.]

NOTE. — By the above table it is seen that any given quantity of steam having a temperature of 212° F., occupies a space, under the ordinary pressure of the atmosphere, 1694 times greater than it occupied when as water in a natural state. It exerts a mechanical force, consequently, = 1694 times the weight or force of the atmosphere resting on the bulk from which it was generated, or resting on 1–1694th of the space it occupies. A force, if we consider the volume as so many cubic inches, equal to the raising of 2087 lbs. 12 inches high, by a quantity of steam less than a cubic foot, heated only to the temperature of boiling water, and weighing but 248 grains, and that, too, the product of a single cubic inch of water.

The mean pressure of the atmosphere at the earth's surface is equal to the weight of a column of mercury 29.9 inches in height, or to a column of water 33.87 feet in height, = 2116.8 lbs. per square foot, or

14.7 lbs. per square inch. Its density above the earth is uniformly less as its altitude is greater, and its extent is not above 50 miles — its mean altitude is about 45 miles; at 44 miles it ceases to reflect light. Were it of uniform density throughout, and of that at the surface, its altitude would be but $5\frac{1}{4}$ miles. Its weight is to pure water of equal temperature and volume, as 1 to 829. It revolves with the earth, and its average humidity, at 40° of latitude, is 4 grains per cubic foot. Its weight at 60°, b. 30, compared with an equal bulk of pure water at 40°, b. 30, is as 1 to 830.1.

VELOCITY AND FORCE OF WIND.

Appellations.	Mean velocity in Miles per hour.	Mean velocity in Feet per second.	Force in lbs. per square foot.
Just perceptible, . .	$2\frac{1}{2}$	$3\frac{2}{3}$	.032
Gentle, pleasant wind, . .	$4\frac{1}{2}$	$6\frac{2}{3}$	.101
Pleasant, brisk gale, . .	$12\frac{1}{2}$	$18\frac{1}{3}$	.80
Very brisk, " . .	$22\frac{1}{2}$	33	2.52
High wind, . . .	$32\frac{1}{2}$	$47\frac{2}{3}$	5.23
Very high wind, . . .	$42\frac{1}{2}$	$62\frac{1}{3}$	8.92
Storm, or tempest, . .	50	$73\frac{1}{3}$	12.30
Great storm, . . .	60	88	17.71
Hurricane, . . .	80	$117\frac{1}{3}$	31.49
Tornado, moving buildings, &c.,	100	146.7	49.20

The curvature of the earth is 6.99 inches (.5825 foot) in a single statute mile, or 8.05 inches in a geographical mile, and is as the square of the distance for any distance greater or less, or space between two levels; thus, for three statute miles it is

$$1 : 3^2 :: 6.99 : 5\tfrac{1}{4} \text{ feet, nearly.}$$

The horizontal refraction is $\frac{1}{13}$.

Degrees of longitude are to each other in length, as the cosines of their latitudes. At the equator a degree of longitude is 60 geographical miles in length, at 90° of latitude it is 0; consequently, a degree of longitude at

5° . .	= 59.77 miles.	40° . .	= 45.96 miles.
10° . .	= 59.09 "	50° . .	= 38.57 "
20° . .	= 56.38 "	70° . .	= 20.52 "
30° . .	= 51.96 "	85° . .	= 5.23 "

Time is to longitude 4 minutes to a degree, — faster, east of any given point; slower, west.

The mean velocity of sound at the temperature of 33° is 1100 feet a second. Its velocity is increased $\frac{1}{2}$ a foot a second for every degree above 33°, and decreased $\frac{1}{2}$ a foot a second for every degree below 33 .

In water, sound passes at the rate of 4,708 feet a second.

Light travels at the rate of 192,000 miles per second.

GRAVITATION.

GRAVITY, or GRAVITATION, is a property of all bodies, by which they mutually attract each other proportionally to their masses, and inversely as the square of the distance of their centres apart. Practically, therefore, with reference to our Earth and the bodies upon or near its surface, gravity is a constant force centred at the Earth's centre, and is there continually operating to draw all bodies with a uniformly accelerating velocity to that point, and through very nearly equal spaces, in equal intervals of time from rest, at all localities.

Putting R' to represent the Equatorial radius of the earth, and r to represent the Polar, and making $R' = 3962.5$ statute miles, and $r = 3949.5$, which is nearly in accordance with the mean of the most reliable measurements of the arcs of a degree of latitude at different localities, we have $\varepsilon^2 = (R'^2 - r^2) \div R'^2 = .006550751$, the square of the ellipticity of the earth, and $R = 2R' \div (2 + \varepsilon^2 \sin^2 l)$, the radius at any given latitude l.

And since the initial velocity due to gravity at the level of the sea at the Equator is $G = 32.0741$ feet per second, or, in other words, since a body falling in vacuo at the equator, at the level of the sea, describes a space of 16.03705 feet in the first second of time from rest, we have $g = [R' \sqrt{G} \div R]^2$, the initial velocity at the level of the sea at any given radius R; or $g = (22441.2 \div R)^2$.

And finally $g = \left(\frac{22441.2}{R}\right)^2 \times \left(1 - \frac{2h}{5280R}\right)$ at any given radius R, at any given altitude, h, in feet, above the level of the sea.

NOTE. — When l, reckoned from the equator, is higher than 45°, $\sin^2 l = \cos^2 (90 - l)$.

The *momentum*, or force, with which a falling body strikes, is the product of its weight and velocity (the weight multiplied by the square root of the product of the space fallen through and 64.33, or 4 times $16\frac{1}{12}$); thus, 100 lbs., falling 50 feet, will strike with a force,

$$50 \times 64.333 = \sqrt{3216.66} = 56.71 \times 100 = 5671 \text{ lbs.}$$

An entire revolution of the earth, from west to east, is performed in 23 hours, 56 minutes, and 4 seconds. A solar year = 365 days, 5 hours, 48 minutes, 57 seconds.

The area of the earth is nearly 197,000,000 square miles. Its crust is supposed to be about 30 miles in thickness, and its mean density 5 times that of water. About $\frac{3}{4}$ of its area, or 150,000,000 square miles, is covered by water. The portions of land in the several

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divisions, in square miles, are, in round numbers, as follows, viz: —

Asia, . .	16,300,000	Europe, . .	3,700,000
Africa, . .	11,000,000	Australia, . .	3,000,000
America, . .	14,500,000		

America is 9000 miles long, or $\frac{36}{100}$ the circumference of the earth.

The population of the globe is about 1,000,000,000, of which there are, in

Asia, . .	456,000,000	Africa, . .	62,000,000
Europe, . .	258,000,000	America, . .	55,000,000

CHEMICAL ELEMENTS.

The chemical elements — simple substances in nature — as far as has been determined, are 62 in number: 13 non-metallic and 49 metallic.

Of the non-metallic, 5 — *bromine*, *chlorine*, *fluorine*, *iodine*, and *oxygen*, (formerly termed "*supporters of combustion*,") have an intense affinity for all the others, which they penetrate, corrode, and apparently consume, always with the production, to some extent, of light and heat. They are all non-conductors of electricity and negative electrics.

The remaining 8 — *hydrogen*, *nitrogen* or *azote*, *carbon*, *boron*, *silicon*, *phosphorus*, *selenium*, and *sulphur*, are eminently susceptible of the impressions of the preceding five; when acted upon by either of them to a certain extent, light and heat are manifestly evolved, and they are thereby converted into incombustible compounds.

Of the metals, 7 — *potassium*, *sodium*, *calcium*, *barytium*, *lithium*, *strontium*, and *magnesia*, by the action of oxygen, are converted into bodies possessed of *alkaline* properties.

Seven of them — *glucinum*, *erbium*, *terbium*, *yttrium*, *allumium*, *zirconium*, and *thorium*, — by the action of oxygen, are converted into the *earths* proper.

In short, all the metals are acted upon by oxygen, as also by most or all of the non-metallic family. The compounds thus formed are *alkaline*, *saline*, or *acidulous*, or an *alkali*, a *salt*, or an *acid*, according to the nature of the materials and the extent of combination.

Metals combine with each other, forming *alloys*. If one of the metals in combination is mercury, the compound is called an *amalgam*.

Silicon is the base of the mineral world, and *carbon* of the organized.

For a very general list of the metals, see TABLE OF SPECIFIC GRAVITIES.

TABLE

Exhibiting the Elementary Constituents and per cent. by weight of each, in 100 parts of different compounds.

Compounds.	Constituents and per cent. Hydrogen.	Oxygen.	Nitrogen.	Carbon.
Atmospheric air, . . . *a*		19.96	79.84	
Water, pure, . . .	11.1	88.9		
Alcohol, anhydrous, . .	12.9	34.44		52.66
Olive oil, ...	13.4	9.4		77.2
Sperm "	10.97	10.13		78.9
Castor "	10.3	15.7		74.00
Stearine, (solid of fats,) .	11.23	6.3	0.30	82.17
Oleine, (liquid of fats,) .	11.54	12.07	0.35	76.03
Linseed oil, ...	11.35	12.64		76.01
Oil of turpentine, . .	11.74	3.66		84.6
"*Camphene*," (pure spts. turp.)	11.5			88.5
Caoutchouc, (gum elastic,) .	10.			90.
Camphor, ...	11.14	11.48		77.38
Copal, resin, . . .	9.	11.1		79.9
Guaiac, resin, . . .	7.05	25.07		67.88
Wax, yellow, . . .	11.37	7.94		80.69
Coals, cannel, . . .	3.93	21.05	2.80	72.22
" Cumberland, . .	3.02	14.42	2.56	80.
" Anthracite, . . *b*				93.
Charcoal, ...				97.
Diamond, ...				100.
Oak wood, dry, . . . *c*	5.69	41.78		52.53
Beech " " . . .	5.82	42.73		51.45
Acetic acid, dry, . .	5.82	46.64		47.54
Citric " crystals, . .	4.5	59.7		35.8
Oxalic " dry, . .		79.67		20.33
Malic, " crystals, . .	3.51	55.02		41.47
Tartaric " dry, . .	3.	60.2		36.80
Formic " " . .	2.68	64.78		32.54
Tannin, tannic acid, solid, .	4.20	44.24		51.56
Nitric acid, dry, . . .		73.85	26.15	
Nitrous " anhydrous, liquid,		61 32	30.68	
Ammoniacal gas, . .	17.47		82.53	
Carbonic acid " . . .		72.32		27.68
Carb. hydrogen gas, . .	24.51			75.49
Bi-carb. hyd., olefient gas, .	14.05			85.95
Cyanogen " .			53.8	46.2
Nitric oxyde " .		53.	47.00	
Nitrous " " .		36.36	63.64	
Ether, sulphuric, . . .	13.85	21.24		65.05
Creosote, ...	7.8	16.		76.2

Compounds.		Constituents and per cent. Hydrogen.	Oxygen.	Azote.	Carbon.
Morphia,		6.37	16.29	5.	72.34
Quina, — quinine,		7.52	8.61	8.11	75.76
Veratrine,		8.55	19.61	5.05	66.79
Indigo,		4.38	14.25	10.	71.37
Silk, pure white,		3.94	34.04	11.33	50.69
Starch, — farina, dextrine,		6.8	49.7		43.5
Sugar,		6.29	50.33		43.38
Gluten,		7.8	22.	14.5	55.7
Wheat,	*c*	6.	44.4	2.4	47.02
Rye,		5.7	45.3	1.7	47.03
Oats,		6.6	38.2	2.3	52.9
Potatoes,		6.1	46.4	1.06	45.9
Peas,		6.4	41.3	4.3	48.
Beet root,		6.2	46.3	1.8	45.7
Turnips,		6.	45.9	1.8	46.3
Fibrin,	*d*	7.03	20.30	19.31	53.36
Gelatin,	*d*	7.91	27.21	17.	47.88
Albumen,	*d*	7.54	23.88	15.70	52.88

Muriatic acid gas, — Hydrogen 5.53 + 94.47 chlorine.
Sulphuric acid, dry, — Oxygen 79.67 + 20.33 sulphur.
Silicic acid — Silica, dry, — Oxygen 51.96 + 48.04 silicon.
Boracic acid — Borax, dry, — " 68.81 + 31.19 boron.

a. The atmosphere, in addition to its constituents as given in the table, contains, besides a small quantity of vapor, from 1 to 3 parts in a thousand of carbonic acid gas, and a trace merely of ammoniacal gas.

b. Anthracite coal, charcoal, plumbago, coke, &c., have no other constituent than carbon; they are combined, to a small extent, with foreign matters, such as iron, silica, sulphur, alumina, &c.

c. The constituents of woods, grains, &c., are given per cent., without regard to the foreign matters (*metallic*) which they contain. In *oak*, *chestnut*, and *Norway pine*, the ashes amount to about $\frac{4}{10}$ of 1 per cent., and in *ash* and *maple* to $\frac{7}{10}$ of 1. In anthracite coals, at an average, they are about 7 per cent.

d. Fibrin, *Gelatin*, *Albumen* — Proximate animal constituents — Nutritious properties of animal matter.

Fibrin is the basis of the muscle (lean meat) of all animals, and is also a large constituent of the blood.

Gelatin exists largely in the skin, cartilages, ligaments, tendons and bones of animals. It also exists in the muscles and the membranes.

Albumen exists in the skin, glands and vessels, and in the serum of the blood. It constitutes nearly the whole of the white of an egg.

THE RELATIVE QUANTITIES BY VOLUME of the several gases going to constitute any particular compound, are readily ascertained by help of their respective specific gravities, compared with their relative weights, as given per cent. in the preceding table:—thus, the sp. gr. of hydrogen is .0689, and that of oxygen 1.1025, and 1.1025 ÷ .0689 = 16; showing the weight of the latter to be 16 times that of the former per equal volumes, or, relatively, as 16 to 1. The per cent. by weight, as shown by the table, in which these two gases combine to form water, for instance, is 11.1 and 88.9; or 11.1 of hydrogen and 88.9 of oxygen in 100 of the compound; or as 88.9 ÷ 11.1,—as 8 to 1: 16 ÷ 8 = 2: two volumes, therefore, of the lighter gas (hydrogen) combine with one of oxygen to form water. Water, consequently, is a Protoxide of Hydrogen.

Upon the principle of ATOMIC WEIGHTS,—relative quantities, by weight, in which the elements combine in forming compounds, based upon the standard already shown,—we have, for water, $H^1 + O^8 =$ Aq. 9. That is, an atom of hydrogen is represented by 1, an atom of oxygen by 8, and an atom of water by 9.

By the same rule as the preceding, the constituents of atmospheric air are found to be to each other, in volume, as 4 to 1; four volumes of nitrogen and one volume of oxygen make one volume of atmospheric air. The weight of nitrogen to hydrogen, per equal volumes, is as .972 to .0689, as 14.11 to 1. Atomically, therefore, it is as 7.055 to 1; hence, we have $N^4 + O = 36.22$, the atomic weight of atmospheric air.

The vast condensation of the gases which takes place, in some instances, in forming compounds, may be conceived of, and the process for determining the same exhibited by a single illustration. We will take, for example, water. A single cubic inch of distilled water, at 60°, weighs 252.48 grains. Its weight is to that of dry atmosphere, at the same temperature, as 827.8 to 1. A cubic inch of dry atmosphere, therefore, at that density, weighs .305 of a grain. Hydrogen, we find by the table of Specific Gravities, weighs .0689 as much as atmosphere, and oxygen 1.1025 as much. A cubic inch of hydrogen, therefore, weighs .0689 × .305 = .0210145 of a grain, and a cubic inch of oxygen 1.1025 × .305, = .3362625 of a grain. The constituents of water by volume are 2 of the first mentioned gas to 1 of the latter; and .0210145 × 2 + .3362625 = .3782915 of a grain, = weight of three cubic inches of the uncondensed compound, ⅓ of which, .1260972 of a grain, is the weight of a volume 1 cubic inch.

As the weight of a given volume of the uncondensed compound, is to the weight of an equal volume of the condensed compound, so are their respective volumes, inversely: then—

.1260972 : 252.48 :: 1 : 2002.26, the number of cubic inches of the two gases condensed into 1 inch to form water; a condensation of 2001 times. Of this volume of gases, ⅔, or 1334.84 cubic inches, is hydrogen; the remaining third, 667.42 cubic inches, is oxygen.

The foregoing method, though strictly correct, does not exhibit in a general way the most expeditious for solving questions of that nature, the condensation which takes place in the gases on being converted into solids, or dense compounds. It was resorted to, in part, as a means through which to exhibit principles and proportions pertaining thereto.

As before; one cubic inch of water weighs 252.48 grains, $\frac{1}{9}$ of which, or 28.05+ grains, is hydrogen, and $\frac{8}{9}$, or 224.43— grains, is oxygen. The volume of 1 grain of oxygen is 2.97+ cubic inches, and the volume of hydrogen is 16 times as much, or 47.58+ cubic inches. Therefore, $28.05 \times 47.58 = 1334.62$, and $224.43 \times 2.97 = 665.56$, $= 2001.18$, condensation, as before.

Properties of the SIMPLE SUBSTANCES, *and some of their compounds, not given in the foregoing.*

BROMINE, — at common temperatures, a deep reddish-brown volatile liquid; taste caustic; odor rank; boils at 116°; congeals at 4°; exists in sea-water, in many salt and mineral springs, and in most marine plants; action upon the animal system very energetic and poisonous — a single drop placed upon the beak of a bird destroys the bird almost instantly. A lighted taper, enveloped in its fumes, burns with a flame green at the base and red at the top; powdered tin or antimony brought in contact is instantly inflamed; potash is exploded with violence.

CHLORINE, — a greenish-yellow, dense gas; taste astringent; odor pungent and disagreeable; by a pressure of 60 lbs. to the square inch is reduced to a liquid, and thence, by a reduction of the temperature below 32°, into a solid. It exists largely in sea-water — is a constituent of common salt, and forms compounds with many minerals; is deleterious, irritating to the lungs, and corrosive; has eminent bleaching properties, and is the greatest disinfecting agent known; a lighted taper immersed in it burns with a red flame; pulverized antimony is inflamed on coming in contact, so is linen saturated with oil of turpentine; phosphorus is ignited by it, and burns, while immersed, with a pale-green flame; with hydrogen, mixed measure for measure, it is highly explosive and dangerous.

FLUORINE, — a gas, similar to chlorine, — exists abundantly in *fluor-spar*.

OXYGEN, — a transparent, colorless, tasteless, inodorous, innoxious gas; supports respiration and combustion, but will not sustain life for any length of time, if breathed in a pure state. It is by far the most abundant substance in existence; constitutes $\frac{1}{5}$ of the atmosphere;

$\frac{8}{9}$ of water; and nearly the whole crust of the earth is oxidized substances. For further combinations and properties, see tables of *Elementary Constituents* and *Chemical Elements*.

Iodine, — at common temperatures, a soft, pliable, opaque, bluish-black solid; taste acrid; odor pungent and unpleasant; fuses at 225°; boils at 347°; its vapor is of a beautiful violet color; it inflames phosphorus, and is an energetic poison; exists mainly in sea-weeds and sponges.

Hydrogen, — a transparent, colorless, tasteless, inodorous, innoxious gas; if pure, will not support respiration; if mixed with oxygen, produces a profound sleep; exists largely in water; is the basis of most liquids, and is by far the lightest substance known; burns in the atmosphere with a pale, bluish light; mixed with common air, 1 measure to 3, it is explosive; mixed with oxygen, 2 measures to 1, it is violently so.

Nitrogen, or *Azote*, — a transparent, colorless, tasteless, inodorous gas; will not support respiration or combustion, if pure; exists largely as a constituent of the atmosphere — in animals, and in fungous plants; is evolved from some hot springs; in connection with some bodies, appears combustible.

Carbon, — the *diamond* is the only pure carbon in existence; pure carbon cannot be formed by art; *charcoal* is 97 per cent. carbon; *plumbago*, 95; *anthracite*, 93. Carbon is supposed by some to be the *hardest* substance in nature. A piece of charcoal will scratch glass; but it is doubtful if this is not due to the form of its crystals, rather than to the first mentioned quality. It is doubtless the most *durable*. For combinations, &c., see table.

Boron, — a tasteless, inodorous, dark olive-colored solid.

Silicon, — a tasteless, inodorous solid, of a dark-brown color; exists largely in soils, quartz, flint, rock-crystal, &c.; burns readily in air — vividly in oxygen gas; explodes with soda, potassa, barryta.

Phosphorus, — a transparent, nearly colorless solid, of a wax-like texture; fuses at 108°, and at 550° is converted into a vapor; exists mainly in bones — most abundant in those of man — is poisonous; at common temperatures it is luminous in the dark, and by friction is instantly ignited, burning with an intense, hot, white flame; must be kept immersed in water.

Selenium, — a tasteless, inodorous, opaque, brittle, lead-colored solid, in the mass; in powder, a deep-red color; becomes fluid at 216°, boils at 650°; vapor, a deep yellow; exists but sparingly, mainly in combination with volcanic matter; is found in small quantities combined with the ores of lead, silver, copper, mercury.

Ammoniacal gas, — $N + H^3$; transparent, colorless, highly pungent and stimulating; alkaline; is converted into a transparent liquid by a pressure of 6.5 atmospheres, at 50°; does not support respiration; is inflammable.

Carbonic acid gas, — $C + O^2$; transparent, colorless, inodorous, dense; is converted into a liquid by a pressure of 36 atmospheres; exists extensively in nature, in mines, deep wells, pits; is evolved from the earth, from ordinary combustion, especially from the combustion of charcoal, and from many mineral springs; is expired by man and animals; forms 44 per cent. of the *carbonate of lime* called marble; the brisk, sparkling appearance of soda-water, and most mineral waters, is due to its presence. It is neither a combustible nor a supporter of combustion; and, when mixed with the atmosphere to an extent in which a candle will not burn, is destructive of life. Being heavier than atmosphere, it may be drawn up from wells in large open buckets; or it may be expelled by exploding gunpowder near the bottom. Large quantities of water thrown in will absorb it.

The above gas is expired by man to the extent of 1632 cubic inches per hour; it is generated by the burning of a wax candle to the extent of 800 cubic inches per hour; and, by the burning of "*Camphene*," (in the production of light equal to that afforded by 1 wax candle,) to the extent of 875 cubic inches per hour. Two burning candles, therefore, vitiate the air to about the same extent as 1 person.

Carbonic oxide gas, — $C + O$; transparent, colorless, insipid; odor offensive; does not support combustion; an animal confined in it soon dies; is highly inflammable, burning with a pale blue flame; mixed with oxygen, 1 to 2, is explosive — with atmosphere, even in small quantity, is productive of giddiness and fainting.

Carbureted hydrogen gas, — $C + H^2$; transparent, colorless, tasteless, nearly inodorous; exists in marshes and stagnant pools — is there formed by the decomposition of vegetable matter; extinguishes all burning bodies, but at the same time is itself highly combustible, burning with a bright but yellowish flame; it is destructive to life, if respired.

Cyanogen — Bicarburet of Nitrogen — a *gas*, — $N + C^2$; trans parent, colorless, highly pungent and irritating; under a pressure of

3.6 atmospheres, becomes a limpid liquid; burns with a beautiful purple flame.

Hydrochloric acid gas — Muriatic acid gas, — H + Cl. (chlorine); transparent, colorless, pungent, acrid, suffocating; strong acid taste.

Nitrous oxide gas — Protoxide of Nitrogen, "laughing gas," — N + O; transparent, colorless, inodorous; taste sweetish; powerful stimulant, when breathed, exciting both to mental and muscular action; can support respiration but from 3 to 4 minutes; is often pernicious in its effects.

Nitric oxide gas — Binoxide of Nitrogen, — $N + O^2$; transparent, colorless; wholly irrespirable; lighted charcoal and phosphorus burn in it with increased brilliancy.

Olefiant gas — Bicarbureted hydrogen gas — "coal gas," — $C^2 + H^2$; transparent, colorless, tasteless, nearly inodorous, when pure; does not support respiration or combustion; a lighted taper immersed in it is immediately extinguished. It burns with a strong, clear, white light; mixed with oxygen, in the proportion of 1 volume to 3, it is highly explosive and dangerous.

Phosphureted hydrogen gas, — $P + H^3$; colorless; odor highly offensive; taste bitter; exists in the vicinity of swamps, marshes, and grave-yards; is formed by the decomposition of bones, mainly; is highly inflammable; takes fire spontaneously on coming in contact with the atmosphere; mixed with pure oxygen, it explodes. It is the veritable "Will o' the wisp."

Sulphureted hydrogen gas — Hydrosulphuric acid gas, — S + H; transparent, colorless; taste exceedingly nauseous; odor offensive and disgusting; is furnished by the sulphurets of the metals in general — also by filthy sewers and putrescent eggs. It is very destructive to life; placed on the skin of animals, it proves fatal. It burns with a pale blue flame, and, mixed with pure oxygen, it is explosive.

Hydrocyanic acid — Prussic acid, — $N + C^2 + H$; a colorless, limpid, highly volatile liquid; odor strong, but agreeable — similar to that of peach-blossoms; it boils at 79° and congeals at 0; exists in laurel, the bitter almond, peach and peach kernel. It is a most virulent poison, — a drop placed upon a man's arm caused death in a few minutes. A cat, or dog, punctured in the tongue with a needle fresh dipped in it, is almost instantly deprived of life.

Hydrofluoric acid, — F + H; a colorless liquid, in well stopped lead or silver bottles, at any temperature between 32° and 59°. It is obtained by the action of sulphuric acid on fluor-spar. It readily acts upon and is used for etching on glass. It is the most destructive to animal matter of any known substance.

Nitrohydrochloric acid — "*aqua regia*," — (1 part nitric acid and 4 parts muriatic acid, by measure;) — a solvent for gold. The best solvent for gold is a solution of sal ammoniac in nitric acid.

Nitrosulphuric acid, — (1 part nitric acid and 10 parts sulphuric acid, by measure) — a solvent for silver; scarcely acts upon gold, iron, copper, or lead, unless diluted with water; is used for separating the silver from old plated ware, &c. The best solvent for silver, and one which will not act in the least upon gold, copper, iron, or lead, is a solution of 1 part of nitre in 10 parts of concentrated sulphuric acid, by weight, heated to 160°. This mixture will dissolve about $\frac{1}{6}$ its weight of silver. The silver may be recovered by adding common salt to the solution, and the chloride decomposed by the carbonate of soda.

Selenic acid, — $Se + O^3$; obtained by fusing nitrate of potassa with selenium — a solvent for gold, iron, copper, and zinc.

Silicic acid, — (*Silica* — silex; base *Silicon*) — $Si + O^3$; exists largely in sand. Common glass is fused sand and protoxide of potassium (carbonate of potassa — *potash*) in the proportion of 1 part by weight of the former to 3 of the latter.

Manganese, compounded with oxygen, in different proportions, imparts the various colors and tints given to fancy glass ware, now so generally in vogue.

Butylene, — $C^4 + H^4$; sp. gr. 1.9348; a gaseous hydro-carbon derived from the distillation of coal-tar; illuminating power, compared with that of olefient gas, as 2 to 1.

Propylene, — $C^3 + H^3$; sp. gr. 1.4511; a gaseous hydro-carbon derived from the distillation of coal-tar; illuminating power, compared with that of olefient gas, as 1.5 to 1.

Napthaline Vapor, — $C^{10} + H^4$, the vapor of solidified olefient gas.

Turpentine Vapor, — $C^{10} + H^8$.

SECTION III.

PRACTICAL ARITHMETIC.

VULGAR FRACTIONS.

A fraction is one or more parts of a UNIT.

A vulgar fraction consists of two *terms*, one written above the other, with a line drawn between them.

The term below the line is called the *denominator*, as showing the denomination of the fraction, or number of parts into which the unit is broken.

The term above the line is called the *numerator*, as numbering the parts employed. These together constitute the fraction and its value.

A vulgar fraction always denotes *division*, of which the denominator is the *divisor* and the numerator the *dividend*. Its value as a *unit* is the quotient arising therefrom.

A simple fraction is either a proper or improper fraction.

A proper fraction is one whose numerator is less than its denominator, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{21}{50}$, &c.

An improper fraction has its numerator equal to or greater than its denominator, as $\frac{2}{2}$, $\frac{4}{3}$, $\frac{24}{15}$, &c.

A mixed fraction is a compound of a whole number and a fraction, as $1\frac{1}{2}$, $5\frac{11}{32}$, $12\frac{3}{16}$, &c.

A compound fraction is a fraction of a fraction, as $\frac{1}{2}$ of $\frac{3}{5}$; $\frac{3}{4}$ of $\frac{8}{5}$ of $\frac{12}{17}$, &c.

A complex fraction has a fraction for its numerator or denominator, or both, as $\frac{\frac{1}{2}}{3}$, $\frac{4}{\frac{3}{5}}$, $\frac{\frac{1}{2}}{\frac{3}{4}}$, $\frac{5\frac{1}{2}}{4}$, &c., and is read $\frac{1}{2} \div 3$; $4 \div \frac{3}{5}$; $\frac{1}{2} \div \frac{3}{4}$; $5\frac{1}{2} \div 4$, &c.

REDUCTION OF VULGAR FRACTIONS.

To reduce a fraction to its lowest terms.

This consists in concentrating the expression without changing the value of the fraction or the relation of its parts.

It supposes division, and, consequently, by a measure or measures common to both terms.

It is said to be accomplished when no number greater than 1 will divide both terms without a remainder:—therefore, RULE. — Divide both terms by any number that will divide them without a remainder, and the quotient again as before; continue so to do until no number greater than 1 will divide them, — or divide by the greatest common measure at once.

EXAMPLE. — Reduce $\frac{864}{1512}$ to its lowest terms.

$$4\ )\ \frac{864}{1512} = \frac{216}{378} \div 2 = \frac{108}{189} \div 9 = \frac{12}{21} \div 3 = \frac{4}{7}. \quad \textit{Ans.}$$

To reduce an improper fraction to a mixed or whole number.

RULE. — Divide the numerator by the denominator and to the whole number in the quotient annex the remainder, if any, in form of a fraction, making the divisor the denominator as before; then reduce the fraction to its lowest terms.

EXAMPLE. $\frac{5}{4} = 1\frac{1}{4}$; $\frac{16}{12} = 1\frac{4}{12} = 1\frac{1}{3}$; $\frac{26}{13} = 2$.

To reduce a mixed fraction to an equivalent improper fraction.

RULE. — Multiply the whole number by the denominator of the fractional part, and to the product add the numerator, and place their sum over the said denominator.

EXAMPLE. — Reduce $3\frac{1}{4}$ and $12\frac{8}{9}$ to improper fractions.

$3 \times 4 = 12 + 1 = \frac{13}{4}$. *Ans.* $12 \times 9 + 8 = \frac{116}{9}$. *Ans.*

To reduce a whole number to an equivalent fraction having a given denominator.

RULE. — Multiply the whole number by the given denominator, and place the said denominator under the product.

EXAMPLE. — How may 8 be converted into a fraction whose denominator is 12?

$$8 \times 12 = \frac{96}{12}. \quad \textit{Ans.}$$

To reduce a compound fraction to a simple one.

RULE. — Multiply all the numerators together for a numerator, and all the denominators together for a denominator; the fraction thus formed will be an equivalent, but often not in its lowest terms. Or, concentrate the expression, when practicable, by reciprocally expunging, or writing out, such factors as exist or are attainable common to both terms, and then multiply the remaining terms as directed above.

NOTE. — This last practice is called cancellation, or cancelling the terms. It consists, as has been stated, in reciprocally annulling, or casting out, equal values from both terms, whereby the expression is concentrated, and the relation of the parts kept undisturbed; and it may always be carried to the extent of reducing the fraction to its lowest terms, before any multiplication, as final, is resorted to; and often, therefore, to the extent that such multiplication is inadmissible, the terms having been cancelled by each other until but a single number is left in each.

EXAMPLE. — Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{2}$ to a simple fraction.

Operation by multiplication, $\frac{2}{3} \times \frac{3}{4} \times \frac{1}{2} = \frac{6}{24} = \frac{1}{4}$. *Ans.*

Operation by cancellation, $\dfrac{\not{2}\ \not{3}\ 1}{\not{3}\ 4\ \not{2}} = \frac{1}{4}$. *Ans.*

EXAMPLE. — Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{12}{8}$ of $\frac{6}{8}$ of $\frac{5}{9}$ of 2 to a simple fraction.

By multiplication, $\frac{2}{3} \times \frac{3}{4} \times \frac{12}{8} \times \frac{6}{8} \times \frac{5}{9} \times \frac{2}{1} = \frac{4320}{6912} = \frac{5}{8}$. *Ans.*

The last example stated for cancellation, $\left\{ \dfrac{2\ \ 3\ \ 12\ \ 6\ \ 5\ \ 2}{3\ \ 4\ \ 8\ \ 8\ \ 9} \right.$

PROCESS OF CANCELLING THE ABOVE.

1. The 3 in num. equals the 3 in denom., therefore erase both.
2. The first 2 in num. equals or measures the 4 in denom. *twice*, therefore place a 2 under the 4, and erase the 4 and 2 which measured it — (as 4 : 2 : : 2 : 1.)
3. The 2 (remaining factor of 4 and 2 erased) in denom., and the remaining 2 in num., will cancel each other, — erase them.
4. The 12 and 6 in num. = 72, and the 9 and 8 in denom. = 72; these, therefore, in their relations as factors equal each other, and may be erased.

The remaining factors represent the true value of the compound fraction, and will be found = $\frac{5}{8}$, as by multiplication.

EXAMPLE. — Reduce $\frac{18}{13}$ of $\frac{7}{12}$ to a simple fraction.

$$\dfrac{\overset{\overset{3}{\not{9}}}{\not{18}} \times 7}{13 \times \underset{\underset{2}{\not{6}}}{\not{12}}}. \quad \text{Or, } \dfrac{\overset{3}{\not{18}} \times 7}{13 \times \underset{2}{\not{12}}} \; (= 18 \div 6, \text{ and } 12 \div 6) = \frac{3}{2} \times \frac{7}{13} = \frac{21}{26}. \quad \textit{Ans.}$$

To reduce fractions of different denominations to an equivalent simple one, — to a fraction having a common denominator.

RULE. — Multiply each numerator by all the denominators except its own and add the products together for the numerator, and multiply all the denominators together for a denominator.

NOTE. — Whole numbers and fractions other than simple, must first be reduced to simple fractions before they can be reduced to a fraction having a common denominator.

EXAMPLE. — Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to an equivalent simple fraction.

$$\frac{2}{3} \times \frac{3}{4} = 8 + 9 = \frac{17}{12}. \quad \textit{Ans.}$$

EXAMPLE. — Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{8}$, and $\frac{14}{3}$ to an equivalent.

$$\frac{1}{2} + \frac{3}{5} = \frac{11}{10} + \frac{7}{8} = \frac{158}{80} + \frac{14}{3} = \frac{1594}{240} = 6\tfrac{77}{120}. \quad \textit{Ans.}$$

To reduce a complex fraction to a simple one.

RULE — Multiply the numerator of the upper fraction by the denominator of the lower, for the new numerator; and the denominator of the upper by the numerator of the lower for the new denominator.

EXAMPLES. — Reduce $\frac{\frac{1}{2}}{3}$, $\frac{4}{\frac{3}{5}}$, $\frac{\frac{1}{2}}{\frac{3}{4}}$, and $\frac{5\frac{1}{2}}{4}$, each to a simple fraction.

$\frac{1}{2} \div \frac{3}{1} = \frac{1}{6}$; $\frac{4}{1} \div \frac{3}{5} = \frac{20}{3}$; $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3}, = \frac{4}{6}, = \frac{2}{3}$; $5\frac{1}{2} = \frac{11}{2}$, and $\frac{11}{2} \times \frac{1}{4} = \frac{11}{8}, = 1\frac{3}{8}$. *Ans.*

To reduce Vulgar Fractions to equivalent Decimals.

RULE. — Divide the numerator by the denominator; the quotient is the decimal, or the whole number and decimal, as the case may be.

EXAMPLE. — Reduce $\frac{7}{8}$, $4\frac{3}{5}$, $\frac{14}{12}$, to decimals.

$7 \div 8 = 0.875$; $4\frac{3}{5} = \frac{23}{5}, = 4.6$; $14 \div 12 = 1.166+$. *Ans.*

To find the greatest common measure of two or more given numbers.

RULE. — Divide the given numbers by any measure common to them all, and set the quotients in a line beneath; then divide the quotients by any measure common to them, and set the quotients beneath; and so on until the quotients are no longer common multiples of any one number greater than unity; the product of all the divisors or common measures employed will be the greatest common measure.

EXAMPLE. — What is the greatest common measure of 84 and 36? Also of 32, 24, and 16? Also of 182, 104, and 52?

4)84.36	8)32.24.16	2)182.104.52
3)21 . 9	4 . 3 . 2.—8. *Ans.*	13) 91 . 52.26
7 . 3 — $4 \times 3 = 12$. *Ans.*		7 . 4 . 2.=26. *Ans.*

NOTE. — When any number in the series is prime to either of the others, the numbers are collectively incommensurable; that is to say, their greatest common measure is unity, or 1.

To find the least common multiple of two or more given numbers.

RULE. — Divide all the given numbers that are commensurable with each other by any measure that is common to them, and set the quotients, together with the undivided numbers, if any, in a line beneath; then divide the quantities in the second line as before, and so on until no two quantities in the last line are common multiples of any number greater than unity, or 1; the product of all the common measures employed into the product of all the numbers in the last line will be the least common multiple of the given numbers.

Example. — What is the least common multiple of 27 and 36? Also of 182, 104, and 52? Also of 24, 14, 12, and 7?

```
9 ) 27 . 36                                  2 ) 182 . 104 . 52
    ───────                                 ──────────────────
    3 . 4 = 4 × 3 × 9 = 108. Ans.           13 )  91 .  52 . 26
                                            ──────────────────
                                             2 )   7 .   4 .  2
                                            ──────────────────
                                                   7 .   2 .  1. = 728. Ans.

               7 ) 24 . 14 . 12 . 7
               ────────────────────
               2 ) 24 .  2 . 12 . 1
               ────────────────────
               6 ) 12 .  1 .  6
               ────────────────────
                    2 .  1 .  1.    168.  Ans.
```

ADDITION OF VULGAR FRACTIONS.

Sum of the products of each numerator with all the denominators except that of the numerator involved, forms numerator of sum.

Product of all the denominators forms denominator of sum.

Rule. — Arrange the several fractions to be added, one after another, in a line from left to right; then multiply the numerator of the first by the denominator of the second, and the denominator of the first by the numerator of the second, and add the two products together for the numerator of the sum; then multiply the two denominators together for its denominator; bring down the next fraction, and proceed in like manner as before, continuing so to do until all the fractions have been brought down and added. Or, reduce all to a common denominator, then add the numerators together for the numerator of the sum, and write the common denominator beneath.

Examples. — Add together $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{3}$.

$\frac{1}{2} \times \frac{2}{3} = \frac{7}{6} \times \frac{3}{4} = \frac{46}{24} \times \frac{4}{3} = \frac{234}{72} = \frac{13}{4} = 3\frac{1}{4}$. *Ans.*

$\frac{1}{2} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4} = \frac{15}{12}$, and $\frac{2}{3} + \frac{4}{3} = \frac{6}{3} = \frac{24}{12}$, and $\frac{15}{12} + \frac{24}{12} = \frac{39}{12}$
$= \frac{13}{4} = 3\frac{1}{4}$. *Ans.*

SUBTRACTION OF VULGAR FRACTIONS.

Product of numerator of minuend and denominator of subtrahend, forms numerator of minuend, for common denominator.

Product of numerator of subtrahend and denominator of minuend, forms numerator of subtrahend, for common denominator.

Product of denominators forms common denominator.

Difference of new found numerators forms the numerator, and common denominator the denominator, of the difference, or remainder sought.

Rule. — Write the subtrahend to the right of the minuend, with the sign (—) between them; then multiply the numerator of the minuend by the denominator of the subtrahend, and the denominator of the minuend by the numerator of the subtrahend; subtract the latter product from the former, and to the remainder or difference affix the product of the two denominators for a denominator; the sum thus formed is the answer, or true difference.

Examples. — Subtract $\frac{1}{2}$ from $\frac{3}{4}$, also $\frac{3}{5}$ from $\frac{11}{17}$.

$$\frac{3}{4} - \frac{1}{2} = \frac{6-4}{8}, = \frac{2}{8} = \frac{1}{4}. \quad \textit{Ans.}$$

$$\frac{11}{17} - \frac{3}{5} = \frac{55-51}{85} = \frac{4}{85}. \quad \textit{Ans.}$$

DIVISION OF VULGAR FRACTIONS.

Product of numerators of dividend and denominators of divisor, forms numerator of quotient.

Product of denominators of dividend and numerators of divisor, forms denominator of quotient; therefore,

Rule. — Write the divisor to the right of the dividend with the sign (÷) between them; then multiply the numerator of the dividend by the denominator of the divisor, for the numerator of the quotient, and the denominator of the dividend by the numerator of the divisor, for the denominator of the quotient. Or, invert the divisor, and multiply as in multiplication of fractions. Or, proceed by cancellation, when practicable.

Examples. — Divide $\frac{1}{2}$ by $\frac{3}{4}$; $\frac{3}{4}$ by $\frac{1}{2}$; $\frac{4}{3}$ by $\frac{11}{13}$; and $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{4}{3}$ by $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{3}$.

$\frac{1}{2} \div \frac{3}{4} = \frac{4}{6}$; $\frac{3}{4} \div \frac{1}{2} = \frac{6}{4}$; $\frac{4}{3} \div \frac{11}{13} = \frac{52}{33}$; or $\frac{4}{3} \times \frac{13}{11} = \frac{52}{33}$. *Ans.*

$\frac{1}{2} \times \frac{3}{4} \times \frac{5}{6} \times \frac{4}{3} = \frac{60}{144} = \frac{5}{12}$, and $\frac{1}{4} \times \frac{1}{2} \times \frac{3}{4} \times \frac{2}{3} = \frac{6}{96} = \frac{1}{16}$, and $\frac{5}{12} \div \frac{1}{16} = \frac{80}{12} = \frac{20}{3} = 6\frac{2}{3}$. *Ans.*

FORM FOR CANCELLATION. — EXAMPLE LAST GIVEN.

$$\frac{1 \quad 3 \quad 5 \quad 4 \qquad 4 \quad 2 \quad 4 \quad 3}{2 \quad 4 \quad 6 \quad 3 \qquad 1 \quad 1 \quad 3 \quad 2} = \frac{20}{3}. \quad \textit{Ans., as above.}$$

Note. — The foregoing example can be cancelled to the extent of leaving but a 4 and a 5 (= 20) numerators, and a 3 denominator. Units, or 1's, in the expressions, are valueless, as a sum multiplied by 1 is not increased.

MULTIPLICATION OF VULGAR FRACTIONS.

Product of numerators of multiplier and multiplicand, forms numerator of product.

Product of denominators of multiplier and multiplicand, forms denominator of product.

Rule. — Multiply the numerators together for a numerator, and the denominators together for the denominator.

Examples. — Multiply $\frac{1}{2}$ by $\frac{1}{2}$; $\frac{3}{4}$ by 7; $\frac{11}{12}$ by $\frac{12}{7}$; $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{2}{3}$.

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$; $\frac{3}{4} \times \frac{7}{1} = \frac{21}{4}$; $\frac{11}{12} \times \frac{12}{7} = \frac{132}{84} = \frac{11}{7}$; $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{6}{24} = \frac{1}{4}$, and $\frac{3}{4} \times \frac{1}{2} \times \frac{2}{3} = \frac{6}{24} = \frac{1}{4}$, and $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ *Ans.*

MULTIPLICATION AND DIVISION OF FRACTIONS, COMBINED.

It has been seen that a compound fraction is converted into an equivalent simple one, by multiplying the numerators together for a numerator, and the denominators together for a denominator; and it has also been seen that a series of simple fractions are converted into a product, by the same process. It is therefore evident that compound fractions and simple, or a series of compound and a series of simple, may be multiplied into each other, for a product, by multiplying all the numerators of both together for a numerator, and all the denominators of both together for a denominator; and that the product will be the same as would be obtained, if the compound were first converted into an equivalent simple fraction, and the simple fractions into a product or factor, and these multiplied together for a product.

It has also been seen that a fraction is divided by a fraction by multiplying the numerator of the dividend by the denominator of the divisor, for the numerator of the quotient, and the denominator of the dividend by the numerator of the divisor, for the denominator of the quotient; and that this multiplication becomes direct as in multiplying for a product, if the divisor is inverted. And it is clear that a compound divisor, or a series of simple divisors, or both, may be used instead of their simple equivalent, and with the same result, if all are inverted.

It is therefore evident that any proposition, or problem, in fractions, consisting of multiplications and divisions both, and these only, no matter how extensive and numerous, or whether in compound fractions, or simple, or both, may be solved, and the true result obtained, as a product, by simply multiplying all the numerators in the statement together for a numerator, and all the denominators in the statement for a denominator, all the divisors in the statement being inverted; that is, all the numerators of the divisors being made denominators in the statement, and all the denominators of the divisor being made numerators in the statement. And it is further evident that a proposition stated in this way, admits of easy cancellation as far as cancellation is practicable, which is often to great extent.

EXAMPLE.—It is required to divide 12 by $\frac{2}{3}$ of $\frac{3}{4}$; to multiply the quotient by the product of 4 and 8; to divide that product by $\frac{7}{2}$ of $\frac{3}{5}$ of 8; to multiply the quotient by $\frac{7}{8}$ of $\frac{8}{3}$ of $\frac{9}{14}$; and to divide that product by the product of 5 and 9.

STATEMENT.

(Dividends read from right to left, divisors from left to right.)

Numerators of dividends and denominators of divisors.

	num.	den.		num.		den.			num.			den.		
Numerator of statement.	12	3	4	8	4	2	5		7	8	9			Dividend of statement.
Denominator of statement.		2	3			7	3	8	8	3	14	5	9	Divisor of statement.
		div's.				div's.			div'd.			div's.		

Denominators of dividends and numerators of divisors.

The *answer* to the above proposition is $1\frac{11}{24}$, and the proposition as stated may be readily cancelled to its lowest terms. It may be cancelled to the extent of leaving but 4, 4, 2 in the numerator, and 7, 3, in the denominator, $\frac{4\times4\times2}{7\times3}=\frac{32}{21}=1\frac{11}{21}$.

To reduce a fraction in a higher denomination to an equivalent fraction in a given lower denomination.

Rule. — Multiply the fraction to be reduced — numerators into numerator and denominators into denominator — by a fraction whose numerator represents the number of parts of the lower denomination, required to make one of the denomination to be reduced.

Example. — Reduce $\frac{7}{8}$ of a foot to an equivalent fraction in inches.

$$\frac{7}{8}\times\frac{12}{1}=\frac{84}{8}=\frac{21}{2}.\quad \textit{Ans.}$$

Example. — Reduce $\frac{5}{6}$ of a pound to an equivalent fraction in $\frac{2}{3}$ ounces.

$$\frac{5}{6}\times\frac{16}{1}=\frac{80}{6}\div\frac{2}{3}=\frac{240}{12}=\frac{20}{1}.\quad \textit{Ans.}$$

$$\text{Or, }\frac{5}{6}\times\frac{16}{1}\times\frac{3}{2}=\frac{240}{12}=\frac{20}{1}.\quad \textit{Ans.}$$

To reduce a fraction in a lower denomination to an equivalent fraction in a given higher denomination.

Rule. — Multiply the fraction to be reduced — numerator into denominator and denominator into numerator — by a fraction whose numerator represents the number of parts required of the lower denomination to make 1 of the higher.

Example. — Reduce $\frac{21}{2}$ inches to an equivalent fraction in feet.

$\frac{21}{2}\div\frac{12}{1}=\frac{21}{24}=\frac{7}{8}$. *Ans.* Or, $\frac{21}{2}\times\frac{1}{12}=\frac{21}{24}=\frac{7}{8}$. *Ans.*

EXAMPLE. — Reduce $\frac{40}{2}$ two third ounces to an equivalent fraction in pounds.

$$\tfrac{40}{2} \times \tfrac{2}{3} = \tfrac{80}{6} \div \tfrac{16}{1} = \tfrac{80}{96} = \tfrac{5}{6}. \quad \textit{Ans.}$$

$$\text{Or, } \tfrac{40}{2} \times \tfrac{2}{3} \times \tfrac{1}{16} = \tfrac{80}{96} = \tfrac{5}{6}. \quad \textit{Ans.}$$

To reduce a fraction in a higher to whole numbers in lower denominations.

RULE. — Multiply the numerator of the given fraction by the number of parts of the next lower denomination that make ONE of the given fraction, and divide the product by the denominator. Multiply the numerator of the fractional part of the quotient thus obtained by the number of parts in the next lower denomination that make 1 of the denomination of the quotient, and divide by its denominator for whole numbers as before; so proceed until the whole numbers in each denomination desired are obtained.

EXAMPLE. — How many hours, minutes, and seconds, in $\frac{9}{14}$ of a day?

$$\tfrac{9 \times 24}{14} = \tfrac{216}{14} = 15, \tfrac{3 \times 60}{7} = \tfrac{180}{7} = 25, \tfrac{5 \times 60}{7} = \tfrac{300}{7} = 42\tfrac{6}{7}, =$$
$$15 \text{ h., } 25 \text{ m., } 42\tfrac{6}{7} \text{ sec.} \quad \textit{Ans.}$$

EXAMPLE. — How many minutes in $\frac{9}{14}$ of a day?

$$\tfrac{9 \times 24 \times 60}{14} = \tfrac{12960}{14} = 925\tfrac{5}{7}. \quad \textit{Ans.}$$

To reduce fractions, or whole numbers and fractions, in lower denominations, to their value in a higher denomination.

RULE. — Reduce the mixed numbers to improper fractions, find their common denominator, and change each whole number and numerator to correspond therewith. Then reduce the higher numbers to their values in the lowest denomination, add the value in the lowest denomination thereto, and take their sum for a numerator. Multiply the common denominator by the number required of the lowest denomination to make ONE of the next higher, that product by the number required of that denomination to make 1 of the next higher, and so on, until the highest denomination desired is reached, and take the product for a denominator, and reduce to lowest terms.

EXAMPLE. — Reduce $5\frac{1}{3}$ oz., $3\frac{1}{5}$ dwts., $2\frac{1}{2}$ grs., troy, to lbs.

$\frac{16}{3} \cdot \frac{16}{5} \cdot \frac{5}{2} = \frac{160 \cdot 96 \cdot 75}{30}$; therefore,

$$160 \times 20 = 3200$$
$$96$$
$$3296 \times 24 = 79104$$
$$75$$
$$\left.\begin{array}{r} 79179 \\ 30 \times 24 \times 20 \times 12 = 172800 \end{array}\right\} = .458 + \text{lbs.} \quad \textit{Ans.}$$

EXAMPLE. — Reduce 11 hours, 59 minutes, 60 seconds, to the fraction of a day.

$$
\begin{array}{r}
11 \times 60 = 660 \\
59 \\
\hline
719 \times 60 = 43140 \\
60 \\
\hline
43200 \\
60 \times 60 \times 24 = 86400
\end{array}
\Bigg\} = \tfrac{1}{2}. \quad Ans.
$$

EXAMPLE. — Reduce 15 h., 25 m., $42\frac{6}{7}$ sec., to the fraction of a day.

$$
\begin{array}{r}
15 \times 60 \times 60 = 54000 \\
25 \times 60 = 1500 \\
42\frac{6}{7} \\
\hline
55542\frac{6}{7} \\
7 \\
\hline
388800 \\
7 \times 60 \times 60 \times 24 = 604800
\end{array}
\Bigg\} = \tfrac{9}{14}. \quad Ans.
$$

To work fractions, or whole numbers and fractions, by the Rule of Three, or Proportion.

RULE. — Reduce the mixed terms to simple fractions, state the question as in whole numbers, invert the divisor, and multiply and divide as in whole numbers.

EXAMPLE. — If $2\frac{1}{2}$ yards of cassimere cost $\$4\frac{1}{4}$, what will $\frac{3}{4}$ of a yard cost? $2\frac{1}{2} = \frac{5}{2}$; $4\frac{1}{4} = \frac{17}{4}$; then,

$$\tfrac{5}{2} : \tfrac{17}{4} :: \tfrac{3}{4} : x, = \tfrac{17 \times 3 \times 2}{4 \times 4 \times 5} = \tfrac{102}{80} = \$1.27,5. \quad Ans.$$

DECIMAL FRACTIONS.

A decimal fraction is written with its numerator only. Its denominator is understood. It occupies one or more places of figures, and has a point or dot (.) prefixed or placed before it. The dot (.) alone distinguishes it from an integer or whole number. It supposes a denominator whose value is a UNIT broken into parts, having a tenfold relation to the number of places the numerator occupies. The denominator, therefore, of any decimal is always a unit (1) with as many ciphers annexed as the numerator has places of figures. Thus, the denominator of .1, .2, .3, &c., is 10, and the fractions are read, *one tenth*, *two tenths*, *three tenths*, &c. The denominator of .01, .11, .12, &c., is 100, and these are read, *one hundredth*, *eleven hundredths*, *twelve hundredths*, &c. The denominator of .001, .101, .125, &c., is 1000, and these are read *one thousandth*, *one hundred and one thousandths*, *one hundred and twenty-five thousandths*, &c. The denominator of a decimal occupying four places of figures as .7525 is 10000, and so on continually.

The first figure on the right of the decimal point is in the place of *tenths*, the second in the place of *tenths* of *tenths*, or *hundredths*, the third in the place of *tenths* of *tenths* of *tenths*, or *thousandths*, &c. Thus the value of a decimal occupying four places of figures, as .7525, for example, is $\frac{7525}{10000}$, $= \frac{.752\frac{1}{2}}{1000}$, $= \frac{75\frac{1}{4}}{100}$, $= \frac{7\frac{1}{2}}{10} + \frac{\frac{1}{4}}{100} = \frac{\frac{3}{4}}{1} + \frac{\frac{1}{4}}{100}$. A decimal is converted into a vulgar fraction of equal value, by affixing its denominator.

Ciphers placed on the right of decimals do not change their value. Thus, .1850 = .185, plainly for the reason that the denominator of the latter bears the same relation to that of the former that 185 bears to 1850; from both terms of the fraction a ten fold has been dropped.

Ciphers placed on the left of decimals *decrease* their value ten fold for every cipher so placed. Thus, .1 = $\frac{1}{10}$, .01 = $\frac{1}{100}$, .001 = $\frac{1}{1000}$, &c.

A mixed number is a whole number and a decimal. Thus, 4.25 is a mixed number. Its value is 4 units, or *ones*, and $\frac{25}{100}$ of 1, = $4\frac{25}{100} = 4\frac{1}{4}$. The number on the left of the separatrix is always a whole number — that on its right, always a decimal.

ADDITION OF DECIMALS.

RULE. — Set the numbers directly under each other according to their values, whole numbers under whole numbers, and decimals under decimals; add as in whole numbers, and point off as many places for decimals in the sum as there are figures in that decimal occupying the greatest number of places.

EXAMPLES. — Add together .125, .34, .1, .8672. Also, 125, 34.' .235. 1.4322.

.125	125.
.34	34.11
.1	.235
.8672	1.4322
1.4322 *Ans.*	160.7772 *Ans.*

SUBTRACTION OF DECIMALS.

RULE. — Set the numbers, the less under the greater, and in other respects as directed for addition; subtract as in whole numbers, and point off as many places for decimals in the remainder as the decimal having the greatest number of figures occupies places.

EXAMPLES. — Subtract .2653 from .8. Also, 11.5 from 238.134.

```
 .8                 238.134
 .2653               11.5
 -----              -------
 .5347  Ans.        226.634  Ans.
```

MULTIPLICATION OF DECIMALS.

RULE. — Multiply as in whole numbers, and point off as many places for decimals in the product as there are decimal places in the multiplicand and multiplier both. If the product has not so many places, prefix ciphers to supply the deficiency.

EXAMPLES. — Multiply 14.125 by 3.4. Also, 5.14 by .007.

```
  14.125                      5.14
     3.4                      .007
  ------                    ------
   56500                    .03598  Ans.
  42375
 -------
 48.0250 = 48.025.  Ans.
```

NOTE. — Multiplying by a decimal is equivalent to dividing by a whole number that bears the same relation to a UNIT that a unit bears to a decimal. Multiplying by a decimal, therefore, is equivalent to dividing by the denominator of a fraction of equal value whose numerator is 1, or of dividing by the denominator of a fraction of equal value whose numerator is more than 1, and multiplying the quotient by the numerator. Thus, the decimal $.25 = \frac{25}{100} = \frac{1}{4}$, and the decimal $.875 = \frac{875}{1000} = \frac{7}{8}$. And $14.23 \times .25 = 3.5575$, and $14.23 \div 4 = 3.5575$. So, also, $14.23 \times .875 = 12.45125$, and $14.23 \div 8 = 1.77875 \times 7 = 12.45125$. It is sometimes a saving of labor and matter of convenience to achieve multiplication by this process.

DIVISION OF DECIMALS.

RULE. — Write the numbers as for division of whole numbers, then remove the separatrix in the dividend as many places of figures to the right, (supplying the places with ciphers if they are not occupied,) as there are decimal figures in the divisor; consider the divisor a whole number and divide as in division of whole numbers.

EXAMPLE. — Divide .5 by .17. Also, .129 by 4.

```
.17).50(2.94+.  Ans.        4).129(.032+.  Ans.
     34                        12
     ---                       --
     160                        9
     153                        8
     ---                        -
       70                       1
       68
       --
        2
```

EXAMPLES. — Divide 16.5 by 1.232. Also, 1.2145 by 12.231.

```
1.232,)16.500, (13.3928+. Ans.   | 12.231,)1.214,50(.09929+ } Ans.
       1232                      |          1 100 79  .0993— }
       ----                      |          --------
        4180                     |            113 710
        3696                     |            110 079
        ----                     |            -------
         4840                    |              3 6310
         3696                    |              2 4462
         -----                   |              -------
         11440                   |              1 18480
         11088                   |              1 10079
         -----                   |              -------
           3520                  |                 8401
           2464                  |
           -----                 |
           10560                 |
            9856                 |
            ----                 |
             704                 |
```

NOTE. — Dividing by a decimal is equivalent to multiplying by a whole number that bears the same proportion to a UNIT that a unit bears to the decimal. Dividing by a decimal, therefore, is equivalent to multiplying by the denominator of a fraction of equal value whose numerator is 1, or multiplying by the denominator of a fraction of equal value whose numerator is more than 1, and dividing the product by the numerator. Dividing by a fraction is equivalent to multiplying by its denominator and dividing the product by its numerator, or dividing by its numerator and multiplying the quotient by its denominator. Thus, $.5 = \frac{5}{10} = \frac{1}{2}$, and $.75 = \frac{75}{100} = \frac{3}{4}$. And $12.24 \div .5 = 24.48$, and $12.24 \times 2 = 24.48$. So, also, $12.24 \div .75 = 16.32$, and $12.24 \times 4 = 48.96 \div 3 = 16.32$. This method of accomplishing division may often be resorted to with convenience.

REDUCTION OF DECIMALS.

To reduce a decimal in a higher to whole numbers in successive lower denominations.

RULE. — Multiply the decimal by that number in the next lower denomination that equals ONE of the denomination of the decimal, and point off as many places for a remainder as the decimal so multiplied has places. Multiply the remainder by the number in the next lower denomination that equals 1 of the denomination of the remainder, and point off as before; so continue, until the reduction is carried to the lowest denomination required.

EXAMPLE. — What is the value of .62525 of a dollar?

```
          .62525
             100
        --------
Cents,  62.52500
              10
        --------
Mills,   5.25000      Ans.  62 cents 5¼ mills.
```

EXAMPLE. — What is the value of .46325 of a barrel?

	.46325
	32
Gallons,	14.82400
	4
Quarts,	3.296
	2
Pints,	.592
	4
Gills,	2.368.

Ans. 14 gals. 3 qts. $2\frac{368}{1000}$ gills.

EXAMPLE. — How many pence in .875 of a pound?

$.875 \times 240 = 210.$ *Ans.*

To reduce decimals, or whole numbers and decimals, in lower denominations, to their value in a higher denomination.

RULE. — Reduce all the given denominations to their value in the lowest denomination, then divide their sum by the number required of the lowest denomination to make ONE of the denomination to which the whole is to be reduced.

EXAMPLE. — Reduce 14 gallons, 3 quarts, 2.368 gills, to the decimal of a barrel.

$14 \times 4 = 56 + 3 = 59 \times 8 = 472 + 2.368 = 474.368.$

$8 \times 4 \times 32 = 1024$) 474.368 (.46325. *Ans.*

To work decimals, or whole numbers and decimals, by the Rule of Three, or Proportion.

RULE. — State the question and work it as in whole numbers, taking care to point off as many places for decimals in the product to be used as the dividend, as there are decimals in the two terms which form it, and to remove the decimal point therein as many places to the right as there are decimals in the term to be used as a divisor, before the division is had.

EXAMPLE. — If .75 of a pound of copper is worth .31 of a dollar how much is 3.75 lbs. worth?

```
.75 : .31 :: 3.75
               .31
              ----
               375
             1125
.75 ) 1.16,25 ($1.55.  Ans.
```

PROPORTION, OR RULE OF THREE.

The Rule of Proportion involves the employment of three terms —a divisor and two factors for forming a dividend—and seeks a quotient, which, when the proposition is written in ratio, bears the same relation to the third term that the second term bears to the first. Two of the terms given are of like name or nature, and the other is of the name or nature of the quotient or answer sought. That of the nature of the answer is always one of the factors for forming the dividend, and, if the answer is to be greater than that term, the larger of the remaining two is the other; but if the answer is to be less than that term, the less of the remaining two is the other—the remaining term is the divisor.

Example.—If $12 buy 4 yards of cloth, how many yards will $108 buy?

$$\frac{\cancel{4} \times 108}{\underset{3}{\cancel{12}}} = \frac{108}{3} = 36 \text{ yards. } \textit{Ans.}$$

Example.—If 4 yards of cloth cost $12, how many dollars will 36 yards cost?

$$\frac{12 \times 36}{4} = 108 \text{ dollars. } \textit{Ans.}$$

Example.—If 30 men can finish a piece of work in 12 days, how many men will be required to finish it in 8 days?

$$\frac{30 \times 12}{8} = 45 \text{ men. } \textit{Ans.}$$

Example.—If 45 men require 8 days to finish a piece of work, how many men will finish the same work in 12 days?

$$\frac{45 \times 8}{12} = 30 \text{ men. } \textit{Ans.}$$

Example.—If 8 days are required by 45 men to finish a piece of work, how many days will be required by 30 men to finish the same work?

$$\frac{8 \times 45}{30} = 12 \text{ days. } \textit{Ans.}$$

Example.—If 12 days are required by 30 men to perform a piece of work, how many days will be required by 45 men to do the same work?

$$\frac{12 \times 30}{45} = 8 \text{ days. } \textit{Ans.}$$

Example.—I borrowed of my friend $150, which I kept 3 months, and, on returning it, lent him $200; how long may he keep the sum that the interest, at the same rate per cent., may amount to that which his own would have drawn?

$$150 \times 3 \div 200 = 2\frac{1}{4} \text{ months. } \textit{Ans.}$$

Example. — A garrison of 250 men is provided with provisions for 30 days, how many men must be sent out that the provisions may last those remaining 42 days?

$$250 \times 30 \div 42 = 179, \text{ and } 250 - 179 = 71. \quad \textit{Ans.}$$

Example. — If to the short arm of a lever 2 inches from the fulcrum there be suspended a weight of 100 lbs., what power on the long arm of the lever 20 inches from the fulcrum will be required to raise it?

$$20 : 2 :: 100 = 10 \text{ lbs. } \quad \textit{Ans.}$$

Example. — At what distance from the fulcrum on the long arm of a lever must I place a pound weight, to equipoise or weigh 20 lbs., suspended 2 inches from the fulcrum at the other end?

$$1 : 2 :: 20 : 40 \text{ inches. } \quad \textit{Ans.}$$

Note. — If we examine the foregoing with reference to the fact, we shall see that every proposition in simple proportion consists of a *term and a half!* or, in other words, of a *compound* term consisting of two factors, and a factor for which another factor is sought that together shall equal the compound. We have only to multiply the factors of the compound together — and a little observation will enable us to distinguish it — and divide by the remaining factor, and the work is accomplished. See Compound Proportion.

COMPOUND PROPORTION, OR DOUBLE RULE OF THREE.

Compound Proportion, like *single* proportion, consists of three terms given by which to find a fourth — a divisor and two factors for forming a dividend — but unlike single proportion, one or more of the terms is a compound, or consists of two or more factors; and sometimes a portion of the fourth term is given, which, however, is always a part of the divisor.

Of the given terms, two are suppositive, dissimilar in their natures, and relate to each other, and to each other only; and upon their relation the whole is made to depend; the remaining term is of the nature of one of the former, and relates to the fourth term, which is of the nature of the other.

The object sought is a number, which, multiplied into the factor or factors of the fourth term given, if any, and if not, which of itself, bears the same proportion to the dissimilar term to which it relates, as the suppositive term of like nature bears to the term to which it relates.

Rule. — Observe the denomination in which the demand is made, and of the suppositive terms make that of like nature the second, and the other the first; make the remaining term the third term; and, if there are any factors pertaining to the fourth term, affix them to the first; multiply the second and third terms together and divide by the first, and the quotient is the answer, term, or portion of a term, sought.

Example.—If 12 horses in 6 days consume 36 bushels of oats, how many bushels will suffice 21 horses 7 days?

$$12 \times 6 : 36 :: 21 \times 7 : x.$$

$$\frac{\overset{3}{\cancel{36}} \times 21 \times 7}{\cancel{12} \times \underset{2}{\cancel{6}}} = \frac{147}{2} = 73\tfrac{1}{2} \text{ bushels. } \textit{Ans.}$$

Example.—If 12 horses in 6 days consume 36 bushels of oats, how many horses will consume $73\frac{1}{2}$ bushels in 7 days?

$$36 : 12 \times 6 :: 73\tfrac{1}{2} : 7 \times x.$$

$$\frac{12 \times 6 \times 73\frac{1}{2}}{36 \times 7} = \frac{147}{7} = 21 \text{ horses. } \textit{Ans.}$$

Example.—If the interest on \$1 is 1.4 cts. for 73 days, (exact interest at 7 per cent.,) what will be the interest on \$150.42 for 146 days?

$$73 : 1.4 :: 150.42 \times 146 : x.$$

$$\frac{1.4 \times 150.42 \times 146}{73} = \$4.21. \textit{ Ans.}$$

Example.—If the interest on \$1 is 1.2 cts. for 73 days, (exact interest at 6 per cent.,) what will be the interest on \$125 for 90 days?

$$73 : 1.2 :: 125 \times 90 : x = \$1.85. \textit{ Ans.}$$

Example.—If \$100 at 7 per cent. gain \$1.75 in 3 months, how much at 6 per cent. will \$170 gain in $11\frac{1}{2}$ months?

$$100 \times 7 \times 3 : 1.75 :: 170 \times 6 \times 11.5 : x.$$

$$1.75 \times 170 \times 6 \times 11.5 \div 100 \times 7 \times 3 = \$9.77{,}5. \textit{ Ans.}$$

Example.—By working 10 hours a day 6 men laid 22 rods of wall in 3 days; how many men at that rate, who work but 9 hours a day, will lay 40 rods of wall in 8 days?

$$22 : 6 \times 3 \times 10 :: 40 : 9 \times 8 \times x.$$

$$6 \times 3 \times 10 \times 40 \div 22 \times 9 \times 8 = 4\tfrac{6}{11}. \textit{ Ans.}$$

Example.—If it costs \$112 to keep 16 horses 30 days, and it costs as much to keep 2 horses as it costs to keep 5 oxen, how much will it cost to keep 28 oxen 36 days?

$$16 \times 30 : 112 :: \tfrac{2}{5} \times 28 \times 26 : x.$$
$$\text{Or, — } 16 \times 30 \times 5 : 112 :: 28 \times 36 \times 2 : x.$$

$$\frac{\overset{7}{\cancel{112}}\ \ 28\ \ \overset{12}{\cancel{36}}\ \ \cancel{2}}{\cancel{16}\ \ \underset{\underset{5}{\cancel{15}}}{\cancel{30}}\ \ 5} = \frac{28 \times 12 \times 7}{5 \times 5} = \$94.08. \quad \textit{Ans.}$$

Example. — If 24 men, in 8 days of 10 hours each, can dig a trench 250 feet long, 8 feet wide, and 4 feet deep, how many men, in 12 days of eight hours each, will be required to dig a trench 80 feet long, 6 feet wide, and 4 feet deep?

$$250 \times 8 \times 4 : 24 \times 8 \times 10 :: 80 \times 6 \times 4 : 12 \times 8 \times x = 5—. \quad \textit{Ans.}$$

Example. — If 120 men in six months perform a given task, working 10 hours a day, how many men will be required to accomplish a like task in 5 months, working 9 hours a day?

$$120 \times 6 \times 10 = 5 \times 9 \times x.$$
$$\text{Or, — } 1 : 120 \times 6 \times 10 :: 1 : 5 \times 9 \times x. = 160. \quad \textit{Ans.}$$

Example. — The weight of a bar of wrought iron, 1 foot in length, 1 inch in breadth, and 1 inch thick, being 3.38 lbs., (and it is so,) what will be the weight of that bar whose length is $12\frac{1}{2}$ feet, breadth $3\frac{1}{4}$ inches, and thickness $\frac{3}{4}$ of an inch?

$$1 : 3.38 :: 12.5 \times 3.25 \times .75 : x.$$
$$\text{Or, — } 1 : 3.38 :: \tfrac{25}{2} \times \tfrac{13}{4} \times \tfrac{3}{4} : x, \text{ and}$$
$$\frac{3.38 \times 25 \times 13 \times 3}{2 \times 4 \times 4} = 102.98+ \text{ lbs.} \quad \textit{Ans.}$$

Example. — The weight of a bar of wrought iron, one foot in length and 1 inch square, being 3.38 lbs., what length shall I cut from a bar whose breadth is $2\frac{3}{4}$ inches, and thickness $\frac{1}{2}$ inch, in order to obtain 10 lbs.? $3.38 : 1 :: 10 : \frac{11}{4} \times \frac{1}{2} \times x.$

$$\frac{1 \times 10 \times 4 \times 2}{3.38 \times 11 \times 1} = 2 \text{ feet } 1\tfrac{8}{10} \text{ inches.} \quad \textit{Ans.}$$

CONJOINED PROPORTION, OR CHAIN RULE.

The Chain Rule is a process for determining the value of a given quantity in one denomination of value, in some other given denomination of value; or the immediate relationship which exists between two denominations of value, by means of a *chain* of approximate steps, circumstances, or equivalent values, known to exist, which connect them. In every instance at least *five* terms or values are employed in the process, and in all instances the number employed will be uneven. A proposition involving but three terms, of this nature, is a question in single proportion. The equivalent values employed are divided into *antecedents* and *consequents*, or causes and effects ; and the value or quantity for which an equivalent is sought, is called the odd term.

RULE. — 1. *When the value in the denomination of the first antecedent is sought of a given quantity in the denomination of the last consequent.* — Multiply all the antecedents and the odd term together for a dividend, and all the consequents together for a divisor ; the quotient will be the answer or equivalent sought.

RULE. — 2. *When the value in the denomination of the last consequent is sought of a given quantity in the denomination of the first antecedent.* — Multiply all the consequents and the odd term together for a dividend, and all the antecedents together for a divisor ; the quotient will be the answer required.

EXAMPLE. — I am required to give the value, in Federal money, of 5 Canada shillings, and know no immediate connection or relationship between the two currencies — that of Canada and that of the United States. The nearest that I do know is that 20 Canada shillings have a value equal to 32 New York shillings, and that 12 New York shillings equal in value 9 New England shillings, and that 15 New England shillings equal \$2.50 ; and with this knowledge will seek the value, in Federal money, of the 5 Canada shillings.

$$\frac{2.50 \times 9 \times 32 \times 5}{15 \times 12 \times 20} = \$1. \quad Ans.$$

EXAMPLE. — If \$2½ equal 15 New England shillings, and nine shillings in New England equal 12 shillings in New York, and 32 shillings in New York equal 20 shillings in Canada, how many shillings in Canada will equal \$1 ?

$$\frac{15 \quad \overset{\cancel{3}}{\cancel{12}} \quad \overset{\cancel{8}}{\cancel{20}} \quad 1}{\cancel{2\tfrac{1}{2}} \quad \underset{3}{\cancel{9}} \quad \underset{\cancel{4}}{\cancel{32}}} = \frac{15}{3} = 5 \text{ shillings.} \quad Ans.$$

EXAMPLE. — If 14 bushels of wheat weigh as much as 15 bushels of fine salt, and 10 bushels of fine salt as much as 7 bushels of coarse, and 7 bushels of coarse salt as much as 4 bushels of sand, how many bushels of sand will weigh as much as 40 bushels of wheat ?

$$\frac{15 \times 7 \times 4 \times 40}{14 \times 10 \times 7} = 17\tfrac{1}{7} \text{ bushels.} \quad Ans.$$

PERCENTAGE.

Pure percentage, or PERCENTAGE, is a rate by the hundred of a *part* of a quantity or number denominated the principal, or basis. But percentage, considered as a means, and as commonly applied, is mixed and related in an eminent degree; and in this light may be regarded as divided into orders bearing different names.

Thus *Interest* is percentage related to intervals of time in the past.

Discount is percentage related to interest, and intervals of time in the future.

Profit and Loss is comparative percentage, or percentage related to the positive and negative interests in business, etc., etc.

Pure percentage is commonly called BROKERAGE when paid to a broker for services in his line.

It is called COMMISSION when paid to or received by a factor or commission merchant for buying or selling goods.

It is called PREMIUM by an insurance company, when taken for insuring against loss.

It is called PRIMAGE when it is a charge in addition to the freight of a vessel, etc.

Comparative percentage relates to the differences of quantities, and is confined always to the idea of *more* or *less*. It implies ratio. This description of percentage, though much in practice, seems not to be well understood; and often a quantity is indirectly stated to be many times less than nothing, or many times greater than it is. The difference of two quantities cannot be as great as a hundred per cent. of the greater, however widely unequal the quantities may be, nor as small as no per cent. of the greater or lesser, however nearly equal they may be. No quantity or number can be as small as 1 time less than another quantity or number; and therefore cannot be as small as 100 per cent. less. But, since one quantity may be many by 1 time, or many times greater than another with which it is compared, it may be said to be many by 100 times, or many hundred per cent. greater.

When one of two quantities in comparison is stated to be three times less, or three hnndred per cent. less, for instance, than the other, the expression is incorrect and absurd. The meaning evidently is, that it is two-thirds less, or only one-third as large as the other, — that it is $66\frac{2}{3}$ per cent. less, or only $33\frac{1}{3}$ per cent. as large as the other. In common comparison, 1 is the measuring unit. In percentage, 100 is the measuring unit.

Let $a =$ principal.
$b =$ percentage.
$s =$ amount (sum of the principal and percentage).
$d =$ difference of the principal and percentage.
$r =$ rate of the percentage.
$p =$ rate per cent. of the percentage.

$$a = s - b = b \div r = 100b \div p = 100s \div (100 + p),$$
$$b = s - a = ar = ap \div 100,$$
$$p = 100r = 100b \div a = 100(s - a) \div a,$$
$$r = p \div 100 = b \div a = (s - a) \div a,$$
$$s = a + b = a(1 + r) = a(100 + p) \div 100,$$
$$d = a - b = 2a - s = s - 2b = a(1 - r).$$

To find the Percentage.

EXAMPLES.

What is $\frac{1}{4}$ of 1 per cent. of \$200?

$b = ar = ap \div 100 = \$0.50.$ *Ans.*

$\frac{8}{7}$ of 2 per cent. of 50 is what part of 50?

$$\frac{50 \times 8 \times 2}{7 \times 100} = 1\tfrac{1}{7}. \quad \textit{Ans.}$$

What is $\frac{3}{5}$ of $\frac{5}{3}$ of $\frac{1}{2}$ of 24 per cent. of 150 lbs.?

$150 \times 12 \div 100 = 18$ lbs. *Ans.*

What is $2\frac{3}{8}$ per cent. of 19 bushels?

$\frac{19}{8} \times \frac{19}{100} = 0.45125$ bushels. *Ans.*

Bought a job lot of merchandise for \$850, and sold it the same day, brokerage, $2\frac{1}{2}$ per cent., for \$975; what was the net gain?

$s - sr - a = s - (sr + a) = s(1 - r) - a = 975 - 975 \times .025 - 850 = \$100.625.$ *Ans.*

To find the Rate or Rate Per Cent.

EXAMPLES.

What per cent. of \$20 is \$2?

$r = b \div a, p = 100b \div a = 10$ per cent. *Ans.*

12 dozen is equal to what per cent. of 2 dozen?

$12 \div 2 = 6,$ 600 per cent. *Ans.*

What part of $5\frac{1}{2}$ lbs. is $\frac{3}{4}$ of 2 lbs.?

$$r = \tfrac{3}{4} \times \tfrac{2}{11} = \tfrac{1\cdot5}{5\cdot5} = 0.27\tfrac{3}{11}. \quad \textit{Ans.}$$

$24\frac{1}{2}$ per cent. is what per cent. of $36\frac{3}{4}$ per cent.?

$66\frac{2}{3}$ per cent. *Ans.*

For an article that cost \$4, \$5 were received; what per cent. of \$4 was received?

$$p = 5 \times 100 \div 4 = 125 \text{ per cent.} \quad \textit{Ans.}$$

A farmer sowed 4 bushels of wheat, which produced 48 bushels; what per cent. was the *increase?* 48 is *more* than 4 by what per cent. of 4? The difference of 48 and 4 is what per cent. of 4?

$$r = \frac{a-b}{b} = \frac{a}{b} - 1,\ p = \frac{100(a-b)}{p} = \frac{48-4}{4} = 48 \div 4 - 1 = 100(48-4) \div 4 = 1100 \text{ per cent.} \quad \textit{Ans.}$$

What per cent. would have been the *decrease*, if he had sowed 48 bushels, and harvested only 4 bushels? 4 is *less* than 48 by what rate of 48? The difference of 48 and 4 is what per cent. of 48?

$$r = (a-b) \div a = 1 - \frac{b}{a} = 0.91\tfrac{2}{3}, \text{ or } 91\tfrac{2}{3} \text{ per cent.} \quad \textit{Ans.}$$

Since water is composed of 8 atoms of oxygen and 1 atom of hydrogen, what per cent. of it is oxygen? 8 is what per cent. of the sum of 8 and 1?

$$r = \frac{a}{a+b} = 1 - \frac{b}{a+b},\ p = \frac{100a}{a+b} = \frac{8}{8+1} = .8889-, \text{ or } 88.89 - \text{per cent.} \quad \textit{Ans.}$$

What per cent. of it is hydrogen? 1 is what per cent. of the sum of 8 and 1?

$$r = 1 - \frac{a}{a+b} = \frac{b}{a+b},\ p = \frac{100b}{a+b} = \frac{1}{8+1} = .1111+, \text{ or } 11.11 + \text{per cent.} \quad \textit{Ans.}$$

How many volumes of water must be added to 100 volumes of 90 per cent. alcohol to reduce it to 50 per cent. alcohol or common proof? 90 is more than 50 by what per cent. of 50? The difference of 90 and 50 is what per cent. of 50?

$$p = \frac{(a-b)100}{b} = \frac{(90-50)100}{50} = 80. \quad \textit{Ans.}$$

How many volumes of 50 per cent. alcohol must be added to 100 volumes of 90 per cent. alcohol to produce 80 per cent. alcohol? 90 is more than 80 by what per cent. of the difference of 80 and 50? The difference of 90 and 80 is what per cent. of the difference of 80 and 50?

$$p = \frac{(a-b)100}{b-b'} = \frac{(90-80)100}{80-50} = 33\tfrac{1}{3}. \quad \textit{Ans.}$$

How many volumes of 90 per cent. alcohol must be added to 100 volumes of 50 per cent. alcohol to raise it to 80 per cent. alcohol? 50 is less than 80 by what per cent. of the difference of 90 and 80? The difference of 80 and 50 is what per cent. of the difference of 90 and 80?

$$\frac{(b-b')100}{a-b} = \frac{(80-50)100}{90-80} = 300. \quad \textit{Ans.}$$

If to 2 volumes of 95 per cent. alcohol, 1 volume of 50 per cent. alcohol be added, what per cent. alcohol will be the mixture? The sum of 50 and twice 95 is what per cent. of the sum of 2 and 1?

$$\frac{2a+b}{2+1} = \frac{2 \times 95 + 50}{2+1} = 80 \text{ per cent.} \quad \textit{Ans.}$$

In a barrel of apples, the number of sound ones was 60 per cent. *greater* than the number that were damaged. What per cent. *less* was the number that were damaged than the number that were sound? 60 per cent. is what per cent. of the sum of 100 per cent. and 60 per cent.? .6 is what rate of 1 + .6?

$$r = \frac{a}{1+a} = 1 - \frac{100}{1+a} = \frac{0.a}{1+.a} = 1 - \frac{1}{1.a} = \frac{60}{1+60} = .375, \text{ or}$$

$$37\tfrac{1}{2} \text{ per cent.} \quad \textit{Ans.}$$

Since the number of damaged apples was $37\frac{1}{2}$ per cent. less than the number that were sound, what per cent. greater was the number that were sound than the number that were damaged?

$$r = a \div (1-a) = 1 \div (1-a) - 1 = 60 \text{ per cent.} \quad \textit{Ans.}$$

Since the number of sound ones was 60 per cent. greater than the number that were damaged, what per cent. of the whole were sound?

$$r = \frac{a+a^2}{2a} = \frac{1+.a}{2},\ p = \frac{100+60}{2} = 80 \text{ per cent.} \quad \textit{Ans.}$$

What per cent. of the whole were damaged?

$$(100-60) \div 2 = 20 \text{ per cent.} \quad \textit{Ans.}$$

Since 20 per cent. of the apples were damaged, what per cen[t.] less was the number that were damaged than the number that we[re] sound?

$$r = \frac{1 - 2.a}{2 - 2.a} = 1 - \frac{1}{2 - 2.a}, \; p = \frac{100 - 2a}{200 - 2a} = 100 - \frac{100}{200 - 40} =$$

$37\frac{1}{2}$ per cent. *An[s.]*

What per cent. greater was the number that were sound tha[n] the number that were damaged?

$$r = 2 - (1 + 2.a) = 2 - 2.a - 1 = 60 \text{ per cent.} \quad \textit{Ans.}$$

Since 80 per cent. of the whole were sound, what per cent. les[s] was the number that were damaged than the number that wer[e] sound?

$$r = \frac{2.a - 1}{2.a} = 1 - \frac{1}{2.a} = \frac{2 \times .80 - 1}{2 \times .80} = 37\tfrac{1}{2} \text{ per cent.} \quad \textit{An[s.]}$$

Since the number of damaged ones was $37\frac{1}{2}$ per cent. less tha[n] the number that were sound, what per cent. of the whole wer[e] sound?

$$r = \frac{1}{2 - 2.a}, \; p = \frac{100}{2 - 2a} = \frac{100}{2 - 2 \times 37.5} = 80 \text{ per cent.} \quad \textit{Ans[.]}$$

Since 80 per cent. of the whole were sound, what per cent[.] greater was the number that were sound than the number tha[t] were damaged?

$$r = \frac{2 - .a}{2} = 2.a - 1 = 2 \times .80 - 1 = 60 \text{ per cent.} \quad \textit{Ans.}$$

Lost 20 per cent. of a cargo of coal by jettison, and 5 per cent[.] of the remainder by screening, what per cent. of the coal wa[s] saved?

$$\left.\begin{array}{l} a - b' = d' \\ d' - b'' = d'' \end{array}\right\} \; r = (1 - r')(1 - r'') = (1 - .20) - (1 - .20) \times .05 = (1 - .20)(1 - .05) = 76 \text{ per cent.} \quad \textit{Ans[.]}$$

$d'' - b''' = d'''$, &c.

Yesterday drew 12 per cent. of my balance of \$4,273 in th[e] bank, and deposited \$1,000; and to-day have drawn $31\frac{1}{4}$ per cent[.] of the balance left over, or as it stood last night. What per cent. o[f] the sum of the first-mentioned balance and deposit of yesterda[y] have I drawn?

$$r = \frac{b' + b''}{a + m} = \frac{512 + 1487.575}{4273 + 1000} = 37.9354 + \text{ per cent.} \quad \textit{Ans.}$$

What per cent. of the said sum is remaining in the bank?

$$-\frac{b'+b''}{a+m}=\frac{a+m-b'-b''}{a+m}=\frac{a+m-(b'+b'')}{a+m}=62.0646-$$ per cent. *Ans.*

What per cent., predicating it upon the first-mentioned balance, ve I drawn?

$$r=\frac{b'+b''}{a}=\frac{512.76+1487.576}{4273}=46.8134-$$ per cent. *Ans.*

What per cent. have I drawn, predicating it upon what I now ve in the bank?

$$=\frac{b'+b''}{a-b'+m-b''}=\frac{b'+b''}{a+m-(b'+b'')}=61.1225+$$ per cent. *Ans.*

What amount of money must I deposit to make good $62\frac{1}{2}$ per nt. of the aforementioned sum?

$$=r(a+m)+b'+b''-(a+m)=r(a+m)-d''=$$ \$22.96. *Ans.*

To find the Principal or Basis.

EXAMPLES.

The percentage being 250, and the rate .06, what is the ncipal?

$$=b\div r=100b\div p=250\div .06=25,000\div 6=4,166\tfrac{2}{3}.$$ *Ans.*

A tax at the rate of $\frac{5}{6}$ of 1 per cent. on the valuation was \$27.50. at was the valuation?

$$a=\frac{b\times 6\times 100}{5}=\$3,300.$$ *Ans.*

old 120 barrels of flour, which amounted to 12 per cent. of a ain consignment. The consignment consisted of how many rels?

$$120\div 0.12=1,000.$$ *Ans.*

16 bushels is *more* by 8 per cent., or 8 per cent. more, than what nber of bushels? 8 per cent. more than what number is equal 16? What number, plus 8 per cent. of it, will make 216?

$$a=s\div(1+r)=216\div 1.08=200.$$ *Ans.*

)0 lbs. is *less* by 8 per cent., or 8 per cent. less, than what number of lbs.? 8 per cent. less than what number is 200? What number, minus 8 per cent. of it, is equal to 200?

$$a = d \div (1 - r) = 200 \div (1 - .08) = 217\tfrac{9}{23}. \quad \textit{Ans.}$$

$$\therefore 217\tfrac{9}{23} - 217\tfrac{9}{23} \times .08 = 200 = a - b = d = a\,(1 - r).$$

To a quantity of silver, a quantity of copper equal to 20 per cent. of the silver is to be added, and the mass is to weigh 22 ounces. What weight of silver is required?

$$a = s \div (1 + r) = 22 \div 1.2 = 18\tfrac{1}{3} \text{ ounces.} \quad \textit{Ans.}$$

What weight of copper is required?

$$s - \frac{s}{1+r} = \frac{sr}{1+r} = 3\tfrac{2}{3} \text{ ounces.} \quad \textit{Ans.}$$

To a quantity of copper, a quantity of nickel equal to $62\frac{1}{2}$ per cent. of the copper, a quantity of zinc equal to $33\frac{1}{3}$ per cent. of the copper, and a quantity of lead equal to 5 per cent. of the copper, are to be added; and the whole is to weigh $40\frac{1}{6}$ pounds. The weight of each constituent of the alloy is required.

$$a = \frac{s}{1 + r + r' + r''} = \frac{40\frac{1}{6}}{1 + .62\frac{1}{2} + .33\frac{1}{3} + .05}$$

$$\left.\begin{aligned} &= 20 \text{ lbs. of copper,} \\ b &= 20\,r = 12\tfrac{1}{2} \text{ lbs. of nickel,} \\ b' &= 20\,r' = 6\tfrac{2}{3} \text{ lbs. of zinc,} \\ b'' &= 20\,r'' = 1 \text{ lb. of lead.} \end{aligned}\right\} \textit{Ans.}$$

INTEREST.

Universal for any rate per cent.

T = time in months and decimal parts of a month; t = time in days; P = principal; r = rate per cent., expressed decimally; i = interest.

$$i = \frac{\mathrm{P} \times \mathrm{T} \times r}{12} = \frac{\mathrm{P} \times t \times r}{365}.$$

$$\mathrm{P} = \frac{12\,i}{\mathrm{T}r} = \frac{365\,i}{t\,r}. \quad \mathrm{T} = \frac{12\,i}{\mathrm{P}\,r}. \quad t = \frac{365\,i}{\mathrm{P}\,r}. \quad r = \frac{12\,i}{\mathrm{PT}} = \frac{365\,i}{\mathrm{P}t}.$$

Example. — A promissory note, made April 27, 1864, for

$825\frac{25}{100}$ and interest at 6 per cent., matured Oct. 6, 1865: what was the interest?

Oct. is 10th month.			
April is 4th month.			
	Y.	*m.*	*d.*
	1865 .	10 .	6
	'64 .	4 .	27
Time =	1 .	5 .	9

Time from April 27 to Oct. 6 (one of the dates always included) = 162 days, which, added to the 365 days in the year preceding = 527 days.

NOTE.—One day's interest at least is generally lost by computing the time in years and months, or months, instead of days.

$$825.25 \times 17.3 \times .06 \div 12 = \$71.38. \quad \textit{Ans.}$$

$$825.25 \times 527 \times .06 \div 365 = \$71.49. \quad \textit{Ans.}$$

To find a constant divisor, k, for any given rate per cent.

When the time is taken in months, $k = 12 \div r$.
When the time is taken in days, $k = 365 \div r$; thus,

When the RATE *is* 6 *per cent.* $\frac{P \times t}{6083} =$ Interest.

When the RATE *is* 7 *per cent.* $\frac{P \times t}{5214} =$ Interest, &c.

EXAMPLE.—Required the interest on \$750 for 93 days, at 7 per cent.

$$750 \times 93 \div 5214 = \$13.38. \quad \textit{Ans.}$$

EXAMPLE.—What is the rate per cent. when \$450 gains \$94½ in 3 years?

$$450 : 100 :: 94.5 : 3x = 7 \text{ per cent.} \quad \textit{Ans.}$$

$$94.5 \div 3 \times 450 = .07. \quad \textit{Ans.}$$

EXAMPLE.—In what time will \$125 at 6 per cent. gain \$18¾?

$$6 : 100 :: 18.75 : 125 \times x = 2\tfrac{1}{2} \text{ years.} \quad \textit{Ans.}$$

$$18.75 \div 125 \times .06 = 2\tfrac{1}{2} \text{ years.} \quad \textit{Ans.}$$

EXAMPLE.—What principal at 5 per cent. interest will gain \$16⅞ in 18 months?

$$5 : 100 :: 16.875 : 1.5 \times x = \$225. \quad \textit{Ans.}$$

$$16.875 \times 12 \div 18 \times .05 = \$225. \quad \textit{Ans.}$$

When partial payments have been made.

RULE. — Find the amount (sum of the principal and interest) up to the time of the first payment, and deduct the payment therefrom; then find the interest on the remainder up to the next payment, add it to the remainder, or new principal, and from the sum subtract the next payment; and so on for all the payments; then find the amount up to the time of final payment for the final amount.

COMPOUND INTEREST.

If we calculate the interest on a debt for one year, and then on the same debt for another year, and again on the same debt for still another year, the sum will be the *simple* interest on the debt for three years. But, on the contrary, if we calculate the interest on the debt for one year, and then on the *amount* (sum of the principal and interest) for the next year, and then on the second amount for the third year, the sum of the interest so calculated will be the *compound* interest, or yearly compound interest, on the debt for three years; equal to the simple interest on the debt for three years, plus the yearly compound interest on the first year's interest for two years, plus the simple interest on the second year's interest for one year. So, if we divide the time into shorter periods than a year, and proceed for the interest as last suggested, the interest will be compound. Thus we have half-yearly compound interest, or compound interest semi-annually, quarter-yearly compound interest, or compound interest quarterly, &c.

This method of computing interest is predicated upon the natural idea, that interest, when it becomes due by stipulation and is withheld, commences to draw interest, and continues at use to the holder, at the same rate as the principal, until it is paid, like other over-due demands; and that the interest so made matures and becomes due as often, and at the same periods, as that on the principal.

It will be perceived by the foregoing that the *working-time* in compound interest is the interval between the stipulated payments of the interest, or between one stipulated payment of the interest and that of another; and that the *working-rate* is pro rata to the rate per annum.

Thus the *amount* of $100 at semi-annual compound interest for 2 years, at 6 per cent. per annum, is

$100 \times (1.03)^4 = \$112.550881 = \$112.55$, or

$$
\begin{array}{r}
100. \\
.03 \\
\hline
3. \\
100. \\
\hline
103. \\
.03 \\
\hline
3.09 \\
103. \\
\hline
106.09 \\
.03 \\
\hline
3.1827 \\
106.09 \\
\hline
109.2727 \\
.03 \\
\hline
3.278181 \\
109.2727 \\
\hline
\$112.550881
\end{array}
$$

$112.550881, as before.

If we let $P =$ principal or debt at interest,
$r =$ working-rate of interest,
$n =$ number of intervals into which the whole time is divided for the payment of interest, or number of consecutive intervals for the payment of interest that have transpired without a payment having been made,
$i =$ compound interest,
$A = P + i$ or amount, then

$$A = P(1 + r)^n;\ P = \frac{A}{(1+r)^n};\ r = \sqrt[n]{\frac{A}{P}} - 1;$$

$$\frac{A}{P} = (1 + r)^n;\ i = A - P.$$

EXAMPLE. — What is the compound interest, or yearly compound interest, on \$100 for $1\frac{1}{2}$ years, at 6 per cent. a year?

$$100 \times 1.06 \times 1.03 = 109.18 - 100 = \$9.18. \quad \textit{Ans.}$$

EXAMPLE. — What is the amount of \$560.46, at 7 per cent. compound interest per year, for 6 years and 57 days?

$$560.46 \times (1.07)^6 \times \left(1 + \frac{.07 \times 57}{365}\right) = \$850.29. \quad \textit{Ans.}$$

EXAMPLE. — The *principal* is \$250, the *rate* 8 per cent. a year, the *time* 2 years, and the *interest* compound per quarter year: required the *amount.*

$$250 \times \left(1.\frac{.08}{4}\right)^8 = \$292.91. \quad Ans.$$

When Partial Payments have been made.

RULE. — Find the amount up to the first payment, and deduct the payment therefrom; then find the amount up to the next payment, and therefrom deduct that payment; and so on for all the payments; then find the amount up to the time of final payment, for the final amount.

EXAMPLE. — A note of hand for \$500 and interest from date, at 6 per cent. a year, has been paid in part as follows; viz., two years and four months from the date of the note, by an indorsement of \$50; and three years from that indorsement, by an indorsement of \$150. It is now eight months since the last payment was made, and the demand is to be settled in full: required the amount at the present time, interest being compound per year.

$$
\begin{array}{lr}
500 \times (1.06)^2 \times 1.02 - 50 = & 523.036 \\
 & (1.06)^3 \\
\hline
 & 622.944 \\
 & 150 \\
\hline
 & 472.944 \\
 & 1.04 \\
\hline
 & \$491.86. \quad Ans.
\end{array}
$$

The following table shows $(1+r)$ raised to all the integer powers from 1 to 12 inclusive; r being taken at 4, 5, 6, 7, 8, and 10 per cent. If the numbers in the column headed years are taken to represent years, then 4 per cent., 5 per cent., &c., at the head of the columns of powers, will stand for per cent. per annum: if they are taken to represent half-years, then 4 per cent., 5 per cent., &c., will stand for per cent. per half-year, &c. The quantities in the columns are powers of $(1+r)$, of which the numbers referred to and standing opposite, respectively, are the exponents. Thus, 1.26248, in the 6 per cent. column, and against 4 in the column marked years, $= (1.06)^4$; and so with the others. The powers or quantities in the columns are co-efficients in the calculations.

Years.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	10 per cent.
1	1.04	1.05	1.06	1.07	1.08	1.10
2	1.0816	1.1025	1.1236	1.1449	1.1664	1.21
3	1.12486	1.15762	1.19102	1.22504	1.25971	1.331
4	1.16986	1.21551	1.26248	1.3108	1.36049	1.4641
5	1.21665	1.27628	1.33823	1.40255	1.46933	1.61051
6	1.26532	1.3401	1.41852	1.50073	1.58687	1.77156
7	1.31593	1.4071	1.50363	1.60578	1.71382	1.94872
8	1.36857	1.47746	1.59385	1.71819	1.85093	2.14359
9	1.42331	1.55133	1.68948	1.83846	1.999	2.35795
10	1.48024	1.62889	1.79085	1.96715	2.15892	2.59374
11	1.53945	1.71034	1.8983	2.10485	2.33164	2.85312
12	1.60103	1.79586	2.0122	2.25219	2.51817	3.13843

NOTE.—If a co-efficient is wanted for a greater number of years or intervals of time than is given in the table, square the tabular co-efficient opposite half that number of intervals, or cube the tabular co-efficient opposite one-third that number of intervals, &c., for the co-efficient required. Thus,

$$1.999^2 = 1.58687^3 = 1.08^{12} \times 1.08^6 = 1.08^{18} = 3.996,$$

the co-efficient for 18 years or intervals at 8 per cent. per interval, &c.

If the compound interest alone is sought on a given principal, subtract 1 from the tabular power corresponding to the time and rate, and multiply the remainder by the given principal; the product will be the compound interest. Thus $(1.26532 - 1) \times 100 = \$26.532$, the yearly compound interest, at 4 per cent. per annum, on $100 for 6 years, or the half-yearly compound interest, at 8 per cent. per annum, on $100 for 3 years, or the half-yearly compound interest, at 4 per cent. per half year, on $100 for 6 half-years.

EXAMPLE.—What is the amount of $125.54, at 5 per cent. compound interest, for 7 years, 21 days?

$1 + \frac{21 \times .05}{365} = 1.00288$, the co-efficient for the odd days; and, turning to the 5 per cent. column in the table, we find against 7, in the column of years, 1.4071, the co-efficient for 7 years: then

$$125.54 \times 1.4071 \times 1.00288 = \$178.20. \quad \textit{Ans.}$$

EXAMPLE.—In what time, at 7 per cent. compound interest per annum, will $1000 gain $462? $A \div P = (1 + r)^n$: then $1462 \div 1000 = 1.462$, the co-efficient demanded. Turning now to the 7 per cent. column in the table, we find the nearest less co-efficient there (there being none that exactly corresponds) to be that for 5 years; viz., 1.40255. And $\left(\frac{1.462}{1.40255} - 1\right) \div .07 =$ .60553, the fraction of a year over 5 years to the answer.

$$.60553 \times 365 = 221 \text{ days}: \text{ 5 years, 221 days.} \quad \textit{Ans.}$$

The following TABLE is of the same nature as the preceding, and is applicable when the interest becomes due at regular intervals short of a year, or when the working-rate in compound interest is less than 4 per cent.

The quantities in the $1\frac{3}{4}$ per cent. column apply to quarter-yearly compound interest when the rate is 7 per cent. a year; and those in the $1\frac{1}{4}$ per cent. column, to quarterly compound interest when the rate is 5 per cent. a year; also the former are applicable to monthly compound interest at 21 per cent. per annum, and the latter to monthly compound interest at 15 per cent. per annum; and so relatively, throughout the table.

Times.	3½ per cent.	3 per cent.	2½ per cent.	2 per cent.	1¾ per cent.	1½ per cent.	1¼ per cent.	1 per cent.	½ per cent.
1	1.035	1.03	1.025	1.02	1.0175	1.015	1.0125	1.01	1.005
2	1.07123	1.0609	1.05063	1.0404	1.03531	1.03023	1.02516	1.0201	1.01003
3	1.10872	1.09273	1.07689	1.06121	1.05342	1.04568	1.03797	1.0303	1.01508
4	1.14752	1.12551	1.10381	1.08243	1.07186	1.06136	1.05095	1.0406	1.02015
5	1.18769	1.15927	1.13141	1.10408	1.09062	1.07728	1.06408	1.05101	1.02525
6	1.22925	1.19405	1.15969	1.12616	1.1077	1.09344	1.0774	1.06152	1.03038
7	1.27228	1.22987	1.18869	1.14869	1.12709	1.10984	1.09087	1.07214	1.03553
8	1.31681	1.26677	1.2184	1.17166	1.14681	1.12649	1.10451	1.08286	1.04071
9	1.3629	1.30477	1.24886	1.19509	1.16688	1.14339	1.11831	1.09369	1.04591
10	1.4106	1.34392	1.28008	1.21899	1.1873	1.16054	1.13229	1.10462	1.05114
11	1.45997	1.38423	1.31209	1.24337	1.20808	1.17795	1.14645	1.11567	1.0564
12	1.51107	1.42576	1.34489	1.26824	1.22922	1.19562	1.16078	1.12683	1.06168

EXAMPLE.—What is the amount of $750 for 4 years and 40 days, allowing half-yearly compound interest, at 7 per cent. a year?

In this case, the working-rate for the full periods of time is $3\frac{1}{2}$ per cent., and there are 8 such full periods; then, seeking the co-efficient in the $3\frac{1}{2}$ per cent. column, we find against 8, in the column of times, the quantity or co-efficient 1.31681; and $1+\frac{40 \times .07}{365} =$ 1.00767: therefore

$$750 \times 1.31681 \times 1.00767 = \$995.18. \quad Ans.$$

EXAMPLE.—What is the amount of $1000 at compound interest per quarter-year, at $1\frac{1}{2}$ per cent. per quarter-year, for $4\frac{1}{4}$ years?

$$1000 \times 1.12649^2 \times 1.015 = \$1288.01. \quad Ans.$$

BANK INTEREST OR BANK DISCOUNT.

A bank loans money on a promissory note made payable without interest at a future period. The operation is called *discounting* the note at bank, and is as follows: The bank takes the note, finds the interest on it for three days more time than by its own tenor it has to run, subtracts it from the principal, and hands the balance, called the *avails* of the note, in its own bills, to the party soliciting the loan, or offering the note for discount, as it is called; whereby the note becomes the property of the bank, and the maker and indorsers are held for its payment when it matures.

The three days mentioned are called *days of grace*, and the note does not become due to the bank until three days after it becomes due by its own tenor. These proceedings are sanctioned by usage, and protected by law.

Bank interest, then, is bank discount, and bank discount is bank interest. But bank discount is not *discount*, nor is it what is called *legal* interest on the money loaned. It is the interest on the money loaned, plus the interest on the interest of the loan, plus the interest on the difference of the sum taken and the interest on the loan for the time of the loan! A kind of interest more onerous, if any description of interest be onerous, than compound interest, rate for rate and time for time, as may be readily perceived.

Let $P=$ principal or face of the note.
$r=$ working-rate of the interest for the time of the loan.
$a=$ avails of the note or sum borrowed.
$i=$ bank interest.
$t=$ time of the loan.

$R : r :: T : t$. R being the rate per cent. per annum, and T one year.

$P=a\div(1-r)$. $a=P-Pr$. $i=Pr$. $r=(P-a)\div P$.

If we let n represent the time of the note in months,

$r=\frac{Rn}{12}+\frac{3R}{365}$. But it is the practice with many banks to count the days of grace as so many 360ths of a year.

Putting d to represent the time of the note in days,

$$r=\frac{Rd+3R}{365}, \text{ true time and rate.}$$

With some banks, it is the practice, in calculating interest, to take the time, when it does not exceed 93 days, as so many 360ths of a year.

A note having 3 months to run from Aug. 10, for instance, will fall due Nov. 10–13; but one having 90 days to run from Aug. 10 will fall Nov. 8–11. The time including grace of the former is 3 mo. 3 ds., and that of the latter 3 mo. 2 ds., mean time. Nevertheless, the former embraces 95 days, or one day more than mean time, and the latter but 93 days.

The following table shows $1-r$, mean time, for the intervals of time set down in the left-hand column; R being taken at 4, 5, 6, 7, and 8 per cent. per annum, as set down at the top of the columns.

Time. mo.	ds.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1	3	.996333	.995417	.9945	.993583	.992667
2	3	.993	.99125	.9895	.98775	.986
3	3	.989667	.987083	.9845	.981917	.979333
4	3	.986333	.982917	.9795	.976083	.972667
5	3	.983	.97875	.9745	.97025	.966
6	3	.979667	.974583	.9695	.964417	.959333
7	3	.976333	.970417	.9645	.958583	.952667
8	3	.973	.96625	.9595	.95275	.946
9	3	.969667	.962083	.9545	.946917	.939333
10	3	.966333	.957917	.9495	.941083	.932667
11	3	.963	.95375	.9445	.93525	.926
12	3	.959667	.949583	.9395	.929417	.919333

Putting k to represent the tabular quantity $1-r$,

$$a = Pk,\ P = a \div k,\ i = P - a = P - Pk.$$

Example. — What will be the avails of a note for $1,250 payable in 4 months if discounted at a bank, interest being 7 per cent. a year?

The tabular constant $1-r$, in the 7 per cent. column, against 4 months and 3 days in the time column, is .976083, and

$1,250 × .976083 = $1,220.10. *Ans.*

Example. — For what sum must I make a note having 6 months to run, in order that the avails at bank, if discounted on the day of the date of the note, may amount to $956.38, interest being 6 per cent. per annum?

By the table, $956.38 ÷ .9695 = $986.47. *Ans.*

Example. — What is the rate of bank interest when the nominal or legal rate is 7 per cent.?

$.07 \div (1 - .07) = .07527 = 7\frac{1}{2} + \frac{27}{1000}$ per cent. *Ans.*

Note.—A note having 5 months to run from Feb. 1 will fall due July 1–4; and the time, including grace, is 5 mo. 3 ds. = 155 days, mean time. But the time in days from Feb. 1 to July 4, when February has but 28 days, is 153 days only, or 2 days short of mean time. See Sec. B., p. 3.

DISCOUNT.

DISCOUNT is a deduction of the interest on the present worth or availability of a debt not yet due, in consideration of its present payment. The *principal* is the present nominal value of the debt, interest included, if any interest has accrued. The *time* is the interval from the present to the date at which the debt will become due. The *rate* is the legal rate of interest, if no other rate is specified; and the *present worth* is that sum of money, which, if put at interest at the same rate and for the same time as the discount, will amount to the principal.

Let a represent the principal, d the discount, w the present worth, and i the interest on one dollar for the time and at the rate of the discount.

$$w = a \div (1 + i) = a - d. \quad d = ai \div (1 + i) = a - w.$$
$$a = d\,(1 + i) \div i = d + w.$$

EXAMPLE. — Required the discount on \$250 for 8 months at 6 per cent.

The interest on \$1 for 8 months at 6 per cent. is .04 of a dollar, or 4 cts.; and

$$250 \times .04 \div (1 + .04) = \$9.6154. \quad Ans.$$

EXAMPLE. — Required the present worth of \$1272.62 due 247 days hence, discount 7 per cent.

The interest on \$1 for 247 days at 7 per cent. $= 247 \times .07 \div 365 = 0.04737$, and

$$1272.62 \div 1.04737 = \$1215.06. \quad Ans.$$

NOTE.—"*Taking off*, in common parlance, a certain per centum from the face of a demand, is equal to deducting the interest, at that rate per centum, on the present worth for 1 year, plus the interest on the interest of the present worth, at the same rate per centum for 1 year. See SEC. B., p. 1.

COMPOUND DISCOUNT.

COMPOUND DISCOUNT is to compound interest what simple discount is to simple interest. In both cases of discount, the difference between the principal and the discount is that sum of money, which, if put at interest for the same length of time, at the same rate, and in the same general manner as the discount, will amount to the principal.

RULE. — Add 1 to the rate per cent. of the discount for the working-time, and raise the sum to a power corresponding with the number of working-times; divide the principal by the power, and the quotient will be the present worth; subtract the present worth from the principal, and the remainder will be the compound discount.

NOTE.—The TABLES of the powers of $1 + r$, applicable to compound interest, are equally applicable to compound discount.

EXAMPLE.—Required the present worth of a debt of $250, allowing yearly compound discount, at 7 per cent. a year, for 3 years 84 days.

$1 + \frac{.07 \times 84}{365} = 1.01611$, the working-rate for the 84 days, and

$$250 \div (1.07^3 \times 1.01611) = \$200.84. \quad \textit{Ans.}$$

EXAMPLE.—What is the present worth of a debt of $150.25, due 3 years, 3 months, and 10 days hence, without interest, allowing compound discount per quarter-year, at $1\frac{1}{2}$ per cent. per quarter-year?

$$150.25 \div \left(1.015^{13} \times 1.\frac{.06 \times 10}{365}\right) = \textit{Ans.}$$

By table, $150.25 \div (1.19562 \times 1.015 \times 1.00164) =$ $123.61. *Ans.*

NOTE.—What is here denominated the debt, or principal, represents the debt at the close of the time of the discount; that is, if the debt be on interest, the interest must be included in what is here called the debt, or principal.

PROFIT AND LOSS.

The term "PROFIT AND LOSS," as intimated in treating of PERCENTAGE, relates to the positive and negative interests in business, and embraces the idea of both.

Both profit and loss are absolute quantities, and are expressed by the difference of the cost price and selling price that limit them. They are usually, however, estimated by percentage, predicated upon the first-mentioned price or prime cost.

When the selling price is greater than the cost price, or when the money obtained by the disposal of property exceeds what the property cost, the difference is positive, and denotes increase, profit, or gain. Conversely, when the cost price is greater than the selling price, or when property is disposed of for less money than it cost, the difference is negative, and denotes decrease, loss, or waste. So, the difference of the two prices, divided by the cost price, expresses the rate of gain on the cost when the selling price is the greater, — expresses the rate of loss on the cost when the cost price is the greater.

Let c represent the cost price, purchase price, par value, or sum of money paid for the property; s, the selling price, trade price, premium price, or sum of money received in exchange for the property; r, the rate of the profit or loss; p, the rate per cent. of the profit or loss.

To find the rate or rate per cent. of the profit or loss.

$r = \frac{s \sim c}{c}$. $p = \frac{(s \sim c)\, 100}{c}$. Moreover, when the difference is positive, $r = \frac{s}{c} - 1$; and, when it is negative, $r = 1 - \frac{s}{c}$.

EXAMPLE. — Paid \$4 for an article, and sold it for \$5. What per cent. was gained? 5 is more than 4 by what per cent. of 4? The difference of 5 and 4 is what per cent. of 4? $5 - 4 = \$1$, gained; and $\frac{5 \sim 4}{4} = .25 = \frac{5}{4} - 1$. 25 per cent. *Ans.*

EXAMPLE. — Paid \$5 for an article, and sold it for \$4. What per cent. was lost? 4 is less than 5 by what per cent. of 5? The difference of 4 and 5 is what per cent. of 5? $4 - 5 = -1 = \$1$, lost; and $\frac{5 \sim 4}{5} = .20 = 1 - \frac{4}{5}$. 20 per cent. *Ans.*

EXAMPLE. — A whistle that cost 3 cents was sold for 20 cents! The profit was how much per cent? $(20 \sim 3) \div 3 = 5\frac{2}{3}$ or $566\frac{2}{3}$ per cent. *Ans.*

EXAMPLE. — A fop paid \$10 for a well-made and well-fitting pair of boots for his own wear, that were worth what they cost him; but, being told that they were unfashionably large, sold them for \$4. His vanity cost him what per cent. of the purchase price? $1 - \frac{4}{10} = .6$ or 60 per cent. *Ans.*

To find a price long a given per cent. of the cost, or to find a selling price that shall be the sum of the cost price and a given per cent. of it.

$$s = c + cr = c\,(1 + r) = c\,(100 + p) \div 100.$$

EXAMPLE. — At what price must I sell an article that cost \$2.35 to gain 25 per cent.? 2.35, more 25 per cent. of it, is how much? The sum of \$2.35 and 25 per cent. of it is how much? $2.35 + 2.35 \times .25 = 2.35 \times 1.25 = \$2.93\frac{3}{4}$. *Ans.*

To find a price short a given per cent. of the cost, or to find a selling price that shall be the difference of the cost price and a given per cent. of it.

$$s = c - cr = c\,(1 - r) = c\,(100 - p) \div 100.$$

Example. — I have a damaged article of merchandise that cost \$2.75, and I wish to mark it for sale at 30 per cent. below cost. At what price shall I mark it? 2.75 less 30 per cent. of it is how much? The difference of \$2.75 and 30 per cent. of it is how much? 2.75 (1 —.30) = 2.75 × .7 = \$1.925. *Ans.*

To find the cost price when the selling price and profit per cent. are given.

$$s = c + cr = c\,(1 + r)\;\therefore\; c = s \div (1 + r) = 100\,s \div (100 + p).$$

Example. — What cost that article whose selling price, \$4, is long 25 per cent. of the cost? What price, more 25 per cent. of it, is equal to \$4? \$4 is the sum of what price and 25 per cent. of it? 400 ÷ 125 = \$3.20. *Ans.*

To find the cost price when the selling price and loss per cent. are given.

$$s = c - cr = c\,(1 - r)\;\therefore\; c = s \div (1 - r) = 100\,s \div (100 - p)$$

Example. — What cost that article whose selling price, \$375, is short 7 per cent. of the cost? What price less 7 per cent. of it is equal to \$375? \$375 is the difference of what price and 7 per cent. of it?

375 ÷ (1 — .07) = 375 ÷ .93 = 375 × 100 ÷ (100 — 7) = \$403.226. *Ans.*

EQUATION OF PAYMENTS.

The Equation of Payments, or *Averaging of Accounts*, as it is more frequently called, practically consists in finding the common time of maturity of two or more debts due at different times, and is either special or general; special when it is made in regard to a given interchangeable rate of interest and discount, in which the magnitude of the rate slightly affects the time, since discount consumes more time per dollar, rate for rate, than interest; and general, when it is made disrespectful of rate, or common in the greatest possible degree to all rates.

Rule, for common purposes. — Multiply each debt by the number of days between its own date of maturity and that of the debt earliest due, and divide the sum of the products by the sum of the debts; the quotient will express the common time in days subsequent to the leading date.

The following exhibits the face of an account in the edger, and the time (date) at which it averages due is required.

1860, April 10 ——	$250.26 —	6 mo.	Due Oct.	10.
" June 25 ——	320.56 —	6 "	" Dec.	25.
" July 10 ——	50.62 —	3 "	" Oct.	10.
" Aug. 1 ——	210.84 —	4 "	" Dec.	1.
" " 18 ——	73.40 —	5 "	" Jan.	18.
" Oct. 15 ——	100.	— cash	" Oct.	15.

EXAMPLE. — Practical method of stating and working.

1860. Due Oct. 10,	$301			
" " Dec. 25,	321 ×	76 =	24396.	
" " " 1,	211 ×	52 =	10972.	
" " Jan. 18,	73 ×	100 =	7300.	
" " Oct. 15,	100 ×	5 =	500.	
	1006	)	43168 (	43 days, = Nov. 22, 1860. *Ans.*

COMPOUND AVERAGE.

COMPOUND AVERAGE consists in finding the time at which the *balance* of an account or demand averages due, whose sides — the debit and the credit — average due at different dates.

RULE. — Multiply the less sum or side by the difference in days between the two dates — that at which the debit side averages due and that at which the credit side averages due — and divide the product by the difference of the sums or sides; the quotient will be the number of days that one of the dates must be set back, or the other forward, to mark the time sought; for which last,

SPECIAL RULE.

Earlier date with larger sum, set back from earlier.
Later date with larger sum, set forward from later.

EXAMPLE. — The debit side of an account in the ledger foots up $400, and averages due Oct. 12, 1860; the credit side of the same account foots $300, and averages due Nov. 16, 1860. At what date does the balance or difference between the two sides average due?

400	300	
300	35	
100)	10500 (	105 days earlier than Oct. 12, = June 29, 1860. *Ans.*

EXAMPLE. — The debit side of an obligation foots $250, and averages due May 17, 1860; the credit side of the same obligation foots $175, and averages due May 1, 1860. At what date does the difference of the sides average due?

250	175	
175	16	
75)	2800 (	37⅓ days later than May 17, = June 23, 1860. *Ans.*

GENERAL AVERAGE.

It is the established usage that whatever of either of the three commercial interests — the ship, the cargo, or the freight — is voluntarily sacrificed or destroyed for the general good, or with the view of saving the most that may be saved when all is in imminent danger of being lost, is matter of general loss to the respective interests, and not more especially to the interest voluntarily abandoned than to the others. So, too, the losses and damages incident to the voluntary sacrifice, and collateral therewith, together with the expenditures which the master has been compelled to make for the general good, in consequence of disaster, are matters of general average, or are to be contributed for, *pro rata*, by the several interests.

The contributory interests are the ship, the cargo, and the freight, at their net values, independent of charges, premiums paid for insurance, &c.

The contributory value of the ship, generally, is her value at the port of departure at the time of leaving, less the premium paid for her insurance.

The contributory value of the cargo is its net value, in a sound state, at the port of destination, if the voyage be completed; or its invoice value if the voyage be broken up and the cargo returned to the port whence it was shipped; or its market-value at any intermediate port, where of necessity it is discharged and disposed of. The value of the goods jettisoned, and to be contributed for, is their value after the same manner; and that value is a part of the contributory value of the cargo, as well as a matter of general average.

The contributory value of the freight, generally, is the gross amount or amount per freight-list, less one-third part thereof, in most of the States; but, in the State of New York, one-half thereof, for seamen's wages and other expenses. The loss of freight by jettison, when any freight is earned, is matter of general average. If the cargo is transshipped on board another vessel, and in that way sent to the port of destination, the contributory value of the freight is the gross amount, less the sum paid the other vessel.

The voluntary damage to the ship, with a view to the general good, — such as throwing over her furniture, destroying her equipments, cutting away her masts, breaking up her decks to get at the cargo for the purpose of throwing it over, &c., — is contributed for at two-thirds the cost of repairing and restoring; the new articles being supposed one-half better, or worth one-half more, than the old.

If we let $V =$ contributory value of the vessel,
$C =$ contributory value of the cargo,
$F =$ contributory value of the freight,
$d =$ aggregate amount of losses to be averaged, then
$d \div (V + C + F) = r$, the per cent. of each interest that each must contribute, and

$$V \times r = \text{Vessel's share of the loss,}$$
$$C \times r = \text{Cargo's share of the loss,}$$
$$F \times r = \text{Freight's share of the loss.}$$

When a contributory interest's share of the loss is to be distributed among the several owners of that interest, the same *pro rata* method is to be observed: thus

$$A \times r = \text{sum A must contribute,}$$
$$B \times r = \text{sum B must contribute,}$$
$$D \times r = \text{sum D must contribute;}$$

A, B, and D being A's, B's, and D's respective shares in that interest.

ASSESSMENT OF TAXES.

G = amount of taxable property, real and personal, as per grand list.
A = amount of money to be raised, including the whole poll-tax.
T = amount of money to be raised on property alone.
n = number of ratable polls.
h = poll-tax per head.
r = rate per cent. to be raised on taxable property.
P = an individual's taxable property, as per grand list.
b = P's poll-tax.

$T = A - hn$. $r = T \div G$. $Pr + b$ = P's tax, including poll.

INSURANCE.

Insurance is a written contract of indemnity, called the *policy*, by which one party (the *insurer* or *underwriter*) engages, for a stipulated sum, called the *premium* (usually a per cent. on the value of the property insured), to insure another against a risk or loss to which he is exposed.

Let P = Principal, or amount insured on,
r = rate per cent. of insurance,
a = premium for insurance.

$$a = Pr. \quad r = a \div P. \quad P = a \div r.$$

Example.—What is the premium for insuring on $4500 at $1\frac{1}{2}$ per cent.?

$4500 \times .015 = \$67.50$. *Ans.*

LIFE-INSURANCE.

Life-insurance is predicated upon the even chance in years, called the *expectation of life*, that an individual in general health at any given age appears by the rates of mortality to have of living beyond that age.

The Carlisle Tables of Expectation, column C in the following tables, are used almost or quite exclusively in England, and by some insurance-companies in the United States; while those by Dr. Wigglesworth, column W, computed with special reference to the rates of mortality in this country, are used by others.

The Supreme Court of Massachusetts has adopted the Wigglesworth rates of expectation in estimating the value of life-annuities and life-estates.

TABLE

Of Ages and Expectations from Birth to 103 *Years.*

Age.	C.	W.	Age.	C.	W.	Age.	C.	W.	Age.	C.	W.
0	38.72	28.15	26	37.14	31.93	52	19.68	20.05	78	6.12	6.59
1	44.68	36.78	27	36.41	31.50	53	18.97	19.46	79	5.80	6.21
2	47.55	38.74	28	35.69	31.08	54	18.28	18.92	80	5.51	5.85
3	49.82	40.01	29	35.00	30.66	55	17.58	18.35	81	5.21	5.50
4	50.76	40.73	30	34.34	30.25	56	16.89	17.78	82	4.93	5.16
5	51.25	40.88	31	33.68	29.83	57	16.21	17.20	83	4.65	4.87
6	51.17	40.69	32	33.03	29.43	58	15.55	16.63	84	4.39	4.66
7	50.80	40.47	33	32.36	29.02	59	14.92	16.04	85	4.12	4.57
8	50.24	40.14	34	31.68	28.62	60	14.34	15.45	86	3.90	4.21
9	49.57	39.72	35	31.00	28.22	61	13.82	14.86	87	3.71	3.90
10	48.82	39.23	36	30.32	27.78	62	13.31	14.26	88	3.59	3.67
11	48.04	38.64	37	29.64	27.34	63	12.81	13.66	89	3.47	3.56
12	47.27	38.02	38	28.96	26.91	64	12.30	13.05	90	3.28	3.43
13	46.51	37.41	39	28.28	26.47	65	11.79	12.43	91	3.26	3.32
14	45.75	36.79	40	27.61	26.04	66	11.27	11.96	92	3.37	3.12
15	45.00	36.17	41	26.97	25.61	67	10.75	11.48	93	3.48	2.40
16	44.27	35.76	42	26.34	25.19	68	10.23	11.01	94	3.53	1.98
17	43.57	35.37	43	25.71	24.77	69	9.70	10.50	95	3.53	1.62
18	42.87	34.98	44	25.09	24.35	70	9.18	10.06	96	3.46	
19	42.17	34.59	45	24.46	23.92	71	8.65	9.60	97	3.28	
20	41.46	34.22	46	23.82	23.37	72	8.16	9.14	98	3.07	
21	40.75	33.84	47	23.17	22.83	73	7.72	8.69	99	2.77	
22	40.04	33.46	48	22.50	22.27	74	7.33	8.25	100	2.28	
23	39.31	33.08	49	21.81	21.72	75	7.01	7.83	101	1.79	
24	38.59	32.70	50	21.11	21.17	76	6.69	7.40	102	1.30	
25	37.86	32.33	51	20.39	20.61	77	6.40	6.99	103	0.83	

Thus, by the tables, a man in general good health at 21 years of age has an even chance, by the Carlisle rate of mortality, of living $40\frac{3}{4}$ years longer; by the Wigglesworth rate, of living $33\frac{84}{100}$ years longer. So a man in general good health, at 60 years of age, has, by the Carlisle rate, an even chance of living 14.34 years longer; by the Wigglesworth rate, an even chance of living 15.45 years longer, etc.

FELLOWSHIP.

Fellowship calls for the distribution of a given effect to each of the several causes associated in its production, proportional to their respective magnitudes one with another.

It is a rule, therefore, adapted to the use of partners associated in business, in achieving a *pro rata* distribution among themselves as individuals, of the profits or losses pertaining to the company.

Rule. — Multiply each partner's investment or share of the capital stock, by the whole gain or loss, and divide the product by the sum of all the shares, or gross capital.

Example. — Three men, A, B, and C, enter into partnership. A invests \$500, B \$700, and C \$300. They trade and gain \$400. What is each partner's share of the profits?

A, \$500 | $500 \times 400 \div 1500 =$ \$133.33⅓ = A's share.
B, 700 | $700 \times 400 \div 1500 =$ 186.66⅔ = B's "
C, 300 | $300 \times 400 \div 1500 =$ 80.00 = C's "

\$1500 = gross capital. \$400.00 Proof.

Example. — D's investment of \$600 has been employed eight months; E's, of \$500, five months; and F's, of \$300, five months; the profits of the company are \$500, and are to be divided *pro rata* among the partners. What is each partner's share?

D, $\$600 \times 8 = 4800 \times 500 \div 8800 =$ \$272.73, D's share.
E, $500 \times 5 = 2500 \times 500 \div 8800 =$ 142.05, E's "
F, $300 \times 5 = 1500 \times 500 \div 8800 =$ 85.22, F's "

8800 \$500. Proof.

Example. — Of \$120 distributed, there were given to A, $\frac{1}{3}$; to B, $\frac{1}{4}$; to C, $\frac{1}{5}$; and to D, $\frac{1}{6}$, and there was nothing remaining. What sum did each receive?

$\frac{1}{3}$ of $120 = 40 \times 120 \div 114 = \$42\frac{2}{19}$ = A's share.
$\frac{1}{4}$ of $120 = 30 \times 120 \div 114 = 31\frac{11}{19}$ = B's "
$\frac{1}{5}$ of $120 = 24 \times 120 \div 114 = 25\frac{5}{19}$ = C's "
$\frac{1}{6}$ of $120 = 20 \times 120 \div 114 = 21\frac{1}{19}$ = D's "

114 \$120. Proof.

Example. — Divide the number 180 into 3 parts, which shall be to each other as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

$\frac{1}{2}$ of $180 = 90 \times 180 \div 195 = 83.08$
$\frac{1}{3}$ of $180 = 60 \times 180 \div 195 = 55.38$
$\frac{1}{4}$ of $180 = 45 \times 180 \div 195 = 41.54$

195 180.00 Proof.

EXAMPLE.—$400 are to be divided between A, B, and C, in the ratio of $\frac{1}{2}$ to A, $\frac{1}{2}$ to B, and $\frac{1}{4}$ to C; how much will each receive?

$\frac{1}{2}$ of $400 = 200$, and $200 \times 400 \div 500 = \$160$ = A's share.
$\frac{1}{2}$ of $400 = 200$, and $200 \times 400 \div 500 = 160$ = B's share.
$\frac{1}{4}$ of $400 = \underline{100}$, and $100 \times 400 \div 500 = \underline{80}$ = C's share.
500 $400. Proof.

ALLIGATION.

ALLIGATION *Medial* is a method by which to find the mean price of a mixture or compound, consisting of two or more articles or ingredients, the quantity and price of each being given.

RULE. — Multiply each quantity by its price, and divide the sum of the products by the sum of the quantities; the quotient will be the price per unity of measure of the mixture; and, having found the price of the given quantities as mixed, any quantities of the same materials, taken in like proportions, will be at the same price.

EXAMPLE. — If 20 lbs. of sugar at 8 cents, 40 lbs. at 7 cents, and 80 lbs. at 5 cents per pound, be mixed together, what will be the mean price, or price per pound, of the mixture?

$20 \times 8 = 160$
$40 \times 7 = 280$
$\underline{80} \times 5 = \underline{400}$
140) 840 (6 cents. *Ans.*

The several kinds, then, at their respective prices, taken in the proportion of 1 at 8, 2 at 7, and 4 at 5 cts., will form a mixture worth 6 cts. a pound.

EXAMPLE. — If 10 lbs. of nickel are worth $2, and 24 lbs. of copper are worth $4½, and 8 lbs. of zinc are worth 40 cts., and 1 lb. of lead is worth 5 cts., what are 5 lbs. of *pretty good* German silver worth?

$$\frac{(200+450+40+5)\times 5}{43} = .81 \text{ cents. } \textit{Ans.}$$

ALLIGATION *Alternate* is a method by which to find what quantity of each of two or more articles or ingredients, whose prices or qualities are given, must be taken to form a mixture or compound that shall be at a given price or of a given quality between the two extremes. It also applies to the finding of relative quantities when the quantity of one or more of the articles is limited.

RULE. — Connect the given prices or qualities — a less than the given mean with that one or either one that is greater — and to the extent that all be thus connected; then place the difference between each given and the given mean opposite, not the given, or the given mean, but the given with which it is alligated; the number standing opposite each price or quality will be the quantity that must be taken at that price, or of that quality, to form a mixture or compound at the price or of the quality desired. And, being proportions respectively to each other, they may be taken in ratio greater or less, as desired.

Example. — In what proportions shall I mix teas at 48 cents a pound and 54 cents a pound, that the mean price may be 50 cents a pound?

In the proportions

$$50 \left\{ \begin{array}{l} 48 \\ 54 \end{array} \right. \left\{ \begin{array}{l} 4 \text{ lbs at } 48 \text{ cts.} \\ 2 \text{ lbs. at } 54 \text{ cts.} \end{array} \right\} \textit{Ans.}$$

Or, as 2 at 48 to 1 at 54.

$$\text{Proof.} \left\{ \begin{array}{l} \overline{2 \times 48} + \overline{1 \times 54} = 150. \\ 3 \times 50 \qquad\qquad\quad = 150. \end{array} \right.$$

Example. — In what proportions shall I mix teas at 48, 54, and 72 cents a pound, that the mixture may average 60 cents a pound?

$$60 \left\{ \begin{array}{l} 48 \\ 54 \\ 72 \end{array} \right. \begin{array}{l} 12, \\ 12, \\ 12 + 6, \end{array} \quad \left. \begin{array}{l} 12 \text{ at } 48 \\ 12 \text{ at } 54 \\ 18 \text{ at } 72 \end{array} \right\} = \left\{ \begin{array}{l} 2 \text{ at } 48 \\ 2 \text{ at } 54 \\ 3 \text{ at } 72 \end{array} \right\} \textit{Ans.}$$

Example. — A wine dealer has received an order for a quantity of wine at 50 cts. a gallon. He has none ready *manufactured* at that price. He has it at 40 cts., at 56 cts., and at 80 cents a gallon, and he has water that cost him nothing. He wishes to fill the order with a mixture composed of the four materials — the water and the three different priced wines. In what proportions must he mix them, that the mean or average price may be 50 cents?

$$50 \left\{ \begin{array}{l} 00 \\ 40 \\ 56 \\ 80 \end{array} \right. \overset{\textit{Ans.}}{\left. \begin{array}{r} 6 \\ 30 \\ 50 \\ \underline{10} \end{array} \right\}} \quad \text{Or, } 50 \left\{ \begin{array}{l} 00 \\ 40 \\ 56 \\ 80 \end{array} \right. \begin{array}{l} 30 \\ 6 + 30 \\ 10 \\ 50 + 10 \end{array} \overset{\textit{Ans.}}{\left. \begin{array}{r} = 30 \\ = 36 \\ = 10 \\ = \underline{60} \end{array} \right\}}$$

$$= 96 \text{ gals.} \qquad\qquad = 136 \text{ gals.}$$

$$\text{Or, } 50 \left\{ \begin{array}{l} 00 \\ 40 \\ 56 \\ 80 \end{array} \right. \overset{\textit{Ans.}}{\left. \begin{array}{l} 6 + 30 \\ 30 \\ 50 \\ \underline{50 + 10} \end{array} \right\}} \quad \text{Or, } 50 \left\{ \begin{array}{l} 00 \\ 40 \\ 56 \\ 80 \end{array} \right. \overset{\textit{Ans.}}{\left. \begin{array}{l} 6 \\ 6 + 30 \\ 50 + 10 \\ \underline{10} \end{array} \right\}}$$

$$= 176 \text{ gals.} \qquad\qquad = 112 \text{ gals.}$$

If, now, having found the proportions desired, it is wished to limit one of the articles in quantity — say the best wine to 8 gallons in the mixture — the proportions of the remaining articles thereto are found thus: —

Instance, 1st example, —

$$\left.\begin{array}{l} 10 : 8 :: 50 = 40 \\ 10 : 8 :: 30 = 24 \\ 10 : 8 :: \;\;6 = \;\;4\tfrac{4}{5} \end{array}\right\} \text{And the mixture will consist of } 8 + 40 + 24 + 4\tfrac{4}{5} = 76\tfrac{4}{5} \text{ gallons.}$$

If, instead, it is desired to mix a given quantity, say 100 gallons, and proportioned, say as in first example, the quantity to be taken of each is ascertained by the following

RULE. — As the sum of the relative quantities is to the quantity required, so is each relative quantity to the quantity required of it respectively.

The sum of the relative quantities alluded to is $6 + 30 + 50 + 10 = 96$; then,

$$\begin{array}{l} 96 : 100 :: 6 = 6\tfrac{1}{4} \\ 96 : 100 :: 30 = 31\tfrac{1}{4} \\ 96 : 100 :: 50 = 52\tfrac{1}{12} \\ 96 : 100 :: 10 = 10\tfrac{5}{12} \end{array}$$

INVOLUTION.

INVOLUTION consists in involving, that is, in multiplying a number one or more times into itself. The number so involved is called the *root*, and the product arising from such involution, its *power*.

The *second power*, or *square*, of the root, is obtained by multiplying the root *once* into itself, as $4 \times 4 = 16$; 4 being the root and 16 its square.

The *third power*, or *cube*, of a number, is obtained by multiplying the number twice into itself, as $4 \times 4 \times 4 = 64$; and so on for any power whatever.

When a number is to be involved into itself, a small figure called the *index* or *exponent* is placed at its right, indicating the number of times it is to be so involved, or the power to which it is to be raised. Thus, $3^4 = 3 \times 3 \times 3 \times 3 = 81$; and $4^3 = 4 \times 4 \times 4 = 64$.

EVOLUTION.

EVOLUTION is the opposite of Involution. It consists in finding a root of a given number, instead of a power of a given root.

When the root of a number is required or indicated, the number is written with the $\surd$ before it: and the character or denomination of the root, if it be other than the square root, is defined by an index figure placed over the sign. When the square root of a number is required, the sign ($\sqrt{}$) is placed before the number, but the index (2) is usually omitted. Thus, $\sqrt{25}$, shows that the square root of 25 is required, or to be taken ; and $\sqrt[3]{25}$ shows that the cube root is required. The operation is usually called extracting the root.

TO EXTRACT THE SQUARE ROOT.

RULE — 1. Separate the given number into periods of two figures each, by placing a point over the *first* figure, *third*, *fifth*, &c., counting from right to left — the root will consist of as many figures as there are periods.

2. Find the greatest square in the left hand period, and place its root in the quotient; subtract the square of the root from the left hand period, and to the remainder bring down the next period for a dividend.

3. Multiply the root so far found — the figure in the quotient — by 2, for a divisor ; see how many times the divisor is contained in the dividend, except the right hand figure, and place the result (the number of times it is contained) in the quotient, to the right of the figure already there, and also to the right of the divisor ; multiply the divisor, thus increased, by the last figure in the quotient, and subtract the product from the dividend, and to the remainder bring down the next period for a dividend.

4. Multiply the quotient — the root so far found (now consisting of two figures) — by 2, as before, and take the product for a divisor ; see how many times the divisor is contained in the dividend, except the right hand figure, and place the result in the quotient, and to the right of the divisor, as before ; multiply the divisor, as it now stands, by the figure last placed in the quotient, and subtract the product from the dividend, and to the remainder bring down the next period for a dividend, as before.

5. Multiply the quotient (now consisting of 3 figures) by 2, as before, and take the product for a divisor, and in all respects proceed as when seeking for the last two figures in the quotient. The quotient, when all the periods have been brought down and divided, will be the root sought.

NOTE. — 1. If there is a remainder after finding the integer of a root, annex periods of ciphers thereto, and proceed as when seeking for the integer. The quotient figures will be the decimal portion of the root.

2. If the given number is a decimal, or consists of a whole number and decimal, point off the decimal from left to right, by placing the point over the *second*, *fourth*, *sixth*, &c., figures therein, and fill the last period, if incomplete, by annexing a cipher.

3. If the dividend does not contain the divisor, a cipher must be placed in the quotient, and also at the right of the divisor, and the next period brought down; then the dividend must be divided by the divisor as increased.

4. If the quotient figure, obtained by dividing by the double of the root, is too large, as will sometimes be the case, (see 3d Example) it must be dropped, and a less — one which is the true measure — taken in its stead.

EXAMPLE. — Required the square root of 123456.432.

```
          123456.4320 ( 351.3636+.  Ans.
           9
     65 ) 334
          325
      70. ) 956
            701
     7023 ) 25543
            21069
     70266 ) 447420
             421596
     702723 ) 2582400
              2108169
     7027266 ) 47423100
               42163596
                5259504
```

EXAMPLE. — Required the square root of 10621. Also, of 28561

```
     10621 ( 103.05+.  Ans.
     1
203 ) 00621
        609
20605 ) 120000
        103025
         16975
```

```
     28561 ( 169.  Ans.
     1
26 ) 185
     156
329 ) 2961
      2961
```

TO EXTRACT THE CUBE ROOT.

RULE — 1. Separate the given number into periods of three figures each, by placing a point over the *first*, *fourth*, *seventh*, &c., counting from right to left — the root will consist of as many figures as there are periods.

2. Find the greatest cube in the left hand period, and place its root in the quotient; subtract the cube of the root from the left hand period, and to the remainder bring down the next period for a dividend.

3. Multiply the square of the quotient by 300, for a divisor; see how many times the divisor is contained in the dividend, and place the result (except that the remainder is large, diminished by one or two units) in the quotient.

4. Multiply the divisor by the figure last placed in the quotient, and to the product add the square of the same figure, multiplied by the other figure, or figures, in the quotient, and by 30; and add also thereto

the cube of the same figure, and take the sum for the subtrahend; subtract the subtrahend from the dividend, and to the remainder bring down the next period for a dividend, with which proceed as with the preceding, so continuing until the whole is completed.

NOTE — 1. Decimals must be pointed from left to right, by placing a point over the *third*, *sixth*, &c., figures in that direction.

2. If the divisor is not contained by the dividend, place a cipher in the quotient, and annex two ciphers to the divisor, and bring down the next period for a dividend, and use the divisor, as thus increased, for finding the next quotient figure.

3. If there is a remainder after finding the integer of the root, annex a period of three ciphers thereto, and proceed for the decimal of the root as if seeking for the integer, annexing a period of three ciphers to each remainder until the decimal is carried to as many places of figures as desired.

EXAMPLE. — Required the cube root of 47421875.6324.

$4\dot{7}42\dot{1}87\dot{5}.63\dot{2}40\dot{0}$ (361.959+. *Ans.*

27

$3^2 \times 300 = 2700$) 20421

6

16200

$6^2 \times 3 \times 30 = 3240$

$6^3 = 216 = 19656$

$36^2 \times 300 = 388800$) 765875

1

388800

$1^2 \times 36 \times 30 = 1080$

$1^3 = 1 = 389881$

$361^2 \times 300 = 39096300$) 375994632

9

351866700

$9^2 \times 361 \times 30 = 877230$

$9^3 = 729 = 352744659$

$3619^2 \times 300 = 3929148300$) 23249973400

5

19645741500

$5^2 \times 3619 \times 30 = 2714250$

$5^3 = 125 = 19648455875$

$36195^2 \times 300 = 393023407500$) 3601517525000

9

3537210667500

$9^2 \times 36195 \times 30 = 87953850$

$9^3 = 729 = 3537298622079$

64218902921

EXAMPLE.—Required the cube root of 32768. Also, of 8489664.

```
              32768(32.                          8489664(204.
              27    Ans.                         8          Ans.
3²×300=2700 ) 5768            2²×300 = 120000 ) 489664
          2                                4
       5400                          480000
2²×3×30=360                 4²×20×30 = 9600
    2³=   8=5768                 4³ =     64=489664
```

General Rule for extracting the roots of all powers, or for finding any proposed root of a given number.

1. Point off the given number into periods of as many figures each, counting from right to left, as correspond with the denomination of the root required; that is, if the cube root be required, into periods of *three* figures, if the fourth root, into periods of *four* figures, &c.

2. Find the first figure of the root by inspection or trial, and place it at the right of the number, in the form of a quotient; raise this quotient figure to a power corresponding with the denomination of the root sought, and subtract that power from the left hand period, and to the remainder bring down the first figure of the next period, for a dividend.

3. Raise the root thus far found (the quotient figure) to a power next inferior in denomination to that of the root required, multiply this power by the number or index figure of the root required, and take the product for a divisor; find the number of times the divisor is contained in the dividend, and place the result (except that the remainder is large, diminished by one or two units) in the quotient, for the second figure of the root.

4. Raise the root thus far found (now consisting of two figures) to a power corresponding in denomination with the root required, and subtract that power from the two left hand periods, and to the remainder bring down the first figure of the third period, for a dividend; find a new divisor, as before, and so proceed until the whole root is extracted.

EXAMPLE.—Required the fifth root of 45435424.

```
          45435424(34.   Ans.
     3⁵ = 243.
  3⁴×5 ) 2113
    34⁵ = 45435424
          ........
```

EXAMPLE. — Required the fifth root of 432040.0354.

$$\dot{4}3204\dot{0}.0354\dot{0}\ (\ 13.4+. \quad \textit{Ans.}$$
$$1^5 = 1$$
$$1^4 \times 5\ )\ 33$$
$$13^5 = 371293$$
$$13^4 \times 5\ )\ 607470$$
$$13.4^5 = 43204003424$$
$$\cdots\cdots\cdots 116$$

For instructions touching special cases, see NOTES relative to the extraction of the square root, and to the extraction of the cube root.

The $\sqrt{\ }$ of the $\sqrt{\ }$ of any number $= \sqrt[4]{\ }$ of that number
" $\sqrt{\ }$ of the $\sqrt[3]{\ } = \sqrt[6]{\ }$.
" $\sqrt{\ }$ of the $\sqrt{\ }$ of the $\sqrt{\ } = \sqrt[8]{\ }$.
" $\sqrt[3]{\ }$ of the $\sqrt[3]{\ } = \sqrt[9]{\ }$.
" $\sqrt{\ }$ of the $\sqrt[5]{\ } = \sqrt[10]{\ }$, &c.

ARITHMETICAL PROGRESSION.

A series of three or more numbers, increasing or decreasing by equal differences, is called an *arithmetical progression*. If the numbers progressively increase, the series is called an *ascending arithmetical progression;* and if they progressively decrease, the series is called a *descending arithmetical progression*.

The numbers forming the series are called the *terms* of the progression, of which the first and the last are called the *extremes*, and the others the *means*.

The difference between the consecutive terms, or that quantity by which the numbers respectively increase upon each other, or decrease from each other, is called the *common difference*.

Thus, 3, 5, 7, 9, 11, &c., is an ascending arithmetical progression, and 11, 9, 7, 5, 3, is a descending arithmetical progression. In these progressions, in both instances, 11 and 3 are the *extremes*, of which 11 is the *greater extreme*, and 3 is the *less extreme*. The numbers between these, (9, 7, 5,) are the *means*.

In every arithmetical progression, the sum of the extremes is equal to the sum of any two means that are equally distant from the extremes; and is, therefore, equal to *twice* the middle term, when the series consists of an odd number of terms. Thus, in the foregoing series, $3 + 11 = 5 + 9 = 7 \times 2$.

The *greater extreme*, the *less extreme*, the *number of terms*, the *common difference*, and the *sum of the terms*, are called the *five* properties of an arithmetical progression, of which, any *three* being given, the other two may be found.

Let s represent the sum of the terms.
" E " the greater extreme.
" e " the less extreme.
" d " the common difference.
" n " the number of terms.

The extremes of an arithmetical progression and the number of terms being given, to find the sum of the terms.

$$\frac{(E+e)\times n}{2} = \text{sum of the terms.}$$

EXAMPLE. — What is the sum of all the even numbers from 2 to 100, inclusive?

$$102 \times 50 \div 2 = 2550. \quad \textit{Ans.}$$

EXAMPLE. — How many times does the hammer of a common clock strike in 12 hours?

$$(1 + 12) \times 12 \div 2 = 78 \text{ times.} \quad \textit{Ans.}$$

$$\left(\frac{E-e}{d}+1\right)\times\frac{E+e}{2} = \text{sum of the terms.}$$

$$(E \times 2 - \overline{\overline{n-1} \times d}) \times \tfrac{1}{2}\, n = \text{sum of the terms.}$$

$$(2\,e + \overline{\overline{n-1} \times d}) \times \tfrac{1}{2}\, n = \text{sum of the terms.}$$

The greater extreme, the common difference, and the number of terms of an arithmetical progression being given, to find the less extreme.

$$E - (d \times \overline{n-1}) = \text{less extreme.}$$

EXAMPLE. — A man travelled 18 days, and every day 3 miles farther than on the preceding; on the last day he travelled 56 miles; how many miles did he travel the first day?

$$56 - (\overline{18-1} \times 3) = 5 \text{ miles.} \quad \textit{Ans.}$$

$$\frac{s}{n} - \left(\frac{\overline{n-1} \times d}{2}\right) = \text{less extreme.}$$

$$\frac{s}{n} \times 2 - E = \text{less extreme.}$$

$$\frac{\sqrt{(\overline{E \times 2 + d)^2 - s \times d \times 8}} + d}{2} = \text{less extreme, when}$$

$\sqrt{(\overline{2\,E + d\,)^2 - 8\,s\,d}}$ is equal to, or greater than d.

$$\frac{\sqrt{(\overline{2\,E + d)^2 - 8\,s\,d}} \backsim d}{2} = \text{less extreme, when}$$

$\sqrt{(\overline{2\,E + d)^2 - 8\,s\,d}}$ is less than d.

$$\frac{\sqrt{(\overline{2\,e \backsim d)^2 + 8\,s\,d}} - d}{2} = \text{greater extreme}$$

$$d \times \overline{n - 1} + e = \text{greater extreme.}$$

$$\frac{s}{n} + \frac{\overline{n-1} \times d}{2} = \text{greater extreme.}$$

$$2\,s \div n - e = \text{greater extreme.}$$

The extremes of an arithmetical progression and the common difference being given, to find the number of terms.

$$E - e \div d + 1 = \text{number of terms.}$$

Example. — As a heavy body, falling freely through space, descends $16\frac{1}{12}$ feet in the first second of its descent, $48\frac{3}{12}$ feet in the next second, $80\frac{5}{12}$ in the third second, and so on; how many seconds had that body been falling, that descended $305\frac{7}{12}$ feet in the last second of its descent?

$305\frac{7}{12} - 16\frac{1}{12} = 289\frac{1}{2} \div 32\frac{1}{6} = 9 + 1 = 10$ seconds. *Ans.*

$$\frac{\sqrt{(\overline{2\,e \backsim d)^2 + 8\,s\,d}} - d}{2} - e \div d + 1 = \text{number of terms.}$$

$$2\,s \div \overline{E + \frac{\sqrt{(\overline{2\,E + d)^2 - 8\,s\,d}} + d}{2}} = \text{number of terms when}$$

$\sqrt{(\overline{2\,E + d)^2 - 8\,s\,d}}$ is equal to, or greater than d.

$$2\,s \div \overline{E + \frac{\sqrt{(\overline{2\,E + d)^2 - 8\,s\,d}} \backsim d}{2}} = \text{number of terms when}$$

$\sqrt{(\overline{2\,E + d)^2 - 8\,s\,d}}$ is less than d.

$$\frac{s \times 2}{E + e} = \text{number of terms.}$$

The extremes of an arithmetical progression, and the number of terms being given, to find the common difference.

$$\frac{E - e}{n - 1} = \text{common difference.}$$

EXAMPLE. — One of the extremes of an arithmetical progression is 28 and the other is 100, and there are 19 terms in the series; required the common difference.

$$100 \backsim 28 \div 1 \backsim 19 = 4. \quad \textit{Ans.}$$

$$E - e \div \left(\frac{s \times 2}{E + e} - 1\right) = \text{common difference.}$$

$$\frac{2\,s \div n - 2\,e}{n - 1} = \text{common difference.}$$

$$\frac{2\,E - (2\,s \div n)}{n - 1} = \text{common difference.}$$

EXAMPLE. — The less extreme of an arithmetical progression is 28, the sum of the terms 1216, and the number of terms 19; required the 7th term in the series, descending.

$$1216 \times 2 \div 19 = 128 = \text{sum of the extremes.}$$
$$128 - 28 = 100 = \text{greater extreme.}$$
$$100 - 28 = 72 = \text{difference of extremes.}$$
$$72 \div \overline{n - 1}\ (18) = 4 = \text{common difference.}$$
$$100 - (\overline{7 - 1} \times 4) = 76 = \text{7th term descending.} \quad \textit{Ans.}$$

Required the 5th term from the less extreme, in an arithmetical progression, whose greatest extreme is 100, common difference 4, and number of terms 19.

$$100 - (\overline{19 - 5} \times 4) = 44. \quad \textit{Ans.}$$

To find any assigned number of arithmetical means, between two given numbers or extremes.

RULE. — Subtract the less extreme from the greater, divide the remainder by 1 more than the number of means required, and the quotient will be the common difference between the extremes; which, added to the less extreme, gives the least mean, and, added to that, gives the next greater, and so on.

Or, $\overline{E - e} \div \overline{m + 1} = d$, E being the greater extreme, e the less extreme, m the number of means required, and d the common difference.

And $e + d$, $e + 2\,d$, $e + 3\,d$, &c.; or, $E - d$, $E - 2\,d$, $E - 3\,d$, &c., will give the means required.

EXAMPLE. — Required to find 5 arithmetical means between the numbers 18 and 3.

$$18 - 3 = 15 \div 6 = 2\tfrac{1}{2}, \text{ and}$$

$$3 + 2\tfrac{1}{2} = 5\tfrac{1}{2} + 2\tfrac{1}{2} = 8 + 2\tfrac{1}{2} = 10\tfrac{1}{2} + 2\tfrac{1}{2} = 13 + 2\tfrac{1}{2} = 15\tfrac{1}{2}.$$

$5\frac{1}{2}$, 8, $10\frac{1}{2}$, 13, $15\frac{1}{2}$, therefore, are 5 arithmetical means, between the extremes, 3 and 18.

NOTE. — The arithmetical mean between any two numbers may be found by dividing the sum of those numbers by 2; thus, the arithmetical mean of 9 and 8 is $(9 + 8) \div 2 = 8\frac{1}{2}$.

GEOMETRICAL PROGRESSION.

A series of three or more numbers, increasing by a common multiplier, or decreasing by a common divisor, is called a *geometrical progression*. If the greater numbers of the progression are to the right, the progression is called an *ascending geometrical progression*, but, on the contrary, if they are to the left, it is called a *descending geometrical progression*. The number by which the progression is formed, that is, the common multiplier, or divisor, is called the *ratio*.

The numbers forming the series are called the *terms* of the progression, of which the first and the last are called the *extremes*, and the others the *means*. The greater of the extremes is called the *greater extreme*, and the less the *less extreme*.

Thus, 3, 6, 12, 24, 48, is an ascending geometrical progression, because 48 is as many times greater than 24, as 24 is greater than 12, &c.; and 250, 50, 10, 2, is a descending geometrical progression, because 2 is as many times less than 10, as 10 is less than 50, &c.

In the first mentioned series, (3, 6, 12, 24, 48,) 48 is the *greater extreme*, and 3 is the *less extreme;* the numbers 6, 12, 24 are the means in that progression.

So, too, of the progression 250, 50, 10, 2; 250 and 2 are the extremes, and 50 and 10 are the means.

In the first mentioned progression, 2 is the ratio, and in the last, or in the progression 2, 10, 50, 250, 5 is the ratio.

In a geometrical progression, the product of the two extremes is equal to the product of any two means that are equally distant from the extremes, and, also, equal to the square of the middle term, when the progression consists of an odd number of terms.

Thus, in the progression 2, 6, 18, 54, 162; $162 \times 2 = 54 \times 6 = 18 \times 18$.

When a geometrical progression has but 3 terms, either of the extremes is called a *third proportional* to the other two; and the middle term, consequently, is a *mean proportional* between them.

Thus, in the progression 48, 12, 3, 3 is a *third proportional* to 48 and 12, because 48 divided by the ratio = 12, and 12 divided by the ratio = 3; or 3 × ratio = 12, and 12 × ratio = 48 : 12 is the *mean proportional*, because 12 × 12 = 48 × 3.

Of the 5 properties of a geometrical progression, viz., the *greater extreme*, the *less extreme*, the *number of terms*, the *ratio*, and the *sum of the terms*, any three being given, the other two may be found.

Let s represent the sum of the terms.
" E " the greater extreme.
" e " the less extreme.
" r " the ratio.
" n " the number of terms.
" ^{n} when affixed as an index or exponent, represent that the term, number, or quantity, to which it is affixed, is to be raised to a power equal to the number of terms in the respective progression, &c.

Any three of the five parts of a geometrical progression being given, to find the remaining two parts.

$$\frac{E - e}{r - 1} + E = \text{sum of the terms.}$$

$$\frac{E \times r - e}{r - 1} = \text{sum of the terms.}$$

$$\frac{r^n \times e - e}{r - 1} = \text{sum of the terms.}$$

$$\frac{E - (E \div \overline{r^{n-1}})}{r - 1} + E = \text{sum of the terms.}$$

$$\frac{E - e}{\sqrt[n-1]{(E \div e)} - 1} + E = \text{sum of the terms.}$$

Example.—The greater extreme of a geometrical progression is 162, the less extreme is 2, and there are 5 terms in the progression; required the sum of the series.

$$\frac{162 - 2}{\sqrt[4]{(162 \div 2)} - 1} = 80 + 162 = 242. \quad \textit{Ans.}$$

$$\frac{s \times \overline{r - 1} + e}{r} = \text{greater extreme.}$$

$$\frac{r^n}{r} \times e = \text{greater extreme.}$$

$$r^{\overline{n-1}} \times e = \text{greater extreme.}$$

$$\frac{s \times r^{\overline{n-1}} \times \overline{r-1}}{r^n - 1} = \text{greater extreme.}$$

$$s - \overline{(s - E) \times r} = \text{less extreme.}$$

$$E \div r^{\overline{n-1}} = \text{less extreme.}$$

$$\frac{s \times r^{\overline{n-1}} \times \overline{r-1}}{r^n - 1} \div r^{n-1} = \text{less extreme.}$$

$$\frac{s - e}{s - E} = \text{ratio.}$$

$$\sqrt[n-1]{\frac{E}{e}} = \text{ratio.}$$

$\frac{s \times \overline{r-1}}{e} + 1 = r^n$; n, therefore, is equal to the number of times that r must be multiplied into itself to equal $\frac{s \times \overline{r-1}}{e} + 1$.

$$\frac{s \times \overline{r-1}}{s - \overline{(s - E) \times r}} + 1 = r^n.$$

Example. — A farmer proposed to a drover that he would sell him 12 sheep and allow him to select them from his flock, provided the drover would pay 1 cent for the first selected, 3 cents for the second, 9 cents for the third, and so on; what sum of money would 12 sheep amount to, at that rate?

$$\frac{r^n \times e - e}{r - 1} = s, \text{ then}$$

$$\frac{3^{12} \times 1 - 1}{3 - 1} = \$2657.20. \quad \textit{Ans.}$$

Note. — Ratio^4, cubed $= \text{ratio}^{12}$; ratio^6, squared $= \text{ratio}^{12}$, &c.

When it is required to find a high power of a ratio, it is convenient to proceed as follows, viz.: write down a few of the lower or leading powers of the ratio, successively as they arise, in a line, one after another, and place their respective indices over them; then will the product of such of those powers as stand under such indices whose sum is equal to the index of the required power, equal the power required.

EXAMPLE. — Required the 11th power of 3.

1	2	3	4	5
3	9	27	81	243

Here $5 + 4 + 2 = 11$, consequently,

$$243 \times 81 \times 9 = 11\text{th power of } 3, \text{ or}$$
$$5 \times 2 + 1 = 11, \text{ consequently,}$$
$$243 \times 243 \times 3 = 11\text{th power of } 3, \text{ or}$$
$$4 \times 2 + 3 = 11, \text{ consequently,}$$
$$81 \times 81 \times 27 = 11\text{th power of } 3, \text{ or}$$
$$3 \times 3 + 2 = 11, \text{ consequently,}$$
$$27^3 \times 9 = r^{11} = 177147. \quad \textit{Ans.}$$

To find any assigned number of geometrical means, between two given numbers or extremes.

RULE. — Divide the greater given number by the less, and from the quotient extract that root whose index is 1 more than the number of means required; that is, if 1 mean be required, extract the square root; if two, the cube root, &c., and the root will be the common ratio of all the terms; which, multiplied by the less given extreme, will give the least mean; and that, multiplied by the said root, will give the next greater mean, and so on, for all the means required. Or the greater extreme may be divided by the common ratio, for the greatest mean; that by the same ratio, for the next less, and so on.

EXAMPLE. — Required to find 5 geometrical means between the numbers 3 and 2187.

$$2187 \div 3 = 729, \text{ and } \sqrt[6]{729} = 3, \text{ then —}$$

$3 \times 3 = 9 \times 3 = 27 \times 3 = 81 \times 3 = 243 \times 3 = 729$, that is, the numbers 9, 27, 81, 243, 729 are the 5 geometrical means between 3 and 2187.

NOTE.— The geometrical mean between any two given numbers is equal to the square root of the product of those numbers. Thus the geometrical mean between 5 and 20, $= \sqrt{(5 \times 20)} = 10$.

ANNUITIES.

An annuity, strictly speaking and practically, is a certain sum of money by the year; payable, usually, either in a single payment yearly, or in half, half-yearly, quarter, quarter-yearly, &c., and for a succession of years, greater or less, or forever. *Pensions*, *awards*, *bequests*, and the like, that are made payable in fixed sums for a succession of payments, are commonly rated by the year, and denominated *annuities*.

A current annuity that has already commenced, or that is to commence after an interval of time not greater than that between the stipulated payments, is said to be in *possession*.

One that is to commence or cease on the occurrence of an indeterminate event, as upon the death of an individual, is a *reversionary*, *contingent*, or *life* annuity.

One that is to commence at a given period, and to continue for a given number of years or payments, is a *certain* annuity.

One that is to continue from a given time, forever, is a *perpetual annuity*, or a *perpetuity*.

Annuity payments do not exist fractionally: they mature, and exist only in that state, and are then due.

A current annuity commences with a payment, and terminates with a payment.

One current in the past is measured from a present included payment, closes with an included payment, and is said to be in *arrears* or *forborne*, from a supposed cancelled payment one regular interval or time beyond.

One current in the future is measured from the present to the first included payment of the series, and from thence is said to *continue* to the close; but if the interval from the present to the first included payment is equal to that between the successive payments, it is supposed to *continue* from the present.

Annuities in negotiation are adjusted, with regard to time, by *interest*, or *discount*, or both.

The TABLES applicable to compound interest and compound discount are applicable in adjusting annuities at compound rates.

To find the Amount of a Current Annuity in Arrears.

LEMMA. — The amount of an annuity that has been forborne for a given time is equal to the sum of the several payments that have become due in that time, plus the interest on each, from the time it became due, until the close of the time.

Then the amount of an annuity of \$100, payable in a single payment annually, but delayed of payment 4 years, allowing simple interest at 6 per cent. on the payments, is

$$
\begin{aligned}
100 \times 1.18 &= 118 \\
100 \times 1.12 &= 112 \\
100 \times 1.06 &= 106 \\
100 \times 1 \quad &= 100 = \$436.
\end{aligned}
$$

And at 6 per cent. compound interest on the payments, it is

$$
\begin{aligned}
100 \times (1.06)^3 &= 119.10 \\
100 \times (1.06)^2 &= 112.36 \\
100 \times (1.06)^1 &= 106.00 \\
100 \times \ 1 \quad &= 100.00 = \$437.46.
\end{aligned}
$$

At 6 per cent. simple interest, when payable in half, half-yearly, it is

$$
\begin{aligned}
50 \times 1.21 &= 60.50 \\
50 \times 1.18 &= 59.00 \\
50 \times 1.15 &= 57.50 \\
50 \times 1.12 &= 56.00 \\
50 \times 1.09 &= 54.50 \\
50 \times 1.06 &= 53.00 \\
50 \times 1.03 &= 51.50 \\
50 \times 1 \quad &= 50.00 = \$442.
\end{aligned}
$$

And at 6 per cent. compound interest PER ANNUM, when payable in half-yearly instalments, it is

$$
\begin{aligned}
50 \times (1.06)^3 \times 1.03 &= 61.34 \\
50 \times (1.06)^3 &= 59.55 \\
50 \times (1.06)^2 \times 1.03 &= 57.86 \\
50 \times (1.06)^2 &= 56.18 \\
50 \times (1.06)^1 \times 1.03 &= 54.59 \\
50 \times \ 1.06 &= 53.00 \\
50 \times \ 1.03 &= 51.50 \\
50 \times \ 1. &= 50.00 = \$444.02.
\end{aligned}
$$

From the foregoing, we derive the following general RULES:—

Let P = annuity or yearly sum,
r = rate of interest per annum,
a = rate of discount per annum,
n or ^{n} = nominal time of the annuity in full years,
A = *amount* for the full years,
D = *present worth* for the full years.

When the annuity is payable in a single payment yearly,

$$A = Pn\left(1 + \frac{r(n-1)}{2}\right), \text{ Simple Interest.}$$

$$A = P\,\frac{(1+r)^n - 1}{r}, \text{ Compound Interest.}$$

When payable in equal half-yearly instalments,

$$A = Pn\left(1 + \frac{r(n-1)}{2} + \frac{r}{4}\right), \text{ Simple Interest.}$$

$$A = P \times \frac{(1+r)^n - 1}{r} \times \left(1 + \frac{r}{4}\right), \text{ Compound Interest.}$$

When payable in equal third-yearly instalments,

$$A = Pn\left(1 + \frac{r(n-1)}{2} + \frac{r}{3}\right), \text{ Simple Interest.}$$

$$A = P\,\frac{(1+r)^n - 1}{r}\left(1 + \frac{r}{3}\right), \text{ Compound Interest.}$$

When payable in quarter-yearly instalments,

$$A = Pn\left(1 + \frac{r(n-1)}{2} + \frac{3r}{8}\right), \text{ Simple Interest.}$$

$$A = P\,\frac{(1+r)^n - 1}{r}\left(1 + \frac{3r}{8}\right), \text{ Compound Interest.}$$

When there are odd payments, to find the amount, S.

When 1 half-yearly,	$S = A(1 + \frac{1}{2}r) + \frac{1}{2}P.$
1 third-yearly,	$S = A(1 + \frac{1}{3}r) + \frac{1}{3}P.$
2 "	$S = A(1 + \frac{2}{3}r) + \frac{1}{3}P(1 + \frac{1}{3}r) + \frac{1}{3}P$
	$= A(1 + \frac{2}{3}r) + P(6 + r) \div 9.$
1 quarter-yearly,	$S = A(1 + \frac{1}{4}r) + \frac{1}{4}P.$
2 "	$S = A(1 + \frac{1}{2}r) + P(8 + r) \div 16.$
3 "	$S = A(1 + \frac{3}{4}r) + P(3 + \frac{3}{4}r) \div 4.$

For any number of equal and regular payments at compound interest per interval between the payments, $S = P'\left(\frac{(1+r')^{n'} - 1}{r'}\right)$, and for any number of equal and regular payments at simple interest per interval between the payments, $S = P'n'\left(1 + \frac{r'(n'-1)}{2}\right)$; P' being a payment, n' or $^{n'}$ the number of payments, and r' the rate of interest per interval between the payments. But this must not be confounded with compound interest *annually*, on payments occurring semi-annually, quarterly, &c.

EXAMPLE. — What is the amount of an annuity of \$150, payable in half, half-yearly, but delayed of payment 2 years and 72 days, allowing compound interest per annum at 7 per cent. ?

$150 \times \frac{(1.07)^2 - 1}{.07} = \$310.50$, the amount for 2 years, if payable in yearly payments, and

$310.50 \times (1.\frac{.07}{4}) = \$315.93$, the amount for 2 years, if payable in half-yearly payments, and

$315.93 \times (1.\frac{.07 \times 72}{365}) = \$320.29$, the amount for 2 years and 72 days, if payable in half-yearly payments. *Ans.*

EXAMPLE. — What is the amount of an allowance, pension, or award, of $100 a year, payable quarterly, but forborne $3\frac{1}{2}$ years, interest compound per annum at 6 per cent.?

$100 \times \frac{(1.06)^3 - 1}{.06} \times (1 + \frac{.06 \times 3}{8}) = \$325.52$, the amount for 3 years, and

$$325.52\,(1 + .03) + 100 \times 8.06 \div 16 = \$385.66. \quad \textit{Ans.}$$

EXAMPLE. — What is the amount of $100 a year, payable in quarterly payments, and in arrears 4 years, interest being compound per quarter-year, at 6 per cent. a year?

$25\left[(1 + \frac{.06}{4})^{16} - 1\right] \times \frac{4}{.06}$. By tabular powers of $(1 + r)$, page 125, $= \$448.30$. *Ans.*

To find the Present Worth of an Annuity Current.

LEMMA. — The present worth of an annuity that is to *continue* for a given time is equal to that sum of money, which, if put at interest from the present time to the close of the payments, will amount to the amount of the payments at that time; and therefore, the times being full, is equal to the sum of the several payments, discounted, respectively, at the rate of interest for their respective times.

NOTE. — If the foregoing proposition is tenable, it follows, since simple interest is due and payable annually, that the true present worth of an annuity having more than one year to run cannot be found by simple interest and discount. By simple interest and discount, at 6 per cent., predicating the rule upon the foregoing lemma, the *amount* of $100, payable annually, and in arrears for 4 years, is $436; and the *present worth*, at 6 per cent., is

$$\frac{100}{1.24} + \frac{100}{1.18} + \frac{100}{1.12} + \frac{100}{1.06} = \$349.$$

But $349 at 6 per cent. interest for 4 years, with the payments of interest annually, will amount to $440.30; and at interest simply for 4 years it will amount to only $432.76.

Then the present worth of an annuity of $100, payable in a single payment yearly, and to continue 4 years, or to become due 1, 2, 3, and 4 years hence, interest and discount being compound per annum, and each at 6 per cent. =

$$\frac{P}{(1+r)^4}+\frac{P}{(1+r)^3}+\frac{P}{(1+r)^2}+\frac{P}{1+r}=\$346.51=$$

$$\begin{aligned}100 &\times (1.06)^3 = 119.10\\ 100 &\times (1.06)^2 = 112.36\\ 100 &\times (1.06) = 106.00\\ 100 &\times 1 = 100.00 = 437.46 \div (1.06)^4 = \$346.51.\end{aligned}$$

And interest at 6 per cent. and discount at 10, both compound, it is

$$\begin{aligned}100 &\times (1.06)^3 = 119.10\\ 100 &\times (1.06)^2 = 112.36\\ 100 &\times 1.06 = 106.00\\ 100 &\times 1 = 100.00 = 437.46 \div (1.10)^4 = \$298.79.\end{aligned}$$

Therefore, when the annuity is payable in a single payment yearly from the present time,

$$D = P\frac{(1+r)^n-1}{r(1+a)^n}, = \frac{A}{(1+r)^n} \text{ when } r \text{ and } a \text{ are equal.}$$

When payable in half-yearly payments,

$$D = P \times \frac{(1+r)^n-1}{r(1+a)^n} \times (1+\tfrac{1}{4}r).$$

When payable in third-yearly payments,

$$D = \frac{P \times [(1+r)^n-1] \times (1+\frac{1}{3}r)}{r(1+a)^n}.$$

When payable in quarter-yearly payments,

$$D = \frac{P\,[(1+r)^n-1]\,(1+\frac{3}{8}r)}{r(1+a)^n}.$$

When there are odd payments, to find the present worth, S.

There being a half-yearly, $S = \frac{D}{1+\frac{1}{2}a} + \frac{\frac{1}{2}P}{1+\frac{1}{2}a}$.

" 1 third-yearly, $S = \frac{D}{1+\frac{1}{3}a} + \frac{\frac{1}{3}P}{1+\frac{1}{3}a}$.

" 2 " $S = \frac{D}{1+\frac{2}{3}a} + \frac{2P\,(1+\frac{1}{6}r)}{3\,(1+\frac{2}{3}a)}$.

" 1 quarter-yearly, $S = \frac{D}{1+\frac{1}{4}a} + \frac{\frac{1}{4}P}{1+\frac{1}{4}a}$.

" 2 " $S = \frac{D}{1+\frac{1}{2}a} + \frac{P\,(1+\frac{1}{8}r)}{2\,(1+\frac{1}{2}a)}$.

" 3 " $S = \frac{D}{1+\frac{3}{4}a} + \frac{\frac{3}{4}P\,(1+\frac{3}{8}r)}{1+\frac{3}{4}a}$.

For any number of equal payments, at equal intervals between the payments, $S = P' \times \frac{(1+r')^{n'}-1}{r\,(1+a')^{n'}}$; P′ being a payment, n′ the number of payments, and r' and a' the rates per interval between the payments.

NOTE.—Since $\frac{(1+r)^n-1}{r(1+a)^n}$ is the co-efficient of P, for its present worth, at compound interest and discount, for the time n, at the rates r, a, it follows that tables of co-efficients of P for its present worth, at given rates, for any number of years, may be easily made. Thus $(1.06^4-1)\div 1.06^4\times .06=3.46511$, the co-efficient of an annuity, P, for 4 years' continuance, interest and discount being compound per annum, at 6 per cent.; and $(1.06^2-1)\div(1.06^2\times .06)=1.83339$, the co-efficient for 2 years, &c.

If the annuity is deferred, then the difference of two of these co-efficients (one of them that for the time deferred, and the other that for the sum of the time deferred and the time of the annuity) will be the co-efficient of P for its present worth. Thus $3.46511-1.83339=1.63172$, the co-efficient of an annuity, P, for its present worth, when it is to commence two years hence, and to continue 2 years, interest and discount being compound per annum, at 6 per cent. each; or $D=1.63172$ P.

In like manner, tables of other co-efficients, such as the formulæ suggest, may be made that will greatly assist in calculating annuities.

EXAMPLE.—What is the present worth of an award of $500 a year, payable in half-yearly instalments, the 1st payment to mature 6 months hence, and the annuity to continue three years; interest and discount being 7 per cent., compounded yearly?

$$\frac{500\times[(1.07)^3-1]\times\left(1.\frac{.07}{4}\right)}{.07\times(1.07)^3}=\$1335.13.\quad \textit{Ans.}$$

EXAMPLE.—What is the present worth of an annuity of $100, payable in half-yearly payments, and to continue $1\frac{1}{2}$ years; interest and discount being 6 per cent. per annum?

$$D=\frac{100\times[1.06-1]\times 1.\frac{.06}{4}}{.06\times 1.06}=95.755,\ \text{and}$$

$$\frac{95.755}{1.03}+\frac{50}{1.03}=\$141.51.\quad \textit{Ans.}$$

EXAMPLE.—What is the present worth of an annuity of $500, payable in semi-annual instalments, and to continue $10\frac{1}{2}$ years, interest and discount being compound per annum, the former at 6 per cent., and the latter at 8?

$$\frac{500\,[(1.06)^{10}-1]\left(1.\frac{.06}{4}\right)}{.06\,(1.08)^{10}\left(1.\frac{.08}{2}\right)}+\frac{500}{2\left(1.\frac{.08}{2}\right)}=$$

$$\frac{A}{1.08^{10}\times 1.04}+\frac{250}{1.04}=\quad \textit{Ans.}$$

By tabular powers of $1 + r$, page 125 : —

$\frac{500 \times .79085}{.06 \times 2.15892} = \$3052.64$, the present worth for 10 years' continuance, if payable in yearly payments, and

$$3052.64 \times 1.015 = \$3098.43,$$

the present worth for 10 years' continuance, if payable in half-yearly payments, and

$$3098.43 \div 1.04 + 500 \div 2 \times 1.04 = \$3219.64. \quad \textit{Ans.}$$

When the interval of time from the present to the 1st payment is *shorter* than that between the consecutive payments, and the annuity is payable in a single payment yearly,

$$A = \frac{P\left[(1+r)^n - 1\right]\left(1 + \frac{dr}{365}\right)}{r}, \text{ and}$$

$$D = \frac{A}{(1+a)^{(n-1)}\left(1 + \frac{a(365-d)}{365}\right)} = \frac{P\left[(1+r)^n - 1\right]\left(1 + \frac{dr}{365}\right)}{r(1+a)^{(n-1)}\left(1 + \frac{a(365-d)}{365}\right)},$$

d being the time in days from the present to the 1st payment.

So, if the annuity is payable in half-yearly, third-yearly, or quarter-yearly instalments, multiply by $1 + \frac{1}{4}r$, $1 + \frac{1}{3}r$, or $1 + \frac{3}{8}r$, as before directed; and if there are odd payments proceed for the present worth, S, as already directed.

Example. — Required the present worth of an annuity of \$100, payable yearly, to commence 4 months hence, and to continue 4 years; interest and discount being 6 per cent. *annually.*

$$\frac{100 \times (1.06^4 - 1) \times \left(1.\frac{4 \times .06}{12}\right)}{.06 \times 1.06^3 \times \left(1.\frac{.06 \times (12-4)}{12}\right)} = \$360{,}24. \quad \textit{Ans.}$$

To find the Present Worth of a Deferred Current Annuity, or of an Annuity in Reversion.

When the annuity is payable in a single payment yearly, and the deferred time embraces full years only,

$$D = P\,\frac{(1+r)^n - 1}{r(1+a)^{(n+n')}}, \text{ n}' \text{ being the deferred time.}$$

If it is payable in half-yearly, third-yearly, or quarter-yearly instalments, multiply by $1 + \frac{1}{4}r$, $1 + \frac{1}{3}r$, or $1 + \frac{3}{8}r$, as already directed; and, if there are odd payments, find the present worth, S, as already directed.

Example. — What is the present worth of an annuity of \$150, payable yearly, to commence 2 years hence, and to continue 4 years; interest and discount being compound per annum, at 6 per cent.?

$$150 \times (1.06^4 - 1) \div .06 \times 1.06^6 = \$462.59. \quad \textit{Ans.}$$

Example. — Required the present worth of an annuity of \$500, payable in semi-annual instalments, to commence $2\frac{1}{2}$ years hence, and to continue 6 years; allowing compound interest and discount annually at 7 per cent.

$$\frac{500 \times (1.07^6 - 1) \times 1.\frac{.07}{4}}{.07 \times 1.07^8 \times 1.\frac{.07}{2}} = \$2046.44. \quad \textit{Ans.}$$

Example. — Required the present worth of an allowance, pension, or award of \$125 a year, payable in half every half-year, to commence 7 months 24 days hence, and to continue $6\frac{1}{2}$ years; interest and discount being compound per annum at 5 per cent.

$$\frac{125 \times (1.05^6 - 1) \times 1.0125}{.05 \times 1.05^6 \times 1.03247 \times 1.025} + \frac{125}{2 \times 1.025} = \$668. \quad \textit{Ans.}$$

Or $\frac{125 \times [(1.05)^6 - 1]}{.05 \times (1.05)^6} = \$634.47$, the present worth for 6 years' continuance, if payable in yearly instalments; and

$$634.47 \times 1.\frac{.05}{4} = \$642.40,$$

the present worth for 6 years' continuance, if payable in half-yearly instalments; and

$$642.40 \div \left(1 + \frac{.05 \times 237}{365}\right) = 622.20,$$

the present worth for 6 years' continuance, if payable in half-yearly instalments, and to commence 7 months, 24 days hence; and

$$622.20 \div (1 + \tfrac{.05}{2}) + \frac{125}{2 \times 1.025} = \$668. \quad \textit{Ans.}$$

To find the Present Worth of a Perpetuity.

Lemma. — The present worth of an annuity to commence one year hence, and to continue forever, is expressed by that sum of money whose interest for 1 year is equal to the *amount* of the annuity for 1 year; and so, *pro rata*, for perpetuities otherwise regularly affected.

Then when the annuity is to commence 1 year hence, and is payable in a single payment yearly . . . $D = P \div r$.

Payable in half-yearly instalments . . $D = \frac{P(1 + \frac{1}{4}r)}{r}$.

Payable in third-yearly instalments . . $D = \frac{P(1 + \frac{1}{3}r)}{r}$.

Payable in quarter-yearly instalments . $D = \frac{P(1 + \frac{3}{8}r)}{r}$.

EXAMPLE. — What is the present worth of a perpetuity of $150 a year, payable in a single payment yearly from the present time; interest at 6 per cent?

$$150 \div .06 = \$2500. \quad \textit{Ans.}$$

EXAMPLE. — What is the present worth of a perpetuity of $150 a year, payable in semi-annual instalments, and to commence 4 months hence; interest 7 per cent?

$$\frac{P(1 + \frac{1}{4}r)}{r} + P\,\frac{.07(12 - 4)}{12} = \$2187.36. \quad \textit{Ans.}$$

EXAMPLE. — Required the present worth of a perpetuity of $400 a year, payable in quarterly payments, and to commence 6 years hence; interest and discount being 5 per cent., compound per year.

$$D = \frac{P(1 + \frac{3}{8}r)}{r(1 + a)^n} = \frac{400 \times 1.\frac{3 \times .05}{8}}{.05 \times 1.05^6} = \$6081.65. \quad \textit{Ans.}$$

The Amount, Time, and Rate given, to find the Annuity.

When payable in a single payment yearly from the present time,

$$P = \frac{Ar}{(1 + r)^n - 1}; \text{ half-yearly, } P = \frac{Ar}{[(1 + r)^n - 1](1 + \frac{1}{4}r)};$$

$$\text{third-yearly, } P = \frac{Ar}{(1 + \frac{1}{3}r)[(1 + r)^n - 1]}; \text{ quarterly,}$$

$$P = \frac{Ar}{(1 + \frac{3}{8}r)[(1 + r)^n - 1]};$$

and so, *pro rata*, for other fractional units of the integral unit.

Therefore $(1+r)^n - 1 = \frac{Ar}{P}$, or $\frac{Ar}{P(1+\frac{1}{4}r)}$, or $\frac{Ar}{P(1+\frac{1}{3}r)}$

or $\frac{Ar}{P(+\frac{3}{8}r)}$, &c.

EXAMPLE. — What annuity, payable in quarterly payments from the present time, will amount to \$3000 in 12 years; interest, being compound per annum, at 8 per cent.?

$$3000 \times .08 \div \left[(108^{12} - 1) \times 1.\frac{3 \times .08}{8}\right] = \$153.48. \quad \textit{Ans.}$$

EXAMPLE. — What length of time must a current annuity of \$400, payable in quarterly payments, remain unpaid, that it may amount to \$2500; interest being 7 per cent. yearly?

$\frac{2500 \times .07}{400 \times 1.\frac{3 \times .07}{8}} = .4263094 = 5 +$ years, and 5 years by table of $(1+r)^n - 1 = .402552$: therefore $\left(\frac{.4263094}{.402552} - 1\right)\frac{365}{.07} = 308$ days, 5 years, 308 days. *Ans.*

The Present Worth, Time, and Rate given, to find the Annuity.

When payable in a single payment yearly from the present time, $P = \frac{Dr(1+r)^n}{(1+r)^n - 1}$; half-yearly, $P = \frac{Dr(1+r)^n}{[(1+r)^n - 1](1+\frac{1}{4}r)}$; third-yearly, $P = \frac{Dr(1+r)^n}{[(1+r)^n - 1](1+\frac{1}{3}r)}$; quarter-yearly, $P = \frac{Dr(1+r)^n}{[(1+r)^n - 1](1+\frac{3}{8}r)}$, &c. Therefore, $(1+r)^n; = \frac{P}{P - Dr} =$

$$\frac{P(1+\frac{1}{4}r)}{P(1+\frac{1}{4}r) - Dr} = \frac{P(1+\frac{3}{8}r)}{P(1+\frac{3}{8}r) - Dr}, \text{ \&c.}$$

EXAMPLE. — What annuity, payable in half-yearly instalments, and to continue 3 years, is at present worth \$1335.13; discount and interest being compound per year, at 7 per cent?

$$\frac{1335.13 \times .07 \times 1.07^3}{(1.07^3 - 1) \times 1\frac{.07}{.4}} = \$500. \quad \textit{Ans.}$$

OF INSTALMENTS GENERALLY.

Any certain sum of money to be paid on a debt periodically until the debt is paid is called an instalment; and a debt so made payable is said to be payable by instalments.

Let D = principal or debt to be paid,
n = number of years in which the debt is to be paid,
r = rate of interest per annum,
p = instalment or periodical payment.

When the instalments are payable yearly, and the debt is at interest,

$$p=\frac{\mathrm{D}r(1+r)^n}{(1+r)^n-1};\ (1+r)^n=\frac{p}{p-\mathrm{D}r};\ \mathrm{D}=\frac{p[(1+r)^n-1]}{r(1+r)^n}.$$

When payable half-yearly,

$$p=\frac{\mathrm{D}r(1+r)^n}{2[(1+r)^n-1](1+\tfrac{1}{4}r)};$$

$$(1+r)^n=\frac{p(1+\frac{1}{2}r)+p}{[p(1+\frac{1}{2}r)+p]-\mathrm{D}r};\ \mathrm{D}=\frac{2p[(1+r)^n-1](1+\frac{1}{4}r)}{r(1+r)^n}.$$

When the debt is not on interest, and the instalments are payable yearly,

$$p=\frac{\mathrm{D}r}{(1+r)^n-1};\ (1+r)^n=\frac{\mathrm{D}r+p}{p};\ \mathrm{D}=\frac{p(1+r)^n-p}{r}.$$

EXAMPLE. — What yearly instalment will pay a debt of $4000 in 4 years, the debt being on interest the while, at 6 per cent. annually?

$$4000\times .06\times 1.06^4\div(1.06^4-1)=\$1154.37.\quad \textit{Ans.}$$

EXAMPLE. — What semi-annual instalment will pay a debt of $4500 in 3 years, the debt bearing interest at 7 per cent. yearly?

$$\frac{4500\times .07\times 1.07^3}{2\times(1.07^3-1)\times 1.0175}=\$842.62.\quad \textit{Ans.}$$

When the debt is on interest, and is payable in equal yearly instalments, $p=D(1+rn)\div n(1+\frac{r(n-1)}{2})$, at simple interest; but simple interest is not strictly applicable to instalments. See NOTE, p. 157.

When a debt has been diminished at regular intervals by the payment of a constant sum, to find the remaining debt at the close of the last payment.

When the debt is on interest, and the payments have been made yearly from the date of the debt,

$$d = \frac{p - (p - \mathrm{D}r)(1+r)^n}{r};\ (1+r)^n = \frac{p - dr}{p - \mathrm{D}r}$$

$$p = \frac{\mathrm{D}r(1+r)^n - dr}{(1+r)^n - 1}.$$

When the payments have been made half-yearly,

$$d = p + p(1 + \tfrac{1}{2}r) - (1+r)^n[p + p(1+\tfrac{1}{2}r) - \mathrm{D}r] \div r, \text{ \&c.}$$

EXAMPLE. — On a debt of $1000, drawing interest the while at 8 per cent. a year, there has been paid yearly, from the date of the debt, $200 for 6 years: required the unpaid debt at the close of the last payment.

$[200 - 1.08^6(200 - .08 \times 1000)] \div .08 = \$119.69.$ *Ans.*

EXAMPLE. — On a note of hand for $1000, and interest from date, at 8 per cent. annually, the following payments have been made; viz., $100 at the close of every half-year from the date of the note, for 6 years. How much remained unpaid at the close of the last payment?

$[204 - 1.08^6(204 - .08 \times 1000)] \div .08 = \$90.34.$ *Ans.*

NOTE. — In the foregoing, I have treated the terms annual interest and interest payable annually as synonymous in meaning with the terms compound interest and compound interest per annum, and they are so in equity and in fact; besides, simple interest is inapplicable, in equity, to instalment payments. If the debtor stipulates to pay the interest annually on a debt, and abides his contract, he will pay it when it becomes due, and it then becomes a principal in the hands of the creditor, to be let, it is fair to suppose, upon as favorable terms to himself as he let the principal which grew it: whereby he realizes equal to compound interest per annum on the first principal: moreover, if the debtor withholds it from the creditor, it is fair to suppose that he considers it of as much worth to himself as a like part of the principal.

PERMUTATION.

Permutation, in the mathematics, has reference to the greatest number of unlike relative positions, that a given number of things, either wholly unlike, or unlike only in part, may be placed in. It considers the number of changes, therefore, that may be made, in the arrangement of the things, under different given circumstances.

To find the number of changes that can be made in the order of arrangement of a given number of things, when the things are all different.

Rule. — Find the product of the natural series of numbers, from 1 up to the given number of things, inclusive; and that product will be the number of changes or permutations that may be made.

Example. — In how many different relative positions may 12 persons be seated at a table?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 =$ 479,001,600. *Ans.*

To find the number of changes that can be made in the order of arrangement of a given number of things, when that number is composed of several different things, and of several which are alike.

Rule. — Find the number of changes that could be made if the things were all unlike, as in first example. Then find the number of changes that could be made with the several things of each kind, if they were unlike. Lastly, divide the number first found by the product of the numbers last found, and the quotient will be the number of permutations or changes that the collection admits of.

Example. — Required the number of permutations that can be made with the letters *a*, *b b*, *c c c*, *d d d d*, = 10 letters.

$$\begin{array}{r} 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = \\ 1 \times 2 \times 6 \times 24 = \end{array} \frac{3628800}{288} = 12{,}600. \ \textit{Ans.}$$

To find the number of permutations that can be made with a given number of different things, by taking an assigned number of them at a time.

Rule. — Take a series of numbers beginning with the number of things given, and decreasing by 1 continually, until the number of terms is equal to the number of things that are to be taken at a time; then will the product of the series be the number of changes that may be made.

Example. — What number of changes can be made with the numbers 1, 2, 3, 4, 5, 6, taking three of them at a time?

$6 - 1 = 5$, $5 - 1 = 4$, then $6 \times 5 \times 4 = 120$. *Ans.*

What number, by taking 4 of them at a time?

$6 \times 5 \times 4 \times 3 = 360$. *Ans.*

Example. — Arrange the three letters *a*, *b*, *c*, into the greatest number of permutations possible.

abc, *acb*, *bac*, *bca*, *cab*, *cba*, = 6 permutations. *Ans.*

Example. — Arrange the four letters *a*, *b*, *a*, *b*, into the greatest number of permutations possible.

abab, *aabb*, *abba*, *bbaa*, *baba*, *baab*, = 6 permutations. *Ans.*

COMBINATION.

Combination, in the mathematics, has reference to the number of unlike groups, which may be formed from a given number of different things, by taking any assigned number of them, less than the whole at a time. It does not regard the relative positions of the things, one with another, in any of the collections or groups. But it exacts that each group, in all instances, shall have the assigned number of members in it, and that, in every group, in every instance, there shall be a like number of members. It exacts, therefore, that no two groups shall be composed of precisely the same members.

To find the number of combinations that can be made from a given number of different things, by taking any given number of them at a time.

Rule. — Take a series of numbers beginning with that which is equal to the number of things from which the combinations are to be made, and decreasing by 1, continually, until the number of terms is equal to the number of things that are to be taken at a time, and find the product of those numbers or terms. Then take the natural series, 1, 2, 3, 4, &c., up to the number of things that are to be taken at a time, and find the product of that series. Lastly, divide the product first found by the product last found, and the quotient will express the number of combinations that can be made.

Example. — What number of combinations can be made from 8 different things, by taking 4 of them at a time?

$$\frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = \frac{1680}{24} = 70. \quad \textit{Ans.}$$

What number, by taking 5 of them at a time?

$$\frac{8\times7\times6\times5\times4}{1\times2\times3\times4\times5}=\frac{6720}{120}=56. \quad \textit{Ans.}$$

What number, by taking 3 of them at a time?

$$\frac{8\times7\times6}{1\times2\times3}=\frac{336}{6}=56. \quad \textit{Ans.}$$

EXAMPLE.—What number of combinations can be made from 5 different things, by taking three of them at a time?

$$\frac{5\times4\times3}{1\times2\times3}=\frac{60}{6}=10. \quad \textit{Ans.}$$

What number, by taking 2 of them at a time?

$$\frac{5\times4}{1\times2}=\frac{20}{2}=10. \quad \textit{Ans.}$$

EXAMPLE.—Form 5 letters, *a*, *b*, *c*, *d*, *e*, into 10 combinations of 2 letters each; that is, into 10 unlike groups of two letters each.

ab, *ac*, *ad*, *ae*, *bc*, *bd*, *be*, *cd*, *ce*, *de*. *Ans.*

Form them into the greatest number of combinations possible, in collections of three each.

abc, *abd*, *abe*, *acd*, *ace*, *ade*, *bcd*, *bce*, *bde*, *cde*. *Ans.*

PROBLEMS.

Prob. I. — *The sum and difference of two numbers given, to find the numbers.*

Let $a =$ the greater number,
$b =$ the less number ; then —

$$\frac{a+b-a\backsim b}{2}=b, \text{ and } b+a\backsim b=a; \text{ or}$$

$$\frac{a+b+a\backsim b}{2}=a, \text{ and } a-a\backsim b=b.$$

Prob. II. — *The sum of two numbers and their product given, to find the numbers.*

$$\sqrt{(a+b)^2-(a\times b\times 4)}=a\backsim b, \text{ and}$$
$$\frac{a+b-a\backsim b}{2}=b, \text{ and } b+a\backsim b=a.$$

Prob. III. — *The difference of two numbers and their product given, to find the numbers.*

$$\sqrt{(a\backsim b)^2+(a\times b\times 4)}=a+b, \text{ and}$$
$$\frac{a+b-a\backsim b}{2}=b, \text{ and } b+a\backsim b=a.$$

Prob. IV. — *The sum of two numbers and the sum of their squares given, to find the numbers.*

$$\sqrt{(a^2+b^2)\times 2-(a+b)^2}=a\backsim b, \text{ thence, by Prob. I.}$$

Prob. V. — *The difference of two numbers and the sum of their squares given, to find the numbers.*

$$\sqrt{(a^2+b^2)\times 2-(a\backsim b)^2}=a+b, \text{ thence, by Prob. I.}$$

Prob. VI. — *The sum of two numbers and the difference of their squares given, to find the numbers.*

$$\frac{a^2\backsim b^2}{a+b}=a\backsim b; \ \frac{a+b-a\backsim b}{2}=b; \ b+a\backsim b=a; \text{ or}$$

$$\frac{(a+b)^2-a^2\backsim b^2}{(a+b)\times 2}=b, \text{ and } a+b-b=a.$$

Prob. VII. — *The difference of two numbers and the difference of their squares given, to find the numbers.*

$$\frac{a^2\backsim b^2}{a\backsim b}=a+b; \ \frac{a+b-a\backsim b}{2}=b; \ b+a\backsim b=a.$$

PROB. VIII. — *The product of two numbers and the sum of their squares given, to find the numbers.*

$$\surd\,(a^2+b^2-\overline{a\times b\times 2})=a\backsim b,\text{ and}$$
$$\surd\,(a^2+b^2+\overline{a\times b\times 2})=a+b,\text{ and}$$
$$\frac{a+b-a\backsim b}{2}=b,\text{ and } b+a\backsim b=a.$$

PROB. IX. — *The sum of two numbers, and the product of those numbers plus the square of one of the numbers, in another sum given, to find the numbers.*

$$\frac{a\times b+b^2}{a+b}=b;\; a+b-b=a$$

PROB. X. — *The product of two numbers and the relation of those numbers to each other given, to find the numbers.*

$\surd\left(\frac{a\times b}{r\times r'}\right)\times r=a$, or $\times r'=b$; r being the term in relation representing the greater number, and r' being the term in relation representing the less.

PROB. XI. — *The sum of the squares of two numbers, and the relation of those numbers to each other given, to find the numbers.*

$$\surd\left(\frac{a^2+b^2}{r^2+r'^2}\right)\times r=a,\text{ or }\times r'=b.$$

PROB. XII. — *The sum of three numbers which are in arithmetical progression, and the sum of their squares given, to find the numbers.*

Let $a=$ the greatest number,
$b=$ the middle number,
$c=$ the least number; then

$$\frac{a+b+c}{3}=b,\text{ and } a+b+c-b=a+c.$$
$$a^2+b^2+c^2-b^2=a^2+c^2.$$
$$\surd\,\overline{(a^2+c^2)\times 2-(a+c)^2}=a\backsim c.$$
$$\frac{a+c-a\backsim c}{2}=c,\text{ and } c+a\backsim c=a.$$

EXAMPLE. — The sum of three numbers which are in arithmetical progression is 18, and the sum of their squares is 140; what are the numbers?

$18 \div 3 = 6 = b$, and $18 - 6 = 12 =$ sum of a and c.
$140 - 6^2 = 104 =$ sum of a^2 and c^2.
$\sqrt{(104 \times 2 - 12^2)} = 8 = a - c$.
$12 - 8 = 4 \div 2 = 2 = c$, and $2 + 8 = 10 = a$; the numbers, therefore, are 2, 6, and 10. *Ans.*

NOTE. — Half the sum of the first and third of three numbers forming an arithmetical progression is equal to the second number.

PROB. XIII. — *The sum of three numbers which are in arithmetical progression added to the sum of the greatest and least, and the sum of the squares of the numbers given, to find the numbers.*

$$\frac{a + b + c + a + c}{5} = b, \text{ and}$$

$$\frac{a + b + c + a + c - b}{2} = a + c; \text{ thence, by PROB. XII.}$$

PROB. XIV. — *The sum of three numbers which are in arithmetical progression added to the sum of twice the greatest and twice the least, and the sum of the squares of the numbers given, to find the numbers.*

$$\frac{3a + 3c + b}{7} = b, \text{ and}$$

$$\frac{3a + b + 3c - b}{3} = a + c; \text{ thence, by PROB. XII.}$$

To find the altitude of an equilateral four-sided pyramid, the slant height and side of the base being known.

$\sqrt{(S^2 - \overline{\tfrac{1}{2} A}^2)} = h$; S being the slant height, A a side of the base, and h the altitude.

$\sqrt{(F^2 - \overline{(\tfrac{1}{2} A)^2 \times 2})} = h$; F being the linear edge.

$\sqrt{(F^2 - f^2)} = h$; f being half the diagonal.

$\sqrt{(F^2 - \overline{A \times .7071}^2)}* = h$;

To find the altitude of the frustum of an equilateral rectangular pyramid.

$\sqrt{S^2 - \left(\frac{A - a}{2}\right)^2} = h$; S being the slant height, A a side of the greater base, and a a side of the less.

$\sqrt{F^2 - \left(\frac{A - a}{2}\right)^2 \times 2} = h$; F being the slant height measured along an angle.

* Diameter $\times$ .7071 = side of inscribed square. See CENTRES OF SURFACES.

SECTION IV.

GEOMETRY —PRACTICAL AND ILLUSTRATIVE.

Geometry is the science that treats of the properties of figured space. It is the science of magnitude in general, and comprehends the mensuration of solids, surfaces, lines, and their various relations.

DEFINITIONS.

A *Point* has position, but not magnitude.

A *Line* is length without breadth, and is either *Right*, *Curved*, or *Mixed*. When no particular line is specified, a right line is meant.

A *Right Line* is a straight line, or the shortest distance between two points.

A *Mixed Line* is a right line and curved line united.

Lines are *parallel*, *oblique*, *perpendicular*, or *tangentical*, one to another.

An *Area*, *surface*, *superficies*, is the space contained within the outline or perimeter of a figure; it has no thickness, and is estimated in the *square* of some unit of measure, as square *inch*, square *yard*, &c.

A *Solid* has length, breadth, and thickness, and its contents are estimated in the *cube* of some unit of measure.

An *Angle* is the diverging of two lines from each other, and is *right*, *acute*, or *obtuse*.

A *Right Angle* has one line perpendicular to another and resting upon it.

A *Triangle*, or *trigon*, is a figure having three sides.

An *Equilateral Triangle* has all its sides equal.

An *Isosceles Triangle* has two of its sides equal.

A *Scalene Triangle* has no two sides equal.

A *Right-angled Triangle* has one right angle.

An *Obtuse-angled Triangle* has one obtuse angle.

An *Acute-angled Triangle* has all its angles acute.

A *Quadrangle*, *tetragon*, *quadrilateral*, is a figure having four sides.

A *Parallelogram* is a quadrilateral figure whose opposite sides are parallel and equal.

A *Rectangle* is a parallelogram whose opposite sides are equal, its angles right angles, and its length greater than its breadth.

A *Square* is an equilateral rectangle.

A *Rhomboid* is a quadrilateral, having its opposite sides equal and parallel, its angles oblique, and a length greater than its breadth.

A *Rhombus*, or *lozenge*, is an equilateral four-sided figure, having oblique angles.

A *Trapezium* is a quadrilateral having no two sides parallel.

A *Trapezoid* is any four-sided figure having two of its sides parallel, but of unequal lengths.

A *Diagonal* is a line joining any two opposite angles of a figure having four or more sides.

A *Polygon* is a plain figure having more than four sides.

A *Regular Polygon* has all its sides equal.

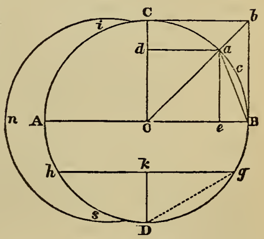
An *Irregular Polygon* has not all its sides equal.

A *Pentagon* has five sides; a *hexagon*, six; a *heptagon*, seven; an *octagon*, eight; a *nonagon*, nine; a *decagon*, ten; an *undecagon*, eleven; a *dodecagon*, twelve.

The *Perimeter* of a figure is its bounds, limits, or outline. It is to other figures what the *circumference* is to the circle, and the perimeter of any portion of a figure is the outline of that portion.

The *Altitude*, or height, of a figure, is a perpendicular let fall from its *vertex*, or highest point, to the opposite side or end, its *base*.

The *Base* of a triangle is that side that is placed parallel to the horizon; and of figures in general the base is that end, or side, upon which the figure is supposed to stand or rest. The sides of a triangle are often called the *legs*. In a right-angled triangle, the longest side, or line which subtends the right angle, is called the *hypotenuse*, and of the other two sides, one is the base, and the other the perpendicular.



A Circle is a plane figure, bounded by a curve line, called the *circumference* or *periphery*, every part of which is equi-distant from a point within called the *centre*, as A C B D, in the *diagram*. The circumference itself is often called a circle.

The *Radius* — *semi-diameter* — is a line drawn from the centre to the circumference, as O A, or O C.

The *Diameter* is a line drawn from the circumference through the centre to the opposite side, as A B.

A *Semicircle* is half a circle, or it is half the circumference of a circle, as A C B.

A *Quadrant* is a quarter of a circle. It is also sometimes a quarter of the circumference, as A C.

An *Arc* is any portion of the circumference, as B *c a*, or *h* C *g*.

A *Chord*, or *subtense*, is a right line joining the extremities of an arc, as B *a*, or *h g*.

A *Segment* is the portion of a circle contained between the arc and its chord, as the space between the arc B *c a* and its chord B *a*, or between the arc *h* D *g*, or *h* C *g*, and the chord *h g*.

A *Sector* is the space between two radii, or lines passing from the centre to the circumference, as the space B O *a*.

A *Secant* is a line that cuts another line. In trigonometry, the secant of an arc is a right line drawn from the centre of a circle through one end of the arc, and terminated by a tangent drawn through the other end; thus, the secant of the arc B *c a* is the line O *b*.

A *Cosecant* is the secant of the complement of an arc, as O *b*.

A *Sine* of an arc is a line drawn from one end of the arc perpendicular to a radius drawn through the other end, as *a e*, and is always equal to half the chord of double the arc; and the sine of an angle is the sine of the arc that measures that angle.

The *Versed Sine* is that portion or part of the radius lying between the foot of the sine and origin of the arc, as *e* B, and the *versed sine of half the arc* is that portion of the radius lying between the chord and the arc, bisecting or dividing both at their centres, as *k* D, in the arc *h* D *g*. It is the height of the arc or segment.

The *Cosine* of an arc, or angle, is that portion of the radius lying between the sine and the centre, as *e* O.

The *Coversed Sine* is the sine of the complement of an arc, or angle, or the coversed sine of the given arc, or angle; thus, the line *a d* is the coversed sine of the arc B *c a*, or of the angle B O *a*.

A *Tangent* is a right line that touches a curve, and which, if produced, will not cut it, — the tangent of the arc B *c a*, is B *b*.

A *Cotangent* is the tangent of the complement of an arc, or the tangent of an arc which is the complement of another arc to ninety degrees; thus, the cotangent to the arc B *c a*, is the line C *b*.

The *Complement* is what remains of the quadrant of a circle, after the angle has been taken therefrom, — the complement of the arc B *c a*, is *a* C.

The *Supplement* is what remains of a semi-circle after taking an angle therefrom, — the supplement of the arc B *c a*, is *a* C A.

A *Gnomon* is the space included between two similar parallelograms, one inscribed within the other, and having one angle common to them both.

A *Zone* is the space between two parallel chords of a circle, — the space included between the lines A B and *h g*.

A *Lune*, or *Crescent*, is the space contained between the intersecting arcs of two *eccentric* circles, as *i n s*.

A *Circular Ring* is the space between the circumferences of two *concentric* circles.

An *Oval* is a figure of an elliptical form made up of arcs of circles.

A *Helix* is a coil or spiral, or it is part of a spiral line.

An *Ogee*, *cyma*, or *talon*, is two circle arcs that tangent each other, and meet two parallel lines, either tangentical to the lines or at right-angles to the lines in given points.

A PRISM is a solid whose bases or ends are any similar, equal plane figures, and whose sides are parallelograms.

A *Parallelopiped* is a solid having six sides, its angles right-angles, and its opposite sides equal. It is a prism, therefore, whose base is a parallelogram.

A *Cube* is a solid having six equal sides and all its angles right-angles. It is a square prism.

A *Prismoid* is a solid whose bases are parallel but unequal, and whose sides are quadrilateral.

A *Pyramid* is a solid having any plane rectilinear figure for its base, and all its sides, more or less, terminating in a point, called its *vertex* or *summit*.

A *Cylinder* is a circular solid, having a uniform diameter, and equal and parallel circles for its ends.

A *Cone* is a solid having a circle for its base, and a true taper therefrom to its vertex.

Conic Sections are the lines formed by the intersections of a plane with a conic surface. They are the *triangle*, *circle*, *ellipse*, *parabola* and *hyperbola*.

A *Conoid* is a solid generated by the revolving of a parabola, or hyperbola around its axis.

A *Spheroid* is a solid generated by the revolving of an ellipse about either of its axes or diameters.

The *Transverse* or *Major* axis of an ellipse is its longest diameter, or the distance, lengthwise, through its centre.

The *Conjugate* or *Minor* axis of an ellipse is the shorter of the two diameters, — a right line bisecting the transverse. If the generating ellipse revolves about its major axis, the spheroid is *prolate*, or *oblong* ; if about its minor axis, it is *oblate*, or flattened.

An *Ordinate* is a right line drawn from any point of the curve of a conic section to either of its diameters, and perpendicular to that diameter.

The *Abscissæ* of a conic section are the parts of either diameter, or axis, lying between their respective vertices and an ordinate.

The *Parameter* — *latus rectum* of a parabola — is a third proportional to any diameter and its conjugate. In the parabola it is a third proportional to any abscissa and its ordinate, or to the altitude of the figure and half the base.

The *Focus* is the point in the axis where the ordinate is equal to half the parameter.

A *Sphere*, or *globe*, is a perfectly round substance — a solid contained under a curved surface, every point of which is equally distant from a point within, called the centre. Its axis, or diameter, is any right line passing from a side through the centre to the opposite side. A hemisphere is half a sphere.

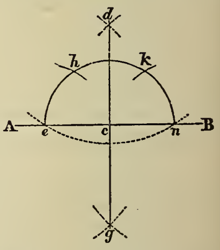
A *Frustum* of any solid figure, as of a cone, pyramid, etc., is the part remaining after a segment has been cut off.

An *Ungula* is the section of a cylinder cut off by a plane oblique to the base.

The *Slant height* of a regular figure is the length of one of its sides, or the distance from the outline of its base to its vertex, or summit.

To bisect a right line, A B, *by a perpendicular.*

Set one foot of the dividers in A, and with the other extended so as to reach somewhat beyond the middle of the line, describe arcs above and below the line ; then, with one foot of the dividers in B, describe arcs crossing the former ; a line drawn from the intersection of the arcs above the line to the intersection of those below, will divide the line into two equal parts.



To erect a perpendicular on a given point in a straight line, or to draw a line at a right angle to another line.

Set one foot of the dividers in the given point, *c*, and with the other extended to any convenient distance, as to A, mark equal distances on each side *c*, as *c* A, *c* B ; and from A and B as centres, with the dividers extended to a distance somewhat greater than that between *c* and A, or *c* and B, describe arcs cutting each other above the line, as at *d*; a line drawn from the intersection of the arcs, *d*, to the point *c*, will be perpendicular to the line A B, or will form a right angle with the line *c* A, or *c* B.

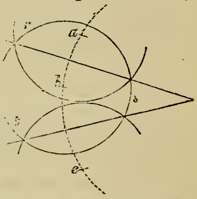
From a point, d, *to let fall a line perpendicular to another line*, A B.

Set one foot of the dividers in *d*, and with the other extended so as to reach beyond the line A B, describe an arc cutting the line A B, in *e* and *n;* then with one foot of the dividers in *e*, and the other extended to more than half the distance between *e* and *n*, describe the arc *g;* then with one foot of the dividers in *n*, describe an arc cutting the arc *g* in *g;* a line drawn from the point *d* through *c* to the intersection of the arcs at *g*, will be the perpendicular required.

To erect a perpendicular upon the end of a line, as at c, *on the line* A c,

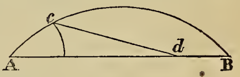
Set one foot of the dividers in *c*, and, at any convenient radius, describe the arc *e h k;* with one foot of the dividers in *e*, cut the arc in *h*, and with one foot in *h*, cut it in *k;* from *h* as a centre, and *k* as a centre, describe arcs cutting each other at *d;* a line drawn from the intersection of the arcs, *d*, to the point *c*, will be perpendicular to the line A *c*.

To draw a circle through any three given points not in a straight line, and to find the centre or radius of a circle, or arc.



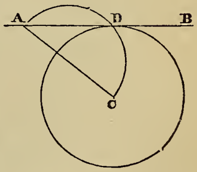
Let the given points be *a*, *b*, *c*. With the dividers opened to any convenient distance, and either point the centre, (as *b*,) describe the portion of a circle *r s t*, and with the same radius and *a* the centre, describe an arc cutting *r s t* in *r* and *s*, and with the same radius and *c* the centre, describe an arc cutting *r s t* in *t* and *s;* draw lines through the points where the arcs cut each other, (the lines *r s* and *t s*,) and their point of union will indicate the centre of the circle or point, from which as a centre a circle, if drawn, will pass through the three given points.

To find the length of an arc of a circle.



Take ¼ the length of the chord of the arc, (A B,) and with the dividers at that radius, and A the centre, cut the arc in *c;* also, with the dividers at the same radius and B the centre, cut the chord in *d;* draw the line *c d*, and twice its length is the length of the arc, nearly.

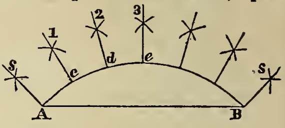
From a given point, to draw a tangent to a circle.



Let A represent the given point, and C, the centre of the circle. Draw a line from A to C; bisect the line, and with the point of bisection as centre, describe the semicircle A D C; then draw a right line, A B, cutting the semicircle at the point where it intersects the circle, which is the tangent sought.

To draw from or to the circumference of a circle, lines tending to the centre of said circle, when the centre is inaccessible.

Divide the circle, or such portion thereof as required, into the desired number of equal parts, and designate the points of division, upon or at an uniform distance from the periphery; then, with any radius less than two of the parts, and the respective points as centres, describe arcs cutting each other, as A 1, *d* 1; *c* 2, *e* 2, &c.; draw the lines *c* 1, *d* 2, *e* 3, &c., which tend to the centre, as required.

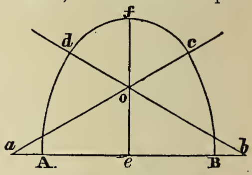


When a portion of a circle or segment only is used, to draw the end lines, A *s*, B *s*.

Produce the arc each way beyond A and B to a distance equal to that between A and *c*, and with the extremes of the extensions as centres, and the second points inward therefrom as centres, describe the intersecting arcs, *s s*. Or, with *c* as a centre, describe the arc *s*, and with the radius *c* 1, and A or B as the centre, describe the intersecting arcs; lines from the intersection of the arcs, *s s*, to their respective points, A and B, will tend to the centre, as required.

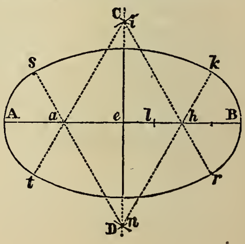
To describe an elliptic arch on a given conjugate diameter.

Let A B be the given diameter, which bisect, and from the point of bisection, *e*, erect the perpendicular *e f*, equal in length, or proportional in length to the height of the intended arch; make *e a*, *e b*, each equal to *e f*, and bisect *e f* in *o*; draw the lines *a o c* and *b o d*, and with the radius *a* B or *b* A, and *a* and *b* as centres, describe the arcs A *d* and B *c*; then, with the radius *o d*, or *o c*, and *o* as the centre, describe the arc *d f c*, and the arch is completed.



To describe an Ellipse of given length and breadth.

Let the line A B equal the given length, or transverse diameter, and the line C D the conjugate, and let these lines bisect each other, forming right angles, on either side, as at *e*. Lay off the distance C D on the line A B, as from A to *l*, and divide the distance *l* B into three equal parts. From *e*, on the line A B, set off two of the parts each way, as *e a*, *e h*; and from *a*, or *h*, designate the distance *a k* on the line C D, as at *i* and *n*; from *i* draw the lines *i t* and *i r*, and from *n*, the lines *n s* and *n k*, passing through the points *a* and *h* and cutting each other

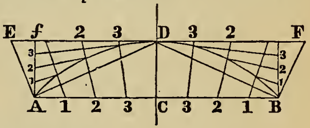


therein. From the point *n*, as a centre, describe the arc *s k*, and from *i*, as a centre, the arc *r t;* also, from *a*, as a centre, describe the arc *t s*, and from *h*, as a centre, the arc *k r*, and the required ellipse is drawn.

Note. — An architrave of any depth desired, may be readily described on the above.

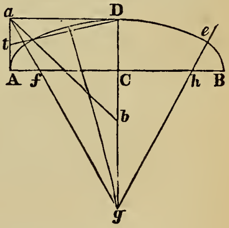
To construct an arc or segment of a circle of large radius.

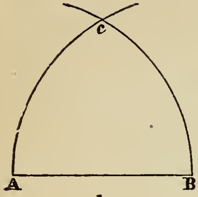
Draw the chord A B equal in length, or proportional in length, to the chord of the arc intended; also draw E F parallel to the chord and at a distance therefrom equal to the height of the intended segment. Bisect the chord, and from the point of bisection erect the perpendicular C D; draw the right lines A D, D B, and draw A E at a right angle to A D, and B F at a right angle to B D, and erect the perpendiculars A *f*, B *f*. Divide A B into any even number of equal parts, and divide E F into the same number of equal parts, and draw the lines 1, 1; 2, 2; 3, 3, &c. Divide A *f*, B *f*, each into half the number of equal parts the chord A B is divided into, and draw lines from D to the points of division, respectively. A curve passing through the intersections of the crossing lines bearing the same number will describe the arc required.



To describe an elliptic arch, the span and rise being given.

Bisect the given chord, or span, A B, with a line at right angles therewith, and let the portion of the line C D be the rise intended. Erect A *a* equal and parallel to C D, and draw the line *a* D equal and parallel to A C. Bisect A *a* in *t*, and draw the line *t* D; make C *b* equal to C D, and draw the line *a b*. Bisect the portion of *t* D lying between the line *a b* and the point D, and from the point of bisection, at right angles to *t* D, draw the line meeting D C in *g;* then draw the line *a g*. Let *f* designate the point where the line *a g* cuts A B, and make *h* B equal to A *f*, and draw the line *g h e*. With the radius *g* D, and *g* as a centre, describe the arc from the line *a g* to *e*, and with the radius A *f*, and *f* and *h* as centres, describe the arcs A *s*, *e* B, which will complete the arch required.





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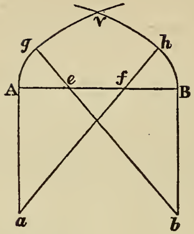

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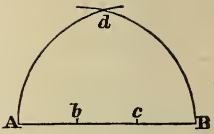


2

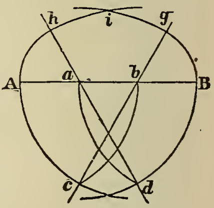
To draw a Gothic arch.

Fig. 1.— With the chord A B as radius, and A and B as centres, describe the arcs A *c* and B *c*.

Fig. 2.—Divide the given chord, A B, into three equal parts, and with two of the parts as radius, and *b* and *c* as centres, describe the arcs A *d*, B *d*.

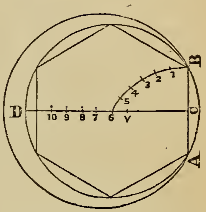
Fig. 3.—Divide the given chord, A B, into three equal parts, *e* and *f*, and from the points A and B let fall the perpendiculars A *a*, B *b*, equal in length to two of the divisions of the chord. Draw the lines *a h* and *b g*, passing through the divisions *e* and *f*, and with one of the divisions as radius, and *e* and *f* as centres, describe the arcs A *g*, B *h;* also, with the radius *a h*, or *g b*, and *a* and *b* as centres, describe the arcs *g v* and *v h*.

Fig. 4.—Divide the given chord, A B, into three equal parts, *a* and *b*, and with the radius two of the parts, and A,*a*,*b*, and B as centres, describe the four arcs *b c*, B *d*, *a d*, A *c;* then, with the radius one of the parts, and *a* and *b* as centres, describe the arcs A *h* and B *g;* then, with the radius *c g*, or *d h*, and *c* and *d* as centres, describe the arcs *h i* and *g i*.



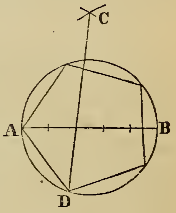
4

To describe a Regular Polygon of any number of sides not exceeding twelve, to a given face or chord line.



Let A B be the given face, which bisect, and from the point of bisection, and at a right angle to A B, draw the line C D. With the radius A B, describe the arc B 6, and divide the arc into six equal parts, and from 6 continue the divisions on the line C D, as v, 7, 8, 9, &c., to 12. A circle whose radius is B v, B 6, B 7, &c., will contain the given face or side (A B) 5, 6, 7, &c., times.

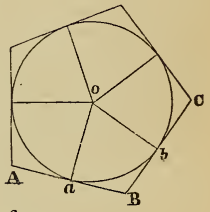
To inscribe any Regular Polygon in a given circle, or to a given diameter.



Let A B be the given diameter, which divide into as many equal parts as the polygon is to have sides; then, with the radius A B, and A and B as centres, describe the arcs cutting each other as at C. From the intersection of the arcs at C, draw a line through the second point of division on the diameter to the periphery, as C D, and the chord of the arc, D A, will be one side of the polygon required nearly.

Let a pentagon be required, See *Fig.*

To circumscribe a Regular Polygon of any given number of sides, about a given circle.

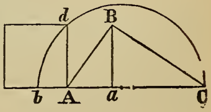


Divide the given circle into as many equal parts as the polygon is to have sides, and define the points of division on the circle; then draw lines from the centre, *o*, to each of the respective points, as *o a*, *o b*, &c. Through these points, and at a right angle to the line leading from the centre thereto, draw the lines A B, B C, &c., which will complete the figure.

Let it be required to circumscribe with a pentagon; — see *Fig.*

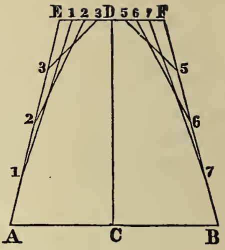
To construct a square whose area shall be that of a given triangle.

Let A B C be the given triangle.



From B let fall the perpendicular B *a*; make A *b* equal to half B *a*; bisect *b* C, and from the point of bisection as a centre, describe the semicircle *b d* C: erect the perpendicular, A *d*, which will be the side of the square required.

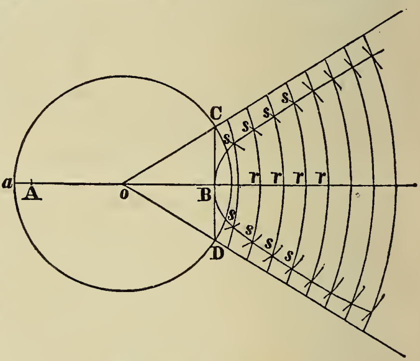
To construct a Parabola.



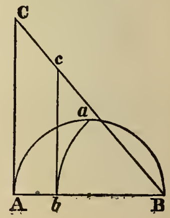
Let A B equal the base, which bisect, and upon the point of bisection erect the perpendicular C D, the altitude; let the line E F be half the length of A B, and let it lie parallel to A B, and at the distance D C from the base, and let it be bisected by C D, as at D. Draw the lines E A, F B, and divide them, together with E D, D F, into four or any number of equal parts, as 1 1, 2 2, &c., and draw the lines connecting the respective points of division. The inner intersections of the said lines with each other define the curve of a parabola.

To construct a Hyperbola.

Let A B equal the longest or transverse diameter, and C D, perpendicular to it, the conjugate, and let the line A B be produced or extended from its respective limits each way, as to *a*, *r*, *r*, &c. Bisect A B in *o*, and with the radius *o* C, or *o* D, and *o* as the centre, describe the circle C *e* D *a*. Divide A B produced from B, into any number of parts, as *r*, *r*, *r*, &c., and with the radii A *r* and B *r*, and the foci *á* and *e* as centres, describe arcs cutting each other as in *s s*, &c. The intersections of the arcs with each other will define the curve of the hyperbola.



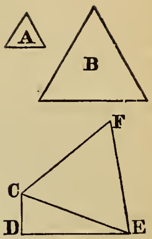
To bisect any given triangle.



Let A B C be the given triangle.

Bisect one of the legs, as A B, and with the point of bisection as a centre describe the semicircle, B *a* A; bisect the semicircle, as at *a*, and with the radius B *a*, and B as a centre, describe the arc *a b;* from *b* erect the line *b c* parallel to A C, which will bisect the given triangle, or divide it into two equal parts, as required.

To draw an equilateral triangle whose area shall be that of two given equilateral triangles.



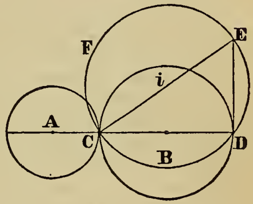
Let the given triangles be A and B.

Draw a line D E equal in length to one side of the larger triangle, and upon one end thereof, and at a right angle therewith, erect a line equal in length to one side of the less triangle, as D C; then draw the line C E, which will be one leg of the triangle required. The equilateral triangle C E F contains an area equal to the triangles A and B.

NOTE. — The same process is applicable to rectangular figures.

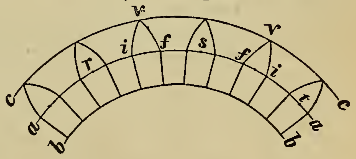
To draw a circle whose area shall be that of two given circles.

Let the circles A and B be the given circles. Draw a line whose length shall be equal to the diameter of the larger circle, and upon the end thereof erect a perpendicular equal in length to the diameter of the lesser circle, as C D E; draw the line E C, and bisect it in *i*, and with the radius *i* C, or *i* E, and *i* as the centre, describe the circle E D F, whose area will be that of the two given circles.



To construct a toothed or cog wheel.

Divide the pitch circle, *a a*, into as many equal parts as there are to be teeth or cogs; then, with the dividers extended to 1¼ times one of those parts, and the point *s* as a centre, describe the arcs *i v*, *v i*; then, with the same radius, and *r* and *t* as centres, describe the arcs *v f* and *f v*, so continuing until the upper sections of all the cogs are defined. The lower sections are bounded by straight lines tending to the centre of the wheel. See TEETH of WHEELS.



NOTE. — The *pitch* of a wheel is the rectilinear distance from the centre of one cog to the centre of the next contiguous, measured upon the pitch circle; and that portion of the length of a tooth lying between the lines *a* and *b* is usually made equal to ⅔ the pitch; and that portion lying between the lines *a* and *c* is usually made equal to ¼ the pitch.

OF THE CONIC SECTIONS.

THE CONIC SECTIONS are the ELEMENTS of geometry. They are lines, and nothing more. The doctrine of their relations is the SCIENCE of geometry. GEOMETRY is lines in position.

The conic sections are the lines formed by the intersections of a plane with a conic surface. They are the *triangle*, *circle*, *ellipse*, *parabola*, and *hyperbola*. With a single exception, they are curved lines or curves.

To the conic sections belong the *foci*, *parameters*, *abscissæ*, and *ordinates*, which are explained in a general way under DEFINITIONS, p. 175, 176.

The *locus*, or place, of a conic section is determined by a point called the *generatrix*, which is supposed to move in accordance with the law of the line.

The *equation* of a line expresses the relation between the ordinate and abscissa of every point in the line, or between the co-ordinates of every point in the line.

The *parameter* is a double ordinate, and passes through the focal point.

The *summit* of an ordinate is a point in the locus of a line, in the *path* of the generatrix. Thus by the plotting of ordinates or double ordinates to different abscissæ, the line is defined; and, by properly connecting the points, it is practically constructed or formed.

The *eccentricity* of a conic section is its deviation from the centre.

The *axis of abscissæ* is the axis, or diameter, in which the abscissæ are taken, or the axis, or diameter, that is divided into abscissæ.

The *axis of ordinates* is the axis, or diameter, that is parallel to the ordinates.

The *origin* of a conic section is in the summit of the axis of abscissæ.

The axis of abscissæ proper is the major axis in any conic section.

The *asymptote* is a right line that continually approaches a curve, but never meets it, however far both may be extended (see CONSTRUCTIONS, *Hyperbola*, p. 182).

In all the conic sections, the parameter is a third proportional to any diameter and its conjugate.

The *radius vector*, or *vector*, is a right line joining the centre of the sun to the centre of a planet. It is an element in astronomy, and has one of its extremities in the focus.

LONGIMETRY AND PLANIMETRY;

OR,

LINES AND SUPERFICIES.

TRIANGLES.

A TRIANGLE is the simplest form in geometry, and the most important. It is a plane, three-sided, rectilinear figure, or is made up of three straight lines. It is the measure chiefly of itself, and is the measure of almost every geometrical form or structure. It is the measure of the resultant of forces acting from different directions upon the same body. Similar triangles are measures of each other.

It is generated, when a plane intersects both sides of a cone, from its apex along the plane of its axis.

As a CONIC SECTION, it is simply two diverging lines having their origin in the same point, and subtended by an ordinate.

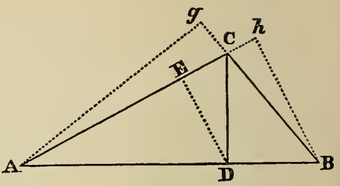
Its equation is expressed by $x : y :: x' : y'$; x being any abscissa on either leg reckoned from the angle opposite the ordinate, y the ordinate, x' any other abscissa on the same leg reckoned from the same angle, y' ordinate to abscissa x'.

In TRIGONOMETRY, it is a figure having three sides and three angles, and is supposed to have one of its angles in the centre of a circle, of which one of the sides is radius.

In GEOMETRY, it is a figure having three sides, three angles, an apothegm, or perpendicular, and an area.

Under its most favorable form (equilateral), it contains less area than a square of the same length of perimeter by 23 per cent.; and less than a circle of the same length of perimeter by nearly 40 per cent.

Right-angled Triangle: — A D C, diagram.



The longest side of this figure, A C, is usually called the hypotenuse, and the other two sides, A D and D C, are called the sides or legs. The legs are perpendicular one to the other, and form the right-angle, D, or angle of 90°.

If one of the legs be made base, the other will be perpendicular, and will be the altitude of the figure; for the altitude of any triangle is a perpendicular, dropped from the vertical angle to the opposite side or base, and either side may be made base; thus, if A D be made base, D C will be perpendicular, and will be the altitude of the figure, and if D C be made base, A D will be perpendicular, and will be the altitude of the figure.

If the hypotenuse be made base, the legs, A D and D C, will still be perpendicular one to the other, but the altitude of the figure, D E or E D, a perpendicular to the hypotenuse, will not be shown.

Whether the legs have equal lengths or unequal, so far as regards the principles of the figure, is immaterial.

$$\sqrt{(\overline{AD}^2 + \overline{DC}^2)} = AC; \quad \sqrt{(\overline{AC}^2 - \overline{DC}^2)} = AD;$$
$$\sqrt{(\overline{AC}^2 - \overline{AD}^2)} = DC.$$

$$\left.\begin{aligned} \overline{AD}^2 \div AC &= AE, \\ \sqrt{(\overline{AD}^2 - \overline{AE}^2)} &= ED. \end{aligned}\right\}$$

Converting the right-angled triangle A D C into two right-angled triangles, A E D and C E D, E D a leg common to both, and perpendicular to A C, and the altitude of the triangle A D C, therefore, A C being base.

$\overline{DC}^2 \div AC = CE.$	$AC \times AE = \overline{AD}^2.$
$\sqrt{(\overline{DC}^2 - \overline{CE}^2)} = ED.$	$AC \times CE = \overline{DC}^2.$
$\overline{AD}^2 \div AE = AC.$	$AE \times CE = \overline{ED}^2.$
$\overline{DC}^2 \div CE = AC.$	$\overline{AD}^2 : \overline{AC}^2 :: AE : AC.$
$\overline{ED}^2 \div AE = CE.$	$\overline{DC}^2 : \overline{AD}^2 :: CE : AE.$

$$\overline{DC}^2 : \overline{AC}^2 :: CE : AC.$$

Twice the area of a right-angled triangle, divided by the hypotenuse, is equal the perpendicular to the hypotenuse, and the respective triangle will thereby be divided into two right-angled triangles having a side common to both.

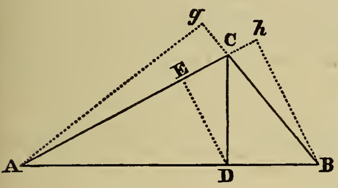
Half the product of the two legs of a right-angled triangle equals the area of that triangle.

$$\frac{2\ \text{area}}{\text{perp. to hypot.}} = \text{hypotenuse.} \qquad \frac{2\ \text{area}}{\text{given leg}} = \text{required leg.}$$

Of Oblique-angled Triangles.

Every triangle not a right-angled triangle is either an acute-angled triangle or an obtuse-angled triangle, and these two (the acute-angled and obtuse-angled) are classed under the general name oblique-angled. The following principles are alike applicable to either.

Let A B C be the triangle.



$$\frac{\overline{AC}^2 - \overline{BC}^2}{2\,AB} + \tfrac{1}{2}\,AB = AD,$$

distance along the base, from the angle formed by the base and longest vertical side, at which a perpendicular dropped from the vertical angle will fall; and

$\sqrt{(\overline{AC}^2 - \overline{AD}^2)} = DC$, the perpendicular alluded to; thus dividing the obtuse-angled triangle A B C into two right-angled triangles, A D C and B D C, D C a leg common to both.

Or, $\sqrt{(\overline{AC + AD} \times \overline{AC - AD})} = DC$; for the sum of any two quantities multiplied by their difference is equal to the difference of their squares.

$\dfrac{\overline{AB}^2 - \overline{AC}^2}{2\,BC} + \tfrac{1}{2}\,BC = B\,g$, and $\sqrt{(\overline{AB}^2 - \overline{Bg}^2)} = A\,g$, perpendicular to B C produced.

$\dfrac{\overline{AB}^2 - \overline{BC}^2}{2\,AC} + \dfrac{AC}{2} = A\,h$, and $\sqrt{(\overline{AB}^2 - \overline{Ah}^2)} = B\,h$, perpendicular to A C produced.

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - 2\,\overline{AB} \times \overline{BD}$$

$$\overline{BD} = \frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2\,\overline{AB}}$$

$$AD = \frac{\overline{AC}^2 + \overline{AB}^2 - \overline{BC}^2}{2(AB)}.$$

$$\overline{AC}^2 - \overline{Ag}^2 = \overline{Cg}^2, \text{ and } Cg + BC = Bg;\ \overline{Ag}^2 + \overline{Cg}^2 = \overline{AC}^2.$$

Twice the area of any triangle divided by the base is equal the perpendicular to the base.

Half the side of any triangle multiplied by the perpendicular to that side is equal the area.

NOTE. — In an obtuse-angled triangle, a perpendicular dropped from either of the acute angles will fall outside the figure.

In an equilateral triangle, a perpendicular dropped from either angle will bisect the side opposite; and the triangle will thereby be divided into two equal and similar right-angled scalene triangles.

In an isosceles triangle, a perpendicular dropped from the angle included by the equal sides will bisect the side opposite, and the triangle will thereby be divided into two equal and similar right-angled scalene triangles, or two equal and similar right-angled isosceles triangles: dropped from an angle opposite one of the equal sides, it will fall outside the figure if the triangle be obtuse; inside, if it be acute.

RECAPITULATIONS.

To find the Perpendicular of an Oblique-angled Scalene Triangle, the Sides being given.

Let l represent the longest side, s the shortest side, m the intermediate side, and h the perpendicular.

When l is made base (B),

$$h = \sqrt{s^2 - \left(\frac{B^2 + s^2 - m^2}{2B}\right)^2} = \sqrt{m^2 - \left(\frac{B^2 + m^2 - s^2}{2B}\right)^2}.$$

When s is made base,

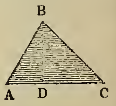
$$h = \sqrt{l^2 - \left(\frac{l^2 + B^2 - m^2}{2B}\right)^2}.$$

When m is made base,

$$h = \sqrt{l^2 - \left(\frac{l^2 + B^2 - s^2}{2B}\right)^2}.$$

To find the Area of a Triangle.

RULE. — Multiply the base by half the perpendicular height, or, the perpendicular height by half the base; or, multiply the base by the altitude, and divide the product by 2, and the quotient will be the area.



EXAMPLE. — The base A C, of the triangle A C B, is 12 feet, and the altitude, D B, is 4 feet and 6 inches; required the area.

$4.5 \times \frac{12}{2} = 27$ square feet. *Ans.*

To find the Area of a Triangle by Means of the Sides.

RULE. — Add the three sides together, and from half the sum subtract each side separately; multiply the half sum and the three remainders into each other, and from the product extract the square root, which will be the area sought.

EXAMPLE. — The sides of a triangle are 30, 40, and 60 rods; what area has the triangle?

$\overline{30 + 40 + 60} \div 2 = 65 = \frac{1}{2}$ sum of the three sides.
$65 - 30 = 35$, first remainder.
$65 - 40 = 25$, second remainder.
$65 - 60 = 5$, third remainder.
$65 \times 35 \times 25 \times 5 = \sqrt{284375} = 533.26$ square rods. *Ans.*

To find the hypotenuse of a triangle, the other two sides being given.

RULE. — Add the square of the base and square of the perpendicular together, and the square root of the sum will be the hypotenuse.

EXAMPLE. — The distance from the base of a building to the sill of an attic window — perpendicular of the triangle, B C — is 40 feet; what must be the length of a ladder, — hypotenuse of the triangle, A C, — placed on a level with the base of the building, and 12 feet therefrom, — base of the triangle, A B, — to reach to the sill of said window?

$\overline{40^2 + 12^2} = \sqrt{1744} = 41\frac{3}{4}$ feet. *Ans.*

The hypotenuse and one of the sides of a right-angled triangle being given, to find the length of the other side.

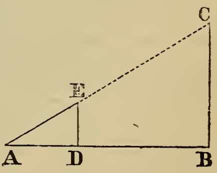
RULE. — Add the hypotenuse and the given leg together, multiply the sum by the difference of their lengths, and the square root of the product will be the side sought.

EXAMPLE. — The sill of a window in a building standing on the edge of a stream is 30 feet above the water, and a line which has been extended therefrom directly across the stream to the opposite shore is found to measure 80 feet; required the width of the stream at that place.

$80 - 30 = 50$, and $(80 + 30) \times 50 = \sqrt{5500} = 74.16$ feet. *Ans.*

To find the height of an inaccessible object, C, *or the length of the perpendicular* B C.

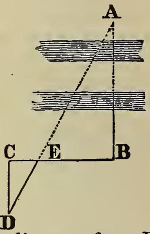
RULE. — Upon a plane, at a right angle to the base of the perpendicular, as A B, erect, at any convenient distance from the base, a perpendicular staff D E, and construct the hypotenuse A E in the direction A C; then, as the base A D is to the perpendicular staff D E, so is the base A B to the height B C.



Example. — The perpendicular D E, of the right-angled triangle A D E, being 3 feet, and the base, A D, 5 feet, what is the perpendicular of a similar triangle, A B C, whose base, A B, is 100 feet?

5 : 3 :: 100 = 60 feet. *Ans.*

To find the distance from a given point, B, *to an inaccessible object, as at* A.

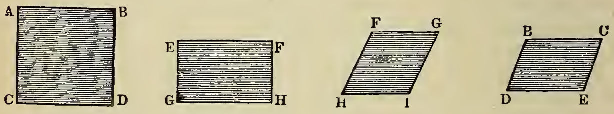


Rule. — Draw a line B in the direction A, and from the point B, at a right angle to B A, draw, to any convenient length, a line B C; form a right angle to B C, from the point C, and convert into a triangle by carrying the hypotenuse, D E, to the line B C, in the direction D A; then, as the distance C E is to the distance C D, so is the distance B E to the distance B A.

Example. — The distance from C to E is 4 rods, the distance from C to D, 15 rods, and the distance from E to B, 22 rods, whereby required the distance from B to A.

4 : 15 :: 22 = 83¾ rods. *Ans.*

OF QUADRILATERAL FIGURES.



Square. Rectangle. Rhombus. Rhomboid.

To find the area of either the above figures.

Rule. — Multiply the length by the breadth or perpendicular height, and the product is the area.

Example. — How many square feet in a floor whose form is a rhomb, or rhombus, each side being 12 feet, and the perpendicular or right-angular distance from the side F G to the side H I being 8 feet?

12 × 8 = 96 square feet. *Ans.*

The above figures are *parallelograms*, for a parallelogram is a quadrilateral whose opposite sides are equal.

A square is an equilateral rectangular parallelogram: it is two equal and similar right-angled isosceles triangles by either of its diagonals; draw both its diagonals, and it will be made up of four equal and similar right-angled isosceles triangles.

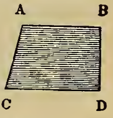
A rectangle is a right-angled parallelogram that has more length than breadth: it is two equal and similar right-angled scalene triangles by either of its diagonals; by both, it is two equal and similar obtuse-angled isosceles triangles, and two equal and similar acute-angled isosceles or equilateral triangles.

A rhombus, rhomb, or lozenge is an equilateral oblique-angled parallelogram: it is two equal and similar obtuse-angled isosceles triangles by its longest diagonal; it is two equal and similar acute-angled isosceles or equilateral triangles by its shortest diagonal; it is four equal and similar right-angled scalene triangles by both its diagonals.

A rhomboid is an oblique-angled parallelogram that has more length than breadth: it is two equal and similar right-angled scalene triangles by either of its diagonals; by both, it is two equal and similar obtuse-angled scalene triangles, and two equal and similar right-angled scalene triangles.

To find the Area of a Trapezoid.

Rule.—Multiply the sum of the two parallel sides by the perpendicular distance between them, and divide the product by 2; the quotient will be the area.

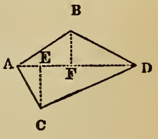


Example.—The side A B, of the trapezoid A B-C D, is $48\frac{5}{12}$ feet, the side C D is $72\frac{4}{12}$ feet, and the perpendicular distance between the sides is $40\frac{1}{2}$ feet; how many feet area has the figure?

$72\frac{4}{12} + 48\frac{5}{12} = 120\frac{3}{4} \times 40\frac{1}{2} = 4890\frac{3}{8} \div 2 = 2445$ ft. $2\frac{1}{4}$ in. *Ans.*

To find the Area of a Trapezium.

Rule—Draw a diagonal through the figure, which will divide it into two triangles, and multiply the length of the diagonal by half the sum of the altitudes of the triangles; the product will be the area.



Example.—The diagonal A D, in the trapezium A B C D, is 54 rods in length, and the altitudes of the triangles formed by the introduction of the diagonal are 20 rods and 26 rods; required the area of the figure.

$\overline{26 + 20} \div 2 = 23 \times 54 = 1242$ square rods. *Ans.*

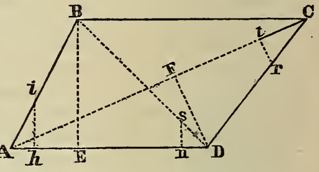
Having the figure of a rhombus, rhomboid, or trapezoid, the perpendicular, whereby to find the area, may be found by the following method.

Suppose the diagram A B C D.

A D and B C sides parallel to each other.

A D base.

A B side inclining to the base and whose length is known.



Then, at any convenient distance from the angle A, on A D, erect a perpendicular to A D that will cut the side A B, as *i h*; then will A *i* be to *i h* as A B to B E, perpendicular required; or A *i* will be to A *h* as A B to A E, distance from the angle A, on A D, at which a perpendicular dropped from B will fall; and

$$\sqrt{(\overline{A\,B}^2 - \overline{A\,E}^2)} = B\,E, \text{ perpendicular required.}$$

Example. — The figure A B C D is that of a field whose side A B is 68 rods: A *i* is 4.2 feet, and *i h* is 3.7 feet; required the perpendicular distance from the side A D to the side B C.

$$4.2 : 3.7 :: 68 : 59.9 \text{ rods. } \textit{Ans.}$$

Example. — A bin in the form of a trapezoid, A B C D (Fig.), has a side A B inclining to A D that is 12 feet in length, and a perpendicular to A D, erected on A D, that cuts the side A B, gives the segment A *i* 2 feet, and the segment A *h* 1½ feet; required the perpendicular distance from the side A D to the side B C.

$$2 : 1.5 :: 12 : 9, \text{ and } \sqrt{(12^2 - 9^2)} = 7.94 \text{ feet. } \textit{Ans.}$$

A *i* : A B :: A *h* : A E, diagram.
A E : A B :: A *h* : A *i*, "
A *h* : E *h* :: A *i* : B *i*, "
i h : A *i* :: B E : AB, "
A *h* : *i h* :: A E : B E, "

To find a diagonal of the above figure A B C D.

$$\sqrt{(\overline{B\,E}^2 + \overline{E\,D}^2)} = B\,D, \text{ diagonal.}$$

Having the area of a rhombus, rhomboid, or trapezoid, and the sum of any two sides of the figure that are parallel to each other, the perpendicular to those sides will be found, if we divide twice the area by the sum referred to.

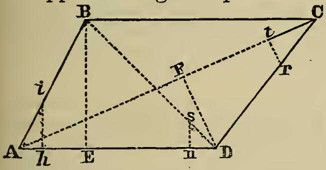
Suppose A D and B C the given sides;
B E the perpendicular required; then

$$\frac{2\ \text{area}}{A D + B C} = B E. \quad \frac{\text{area}}{\frac{1}{2} B E} = A D + B C.$$

$$\frac{\text{area}}{\frac{1}{2} B E} - A D = B C. \quad \frac{2\ \text{area}}{B E} - B C = A D.$$

If through any four-sided figure a diagonal be drawn or be supposed to be drawn, the figure will be converted into two triangles, each of which will have a side, the diagonal, that will be common to them both; and the length of that side or diagonal, which is an indispensable element in calculating the area of a trapezium, may be found, when more simple means may not be resorted to, by one or the other of the following methods:—

Suppose the figure represented by the diagram A B C D.



C to A, diagonal required. Construct a partial diagonal, C t, direction C A, and thereon erect a perpendicular to C t that will cut an adjacent side, as $r\,t$ cutting the side C D; then C r : C t : : C D : C F, and C r : $r\,t$: : C D : D F; and $\sqrt{(\overline{AD}^2 - \overline{DF}^2)} = $ A F, and C F + A F = C A, diagonal sought.

Example.—A structure in the form of a trapezium, A B C D, (Fig.) has C r 8 inches, C t 6 inches, and $r\,t$ 5 inches; the side C D is 16 feet, and the side A D is 20 feet; required the length of a diagonal C A.

8 : 6 : : 16 : 12 feet, C F, or distance from C, on a line C A, at which a perpendicular dropped from D will fall, and

8 : 5 : : 16 : 10 feet, D F, or length of that perpendicular, then

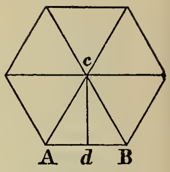
$20^2 - 10^2 = \sqrt{300} = 17.32 + 12 = 29.32$ feet, length of a diagonal C A, or of the side A C of the triangle A C B, or of the side C A of the triangle C A D.

Again, suppose the same diagram, the side A D accessible, and D to B the diagonal required.

D n : D E : : D s : D B, diagonal sought.

OF POLYGONS.

To find the area of a regular polygon.



Rule. — Multiply the length of a side by half the distance from the side to the centre, and that product by the number of sides; the last product will be the area of the figure.

Example. — The side A B of a regular hexagon is 12 inches, and the distance therefrom to the centre of the figure, *d c*, is 10 inches; required the area of the hexagon.

$\frac{10}{2} \times 12 \times 6 = 360$ sq. in. $= 2\frac{1}{2}$ sq. feet. *Ans.*

Table *of angles relative to the construction of Regular Polygons with the aid of the Sector, and of co-efficients to facilitate their construction without it; also, of co-efficients to aid in finding the area of the figure the side only being given.*

Names.	No. of sides.	Angle at centre.	Angle at circum.	Perp'n. side being 1.	Length of side, radius being 1.	Radius of circle, side being 1.	Radius of circ. perp. being 1.	Area, side being 1.
Triangle,	3	120°	60°	0.28868	1.73205	.5773	2.	0.433013
Square,	4	90	90	0.5	1.4142	.7071	1.4142	1.
Pentagon,	5	72	108	0.6882	1.1755	.8506	1.236	1.720477
Hexagon,	6	60	120	0.866	1.	1.	1.155	2.598076
Heptagon,	7	$51\frac{3}{7}$	$128\frac{4}{7}$	1.0382	.8677	1.152	1.11	3.633912
Octagon,	8	45	135	1.2071	.7654	1.3065	1.0823	4.828427
Nonagon,	9	40	140	1.3737	.684	1.4619	1.06	6.181824
Decagon,	10	36	144	1.5388	.618	1.618	1.05	7.694209
Undecagon,	11	$32\frac{8}{11}$	$147\frac{3}{11}$	1.7028	.5634	1.7747	1.04	9.36564
Dodecagon,	12	30	150	1.866	.5176	1.9318	1.035	11,196152

Note. — "Angle at centre" means the angle of radii, passing from the centre to the circumference, or corners of the figure.

"Angle at circumference" means the angle which any two adjoining sides make with each other.

Every circle contains its own radius, as a chord line, exactly six times; therefore,

To describe a polygon with the aid of the sector.

Rule. — Take the chord of 60° on the sector, and describe a circle; then, with the chord, (on the same line of the sector,) of as many degrees as indicated in the table, for the respective polygon — column, "angle at centre" — space off the circle, and each space will be the side of the polygon required. Thus, for a *decagon*, take the chord of 60° on the sector for the radius of the circle, and the chord of 36° on the same line of the sectors for a side.

EXAMPLE. — 1. The radius of a circle is 7 feet; required the side of the greatest regular octagon that may be inscribed therein.

$7 \times .7653 = 5.3571$ feet, or 5 ft. $4\frac{1}{4}$ in., nearly. *Ans.*

2. The sides of a heptagon are to be, each, 5 inches; required the radius of circumscribing circle.

$1.152 \times 5 = 5.760 = 5\frac{3}{4} +$ in. *Ans.*

3. Each side of a hexagon is 12 inches; required the distance (*d c*) from the centre of a side to the centre of the figure.

$12 \times .866 = 10.392$ inches. *Ans.*

4. The side of an equilateral triangle is 12 inches; required its altitude, — the perpendicular.

$12 \times 3 \times .28868 = 10.392$. *Ans.*

5. A PERPENDICULAR, from the centre to either side of a hexagon, is required to be 12 inches; what must be the radius of circumscribing circle?

$12 \times 1.156 = 13.872$ inches. *Ans.*

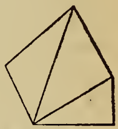
To find the area of a regular polygon, when the side only is given.

RULE. — Multiply the square of the side by the number or factor in the TABLE — (column, Area) — opposite the name of the respective polygon, and the product will be the area.

EXAMPLE. — Each side of a nonagon is $6\frac{1}{2}$ rods; required its area.

$6.5^2 \times 6.181824 = 261.18 +$ square rods. *Ans.*

To find the area of an irregular polygon.



RULE. — Divide the figure into trapeziums and triangles, by drawing diagonals, and find the area of each, separately; the sum of the several areas will equal the area of the figure.

EXAMPLE. — The outline of the above figure defines an irregular polygon; the enclosed lines divide it into three triangles; and the areas of the several triangles, taken collectively, are equal to, or constitute, the area sought. To find the areas of the triangles, see TRIANGLES — *Mensuration of.*

CIRCLE.

A CIRCLE is an endless line equidistant in all its parts from a point within called the *focus*, or centre.

It is generated when a plane revolves about a conic surface perpendicular to the axis, or by a plane cutting through both sides of a cone parallel to the base.

It is equated by means of the triangle, as we shall see.

Either semi-diameter of two diameters drawn at right angles to each other in a circle may be an ordinate.

In TRIGONOMETRY, a circle is the *measure* of angles, and is supposed to be divided into parts called degrees, minutes, and seconds; also into semicircles, quadrants, and arcs. In TRIGONOMETRY, arcs are the measures of angles, and angles are the measures of arcs and sides (see TRIGONOMETRY).

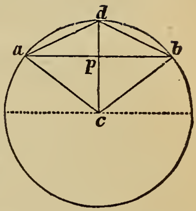
In GEOMETRY, a circle-plane is usually called a circle, and its boundary line, above described, is called the circumference, or periphery; thus triangles, as they relate to the circle, and are drawn within its circumference, may be counted sectional lines of the circle, and treated as such.

Generally, then, a circle is a plane figure having a circumference, a radius, a diameter, an area, &c.

The *ratio* of the circumference to the diameter of a circle is commonly expressed by the Greek letter π (*pi*), and π will be used with that signification and no other in this work. The exact value of π, or the length of the circumference of a circle when the diameter is 1, it is well known has never been found: it can no more be written with numbers, probably, than can the exact root of a surd number. 3.14159265358979323846264338327950288419716939937 . . . $= \pi$. In practice, π is usually taken $= 3.1416$.

A circle contains a greater area than any other figure of the same length of perimeter or outline.

OF THE CIRCLE AND ITS SECTIONS.



Relationship of the sectional lines of the circle, one with another, whereby, any two being given, any other may be found.

To furnish several examples, and to exhibit the proof with the operation, suppose the lines $a\,b$ and $p\,d$ given, and let $a\,b = 20$, inches, feet, yards — any linear measure, — and let $p\,d = 3.51$.

$\underset{}{ab}$ $\quad$ ap
$20 \div 2 = 10$, sine of half the arc, or half the chord of the segment.

$\overline{a\,p}^2 \quad \overline{p\,d}^2 \quad p\,d \quad 2\,c\,a$
$100 + 12.32 \div 3.51 = 32$, diameter.

$2\,c\,a \quad c\,a$
$32 \div 2 = 16$, radius.

$c\,a \quad p\,d \quad c\,p$
$16 - 3.51 = 12.49$, cosine of half the arc.

$c\,a \quad c\,p \quad p\,d$
$16 - 12.49 = 3.51$, versed sine of half the arc, or height of segment.

$c\,p \quad p\,d \quad c\,a$
$12.49 + 3.51 = 16$, radius.

$\overline{a\,p}^2 \quad \overline{p\,d}^2 \quad \overline{a\,d}^2 \quad \overline{a\,d}^2 \quad a\,d$
$100 + 12.32 = 112.32$, and $\sqrt{112.32} = 10.5981$, ch'd of half the arc.

$\overline{c\,a}^2 \quad \overline{a\,p}^2 \quad \overline{c\,p}^2 \quad \overline{c\,p}^2 \quad c\,p$
$256 - 100 = 156$, and $\sqrt{156} = 12.49$, cosine.

$\overline{c\,a}^2 \quad \overline{c\,p}^2 \quad \overline{a\,p}^2 \quad \overline{a\,p}^2 \quad a\,p$
$256 - 156 = 100$, and $\sqrt{100} = 10$, half the chord of seg.

$\overline{c\,p}^2 \quad \overline{a\,p}^2 \quad \overline{c\,a}^2 \quad \overline{c\,a}^2 \quad c\,a$
$156 + 100 = 256$, and $\sqrt{256} = 16$, radius.

Radius and height of segment given, to find chord of segment.

$c\,a - p\,d = c\,p$, and $\overline{c\,a}^2 - \overline{c\,p}^2 = \sqrt{\overline{a\,p}^2} = a\,p$, and $a\,p \times 2 = 2\,a\,p =$ chord, $a\,b$. *Ans.*

Chord and height of segment given, to find diameter.

$\overline{\tfrac{1}{2}\,a\,b}^2 + \overline{p\,d}^2 = \overline{a\,d}^2$, and $\overline{a\,d}^2 \div p\,d = 2\,c\,a$. *Ans.*

Radius and height of segment given, to find chord of half the arc.

$c\,a - p\,d = c\,p$, and $\overline{c\,a}^2 - \overline{c\,p}^2 = \overline{a\,p}^2$, and $\overline{a\,p}^2 + \overline{p\,d}^2 = \sqrt{\overline{a\,d}^2} = a\,d$ *Ans.*

Radius and sine given, to find versed sine, or height.

$\overline{ca}^2 - \overline{ap}^2 = \overline{cp}^2$, and $ca - cp = pd$. *Ans.*

Radius and cosine given, to find chord of half the arc.

$\overline{ca}^2 - \overline{cp}^2 = \overline{ap}^2, ca - cp = pd$, and $\overline{ap}^2 + \overline{pd}^2 \sqrt{\overline{ad}^2} = ad$. *Ans.*

Radius and chord of half the arc given, to find sine of half the arc.

$\overline{ad}^2 \div 2\,ca = pd$, and $\overline{ad}^2 - \overline{pd}^2 = \sqrt{\overline{ap}^2} = ap$. *Ans.*

Chord of half the arc and sine of half the arc given, to find radius.

$\overline{ad}^2 - \overline{ap}^2 = \overline{pd}^2$, and $\overline{ad}^2 \div 2\,pd = ca$. *Ans.*

Chord of half the arc and sine of half the arc given, to find cosine.

$\overline{ad}^2 - \overline{ap}^2 = \overline{pd}^2$, and $\overline{ad}^2 \div 2\,p\,d = c\,a$, and $\overline{ca}^2 - \overline{ap}^2 = \sqrt{\overline{cp}^2} = cp$. *Ans.*

Radius and sine of half the arc given, to find versed sine.

$\overline{ca}^2 - \overline{ap}^2 = \overline{cp}^2$, and $ca - cp = pd$. *Ans.*

Sine and Versed Sine given, to find Cosine.

$\overline{ap}^2 + \overline{pd}^2 = \overline{ad}^2$ and $\overline{ad}^2 \div 2\,pd = ca$; $ca - pd = cp$. *Ans.*

The *equation* of the circle is commonly written $y^2 = a^2 - x^2$, which, referred to the foregoing, is equivalent to the expression $\overline{ap}^2 = \overline{ac}^2 - \overline{cp}^2$, and denotes the same quantities. But, in Analytical Geometry generally, *sines* and *cosines* are not known: they give place to *ordinates* and *abscissæ*. Thus in the equation of the circle, and in constructing it by plotting the double ordinates of different abscissæ, ap is an ordinate and cp is its abscissa. Referring to the diagram, $\overline{ap}^2 = 2\,ac \times \overline{pd} - \overline{pd}^2$, and this is an expression for the equation of the circle: it may be written $y = \sqrt{(dv - v^2)}$. Both $c\,p$ and $d\,p$ are abscissæ to the ordinate $a\,p$. The equation of a curve expresses the relation between the ordinate and corresponding abscissa of every point in the curve.

To find the Diameter, Circumference, and Area of a Circle.

Let d represent the diameter, c the circumference, and Δ the area.

$$d = c \div \pi = c \div 3.1416 = \sqrt{(4\Delta \div \pi)} = 1.12838\sqrt{\Delta}.$$

$$c = \pi d = d \times 3.1416 = \pi\sqrt{(4\Delta \div \pi)} = 3.5449\sqrt{\Delta}.$$

$$\Delta = \pi d^2 \div 4 = d^2 \times .7854 = cd \div 4 = c^2 \div 4\pi = \pi r^2.$$

EXAMPLE. — A line (the chord $a\ b$, diagram) was stretched across a circular railway 46 feet, and the versed sine, $p\ d$, was found to be 1.8 feet; required the diameter, circumference, and area of the circle.

$46 \div 2 = 23$, and $(23^2 + 1.8^2) \div 1.8 = 295.69$ feet, diameter. *Ans.*

$295.69 \times 3.1416 = 928.94$ feet, circumference. *Ans.*

$$\left.\begin{array}{l} 295.69^2 \times 0.7854, \text{ or} \\ \dfrac{928.94}{2} \times \dfrac{295.69}{2} \end{array}\right\} = \left\{\begin{array}{l} 68669.55 \text{ square feet, or} \\ 68669.55 \div 43560 = 1.576 \text{ acres} \end{array}\right\} \textit{Ans.}$$

To find the Length of an Arc of a Circle.

Let r represent the radius of the arc; n, the number of degrees in the arc; d, the diameter of the circle to which the arc belongs; v, the versed sine of half the arc; and l, the length of the arc.

$$l = 2\sqrt{dv}\left(1 + \frac{v}{2.3.d} + \frac{3v^2}{2.4.5.d^2} + \frac{3.5.v^3}{2.4.6.7.d^3} + \frac{3.5.7.v^4}{2.4.6.8.9.d^4} + \right).$$

$$l = \pi rn \div 180 = .01745329252rn.$$

These rules are applicable to all arcs of circles, whether greater or less than half the circumference. But the former is too tedious for general practice, and it is not often that we can know the number of degees in the arc without the aid of trigonometrical tables. We can approximate the length with certainty and considerable precision by other methods.

Let b represent the chord of the arc, $a\ b$, diagram; r, the radius of the arc; c, the cosine of half the arc of the salient angle, $c\ p$, diagram; l, the length of the arc of the salient angle; and

$$k = \frac{(1 + \frac{1}{4}\sqrt{2})45}{\frac{1}{2}\sqrt{2}} = 86.13961: \text{ then}$$

When c is equal to or greater than $\frac{1}{2}b$,

$$l = 1.5034198br \div \left(r + \tfrac{1}{2}c + \tfrac{c - \frac{1}{2}b}{2k}\right) \text{ very nearly,}$$

$$\text{and } n = kb \div \left(r + \tfrac{1}{2}c + \tfrac{c - \frac{1}{2}b}{2k}\right) \text{ very nearly;}$$

$$\text{but } l = \frac{\pi rn}{180};\ \therefore = \frac{\pi r}{180}\left(\frac{86.13961b}{r + \frac{1}{2}c + \frac{c - \frac{1}{2}b}{2k}}\right) \text{ very nearly.}$$

When c is less than $\frac{1}{2}b$,

$$l = \pi r\left(1 - \frac{0.957107c}{r + \frac{1}{4}b + \frac{\frac{1}{2}b - c}{2k}}\right) \text{ very nearly,}$$

$$\text{and } n = 180 - \frac{2kc}{r + \frac{1}{4}b + \frac{\frac{1}{2}b - c}{2k}} \text{ very nearly;}$$

$$\therefore l = \frac{\pi r}{180}\left(180 - \frac{2kc}{r + \frac{1}{4}b + \frac{\frac{1}{2}bc}{2k}}\right) \text{ very nearly.}$$

But $2\pi r =$ circumference of circle containing 360°.

The length of the greater arc, or arc of the re-entrant angle, therefore $= 2\pi r - l$, and contains $360 - n$ degrees, $\therefore$

$$= \pi r\,(360 - n) \div 180.$$

Now, by the foregoing expressions for n, we obtain the calculated angles $kb \div \left(r + \frac{1}{2}c + \frac{c - \frac{1}{2}b}{2k}\right)$, and $2kc \div \left(r + \frac{1}{4}b + \frac{\frac{1}{2}b - c}{2k}\right)$, strictly correct for angles 45°, 90°, 135°, and 180°; and the errors are equal and reciprocal for angles and their supplements of 180°. They are all $-$ from 0 to 45°, and from 90° to 135°, and $+$ from 45° to 90°, and from 135° to 180°. The maximum error is only 2′. It obtains when c and $\frac{1}{2}b$ are to each other as 3 to 4, or at 73° 44′ 22″ and 106° 15′ 38″. The mean error (one minute) occurs at 16° 15′ 38″, 28° 44′ 22″, 57° 28′ 44″, 86° 13′ 6″, 93° 46′ 54″, 122° 31′ 16″, 151° 15′ 38″, and 163° 44′ 22″.

NOTE.—The popular expression for the length of the arc of a circle, when the ratio of the arc to the circumference is unknown: viz. $(8m - b) \div 3$, in which m represents the chord of half the arc, and b the chord of the arc, answers tolerably well when the arc is not greater than a quadrant; for it expresses a quadrant, only 4′ 20″ short, and for a shorter arc the error is less; but it expresses the arc 2⅛° short when it is equal to two quadrants.

EXAMPLE.—The chord of a circle-arc is 24 and the cosine 16. What is the length of the arc?

$$r = \sqrt{[c^2 + (\tfrac{1}{2}b)^2]} = 20, \text{ and } c > \tfrac{1}{2}b;$$

then

$$n = \frac{86.13961 \times 24}{20 + 8 + .0232181} = 73°.772778,$$

$$\text{and } l = \frac{\pi r n}{180} = 25.75162 = \frac{1.5034198 \times 24 \times 20}{20 + 8 + .0232181},$$

and this example involves the maximum error,

(true $l = 25.73998$). *Ans.*

EXAMPLE. — Half the chord of a circle-arc is 9, and the versed sine 5 ; required the length of the arc.

$$(9^2+5^2)\div 2\times 5 = 10.6 = \text{radius of arc,}$$

$$\text{and } 10.6-5 = 5.6 = \text{cosine, } c < \tfrac{1}{2}b\,;$$

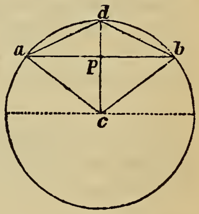
$$\text{then } n = 180 - \frac{172.27922\times 5.6}{10.6+4.5+.0197354} = 116^\circ.191765,$$

$$\text{and } l = \frac{\pi r n}{180} = 21.496 = \pi r\left(1 - \frac{.957107\times 5.6}{10.6+4.5\times .0197+}\right). \quad \textit{Ans.}$$

To find the Area of a Sector of a Circle.

Let Δ represent area of sector of salient angle ($a\,c\,b\,d\,a$, diagram) ; l, length of its arc ; n, number of degrees in its arc ; r, radius of circle.

$$\Delta = \pi r^2 n \div 360 = \tfrac{1}{2}lr.$$



But πr^2 = area of circle containing 360°.

The area of the greater sector, or sector of the re-entrant angle, therefore =

$$\pi r^2(360-n)\div 360 = \pi r^2 - \tfrac{1}{2}lr = \pi r^2 - \Delta.$$

To find the Area of a Segment of a Circle.

Let Δ represent area of segment of salient angle ($a\,b\,d\,a$, diagram) ; l, length of its arc ; n, number of degrees in its arc ; c, cosine of half its arc ($c\,p$, diagram) ; b, chord of arc ; and r, radius : then

$$\Delta = \pi r^2 n \div 360 - \tfrac{1}{2}bc = \tfrac{1}{2}lr - \tfrac{1}{2}bc.$$

But the area of a circle $= \pi r^2$.

The area of the segment of the re-entrant angle therefore =

$$\pi r^2(360-n)\div 360 + \tfrac{1}{2}bc = \pi r^2 + \tfrac{1}{2}bc - \tfrac{1}{2}lr = \pi r^2 - \Delta.$$

NOTE. — When the arc of the segment does not contain more than 90°, $\Delta = \frac{4v}{10}\left(\frac{4m}{3}+b\right)$ nearly, $= \frac{v(4b^2+3v^2)}{6b}$ nearly; m being the chord of half the arc, and v the versed sine. And when the arc contains more than 90°, and not exceeding 180,° $\Delta = \frac{2v}{3}\sqrt{\left(b^2+\frac{8v^2}{5}\right)}$ nearly.

To find the Area of a Zone.

RULE. — Find the area of the circle containing the zone, and the areas of the segments on the sides of the zone, and from the area of the circle subtract the sum of the areas of the segments; the difference will be the area sought. The rule is also applicable in finding the area of a zone of an ellipse.

To find the Diameter of a Circle of which a given Zone is a part.

$d = \sqrt{\left[b^2 + \left(h + \frac{B^2 - b^2}{4h}\right)^2\right]}$, d being the diameter of the circle, B the greater base, and b the less, of the zone, and h the perpendicular distance between the bases.

To find the Area of a Crescent.

RULE. — Find the areas of the two segments formed by the arcs of the crescent and their chord; subtract the less from the greater, and the difference will be the area of the crescent.

To find the Side of a Square that shall contain an Area equal to that of a given Circle.

RULE. — Multiply the diameter of the circle by $\sqrt{(\pi \div 4)} =$ .886228, or multiply the circumference by $\sqrt{\pi} \div 2\pi = .28209447$; the product will be the side.

To find the Diameter of a Circle that shall have an Area equal to that of a given Square.

RULE. — Multiply the side of the square by $\sqrt{(4 \div \pi)} =$ 1.128379, and the product will be the diameter required.

To find the Diameters of Three Equal Circles the greatest that can be inscribed in a given Circle.

RULE. — Multiply the diameter of the circumscribing circle by $3 \div \frac{2}{3}\pi^2 = .45594322$; the product will be the diameters required.

To find the Diameters of Four Equal Circles the greatest that can be inscribed in a given Circle.

RULE. — Multiply the diameter of the given circle by $4 \div \pi^2 =$ .4052829, and the product will be the diameters sought.

To find the Side of a Square inscribed in a given Circle.

A square is said to be inscribed in a circle when all its corners touch the circumference. Its area is half that of a circumscribing square of the same circle.

RULE. — Multiply the diameter of the circle by $\sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2} =$ .7071068, or multiply the circumference by $\sqrt{\frac{1}{2}} \div \pi = .225079$; the product will be the side of the inscribed square.

EXAMPLE. — A log is 28 inches in diameter at the smaller end; required the side of the greatest square that can be hewn from it.

$$28 \times .7071068 = 19.799 \text{ inches. } \quad \textit{Ans.}$$

NOTE. — With reference to circumscribed and inscribed figures generally, see CENTRES OF SURFACES, p. 291.

To find the Diameter of a Circle that will circumscribe a given Triangle.

Let l = longest side of triangle; m, intermediate side; s, shortest side; d, diameter of circle.

$$d = \frac{ms}{\sqrt{\left[m^2 - \left(\frac{l^2 + m^2 - s^2}{2l}\right)^2\right]}}.$$

When two of the sides are equal,

$$d = \frac{2a}{\sqrt{(4a^2 - m^2)}},$$

a being one of the equal sides, and m the unequal side.

When all the sides are equal,

$$d = \frac{2a^2}{\sqrt{(3a^2)}} = \frac{2a}{\sqrt{3}} = 1.1547a.$$

To find the Diameter of the greatest Circle that can be inscribed in a given Triangle.

$$d = \frac{2l\sqrt{\left[m^2 - \left(\frac{l^2 + m^2 - s^2}{2l}\right)^2\right]}}{l + m + s}.$$

When two of the sides are equal,

$$d = \frac{m\sqrt{(4a^2 - m^2)}}{m + 2a}.$$

When all the sides are equal,

$$d = \sqrt{(3a^2)} \div 3 = a \div \sqrt{3} = .57735a.$$

To divide a Circle into any number of Concentric Circles of equal areas.

RULE. — 1. Multiply the square of the radius of the given circle by the number of concentric circles, less 1, required, and divide the product by the number of concentric circles required; the square root of the quotient will be the radius of the given circle, less the breadth of the outermost concentric circle.

2. Multiply the square of the radius of the given circle by the number of concentric circles, less 2, required, and divide the product by the number of concentric circles required; the square root of the quotient will be the radius of the given circle, less the sum of the breadths of the two outermost concentric circles, &c.

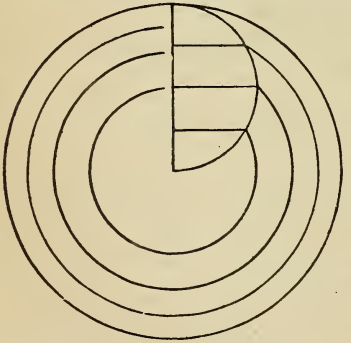
EXAMPLE. — Required to divide a circle of 20 inches radius into two concentric circles of equal areas.

$20^2 \div 2 = \sqrt{200} = 14.142$ inches, radius of inner circle.

$20 - 14.142 = 5.858$ inches, breadth of outer circle.

The following diagram illustrates the principle of the foregoing rule, and exhibits the *mechanical* method of solving the problem. It is perhaps unnecessary to add that it is wholly immaterial into how many concentric circles of equal areas the given circle is to be divided, or whether the concentric circles are to have equal areas or otherwise. The radius of the given circle has only to be divided into the required number of aliquot parts, or proportional parts, the parallel lines struck, the curves drawn, and the division is accomplished.

EXAMPLE. — Four men, A, B, C, and D, own a grindstone, of 15 inches radius, equally between them. A is first to grind off his share, then B, then C; and D is to have the remainder. Required the number of inches that may be ground from the radius by A, B, and C respectively, and the radius of the stone that will be left for D.



$\sqrt{(15^2 \times 3 \div 4)} = 12.99$, and $15 - 12.99 = 2.01$ in. —A.

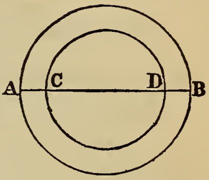
$\sqrt{(15^2 \times 2 \div 4)} = 10.606$, and $12.99 - 10.606 = 2.384$ inches, — B.

$\sqrt{(15^2 \div 4)} = 7.5$, and $10.606 - 7.5 = 3.106$ in., — C.

$15 - (2.01 + 2.384 + 3.106) = 7.5$ inches,—D.

NOTE. — The square root of the quotient obtained by dividing the square of the radius of the given circle by the number of concentric circles required is equal to the radius of the innermost concentric circle.

To find the area of the space contained between two Concentric Circles.



RULE. — Multiply the sum of the two diameters (that of the outer circle and that of the inner) by their difference, multiplied by .7854, and the product will be the area of the ring, or space, referred to.

EXAMPLE. — The diameter of the larger circle, A B, is 36 inches, and the diameter of the less circle, C D, is 30 inches; required the area of the ring, or space, between the two circles.

$36 + 30 = 66$, and $36 - 30 = 6$; then
$66 \times 6 \times .7854 = 311$ square inches. *Ans.*

ELLIPSE.

An ELLIPSE is a single closed curve and an isometrical projection of a circle.

It is formed when a plane cuts through both sides of a cone or a cylinder obliquely to the base.

It has two foci: they are on the transverse diameter, one on each side of the centre of the figure; and they are equally distant from that point.

An ellipse is divisible by either of its right diameters into equal and similar semi-ellipses, and by both into four equal and similar elliptic curves.

The extremities of the two diameters and two parameters of an ellipse are eight points in the circumference. The summit of every ordinate is a point in the curve. The summits of every double ordinate are two points in the circumference.

Either semi-diameter of an ellipse may be an ordinate, though the axis of abscissæ proper is the major axis.

Let $t =$ transverse diameter.
$c =$ conjugate diameter.
$p =$ parameter.
$f =$ distance from vertex to focus.
$g =$ space between the foci.
$\varepsilon =$ eccentricity.

Let $\phi =$ axis of abscissæ.
$\theta =$ axis of ordinates.
$v =$ greater abscissa.
$x =$ lesser abscissa.
$z =$ abscissa from centre.
$y =$ ordinate.

$$\varepsilon^2 = 1 - \frac{c^2}{t^2} = 1 - \frac{p^2}{c^2} = \left(1 - \frac{2f}{t}\right)^2 = 1 - \frac{p}{t} = \frac{g^2}{t^2} = \frac{g^2}{c^2 + g^2}; \text{ also}$$

$$c^2 = tp = t^2 - g^2 = 4(fg + f^2); \; f = \frac{t - \sqrt{(t^2 - c^2)}}{2} = \frac{t - t\varepsilon}{2}.$$

When the transverse axis is the axis of abscissæ,

$$y^2 \div (1-\varepsilon^2) = vx = tv - v^2 = tx - x^2 = \tfrac{1}{4}t^2 - z^2; \text{ also}$$

$$y^2 = \frac{c^2 vx}{t^2} = \frac{c^2}{t^2}(tv - v^2) = \frac{c^2}{t^2}(tx - x^2) = \frac{c^2}{t^2}(\tfrac{1}{4}t^2 - z^2).$$

When the conjugate axis is the axis of abscissæ,

$$y^2(1-\varepsilon^2) = vx = cv - v^2 = cx - x^2 = \tfrac{1}{4}c^2 - z^2; \text{ also}$$

$$y^2 = \frac{t^2 vx}{c^2} = \frac{t^2}{c^2}(cv - v^2) = \frac{t^2}{c^2}(cx - x^2) = \frac{t^2}{c^2}(\tfrac{1}{4}c^2 - z^2);$$

$$\therefore \phi = \frac{\theta v}{\frac{1}{2}\theta + \sqrt{(\frac{1}{4}\theta^2 - y^2)}} = \frac{\theta x}{\frac{1}{2}\theta - \sqrt{(\frac{1}{4}\theta^2 - y^2)}} = v + x = 2z + 2x,$$

$$\theta = \frac{\phi y}{\sqrt{(vx)}} = \frac{\phi y}{\sqrt{(\phi v - v^2)}} = \frac{\phi y}{\sqrt{(\phi x - x^2)}} = \frac{\phi y}{\sqrt{(\frac{1}{4}\phi^2 - z^2)}},$$

$$v = \frac{\phi}{2} + \frac{\phi}{\theta}\sqrt{(\tfrac{1}{4}\theta^2 - y^2)} = \frac{\phi^2 y^2}{\theta^2 x} = \tfrac{1}{2}\phi + z = \phi - x,$$

$$x = \frac{\phi}{2} - \frac{\phi}{\theta}\sqrt{(\tfrac{1}{4}\theta^2 - y^2)} = \frac{\phi^2 y^2}{\theta^2 v} = \tfrac{1}{2}\phi - z = \phi - v,$$

$$z = \frac{\phi}{\theta}\sqrt{(\tfrac{1}{4}\theta^2 - y^2)} = \tfrac{1}{2}\phi - x = v - \tfrac{1}{2}\phi = \tfrac{1}{2}v - \tfrac{1}{2}x,$$

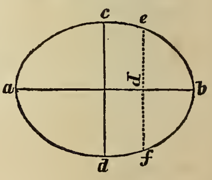
$$y = \frac{\theta}{\phi}\sqrt{(vx)} = \frac{\theta}{\phi}(\sqrt{\phi v - v^2}) = \frac{\theta}{\phi}\sqrt{(\phi x - x^2)} = \frac{\theta}{\phi}(\tfrac{1}{4}\sqrt{\phi^2 - z^2}).$$

EXAMPLE.— The eccentricity of an ellipse is 0.8, an ordinate is 12, and its abscissa, reckoned from the centre, is 9.6; required the axis of abscissa, and axis of ordinate.

$$2\sqrt{(12^2 \times (1 - .8^2) + 9.6^2)} = 24, \text{ axis of abscissæ.} \quad \textit{Ans.}$$

$$\sqrt{\frac{12^2 \times (1 - .8^2) + 9.6^2}{\frac{1}{4}(1 - .8^2)}} = 40, \text{ axis of ordinate.} \quad \textit{Ans.}$$

EXAMPLE.—In an ellipse, *a c b d*, the transverse axis, *a b*, is 40, the conjugate axis, *c d*, 24, the greater abscissa, *a p*, 26, and the lesser abscissa, *b p*, 14; required the length of the ordinate *e p*.



$$\varepsilon^2 = 1 - (24^2 \div 40^2) = 0.64, \text{ and}$$

$$\left.\begin{array}{l} \sqrt{[(1 - .64) \times 26 \times 14]}, \text{ or} \\ \sqrt{\dfrac{24^2 \times 26 \times 14}{40^2}} = \dfrac{24\sqrt{(26 \times 14)}}{40} \end{array}\right\} = 11.4473. \quad \textit{Ans.}$$

To find the Area of an Ellipse.

$$\Delta = \pi\, t\, c \div 4 = 0.7854\; t\, c.$$

EXAMPLE. — The axes of an ellipse are 40 feet and 20 feet. What is the area?

$$40 \times 20 \times .7854 = 628.32 \text{ square feet.} \quad \textit{Ans.}$$

To find the Length of the Circumference of an Ellipse.

$$l = \pi t\left(1 - \frac{1^2.\,\varepsilon^2}{2^2} - \frac{1^2.\,3.\,\varepsilon^4}{2^2.\,4^2} - \frac{1^2.\,3^2.\,5.\,\varepsilon^6}{2^2.\,4^2.\,6^2} - \frac{1^2.\,3^2.\,5^2.\,7.\,\varepsilon^8}{2^2.\,4^2.\,6^2.\,8^2} - \frac{1^2.\,3^2.\,5^2.\,7^2.\,9.\,\varepsilon^{10}}{2^2.\,4^2.\,6^2.\,8^2.\,10^2} - \ldots \&c.\right.$$

But the following empirical formula, which I have been at some pains to devise, expresses the perimeter of an ellipse sufficiently near for practical purposes in all ordinary cases of eccentricity; viz. —

$$l = 3.53\sqrt{\left(\frac{\pi tc}{4} + \frac{(t-c)^2}{\pi}\right)}.$$

And it may be proper to state that the expression furnishes l too short by 1-238 for ellipses of greatest and least possible eccentricities; too short by 1-351 when $c = \frac{3}{4}t$; affords almost strict accuracy when c is not greater than $\frac{2}{3}t$, nor less than $\frac{1}{3}t$; and gives l too long by 1-274 when $c = \frac{1}{4}t$. It may also be proper to state that the empirical formula of the hand-books; viz., $l = \pi\sqrt{\frac{t^2 + c^2}{2}}$, and which, by the way, I have seldom met with given other than as affording strict accuracy, affords very incorrect results for ellipses of much eccentricity. It is correct only for the circle, or when t and c are equal; and therefore gives l in excess in all cases.

To find the Area of an Elliptic Segment.

The area of an elliptic segment is to that of a corresponding circular segment as the axis of ordinate is to the axis of abscissa, nearly.

Let $\phi =$ axis of abscissa.
$\theta =$ axis of ordinate.
$z =$ abscissa from centre.
$x =$ abscissa from vertex.
$B =$ base of corresponding circular segment.
$L =$ length of arc of lesser corresponding circular segment.
$\Delta =$ area of lesser elliptic segment.

$$B = \sqrt{(\phi^2 - 4z^2)} = 2\sqrt{(\phi x - x^2)}\,;\ z = \tfrac{1}{2}\phi - x.$$

When z is equal to or greater than $\frac{1}{2}B$,

$$L = \frac{1.5023\phi B}{\phi + z} \text{ very nearly, } = \frac{3.0046\phi B}{3\phi - 2x} \text{ very nearly.}$$

When z is less than $\frac{1}{2}B$,

$$L = \frac{\frac{1}{2}\pi\phi + \frac{1}{4}\pi B - 3.0046z}{1 + \dfrac{B}{2\phi}} \text{ very nearly, } = \frac{\frac{1}{2}\pi\phi + \frac{1}{4}\pi B - 3.0046(\frac{1}{2}\phi - x)}{1 + \dfrac{B}{2\phi}}$$

very nearly.

$$\Delta = \frac{\theta}{\phi}\left(\tfrac{1}{4}\phi L - \tfrac{1}{2}Bz\right) = \frac{\theta(\phi L + 2Bx - \phi B)}{4\phi}; \text{ and}$$

$$\frac{\pi\phi\theta}{4} - \Delta = \text{area of greater elliptic segment.}$$

Example.—The base b of an elliptic segment is 32; the height, 5; and the axis parallel to the base, 40. What is the area of the segment?

$$\phi = \frac{40 \times 5}{20 - \sqrt{(20^2 - 16^2)}} = 25 = \frac{\theta x}{\frac{1}{2}\theta - \frac{1}{2}\sqrt{(\theta^2 - b^2)}},$$

$$z = (25 - 2 \times 5) \div 2 = 7.5 = \tfrac{1}{2}\phi - x,$$

$$B = \sqrt{(25^2 - 4 \times 7.5)} = 20 = \sqrt{(\phi - 4z)},$$

$$L = \frac{1.5708 \times 25 + .7854 \times 20 - 3.0046 \times 7.5}{1.4} = 23.174,$$

$$\Delta = \frac{40(25 \times 23.174 - 2 \times 20 \times 7.5)}{4 \times 25} = 111.74. \quad \textit{Ans.}$$

Or, in this case (see note p. 201),

$$\Delta = \frac{2\theta x}{3\varphi}\sqrt{\left(\frac{8x^2}{5} + \phi^2 - 4z^2\right)} \text{ nearly.}$$

PARABOLA.

A PARABOLA is a single open curve of two branches. It has but one focus; and its eccentricity is constant, and equal to unity, or 1.

It is formed, when a plane intersects a conic surface, in the direction of the base, parallel to an element of the surface, or parallel to one of the sides.

It is equated upon the principle of opposite parabolas having their vertices towards one another, and has both a transverse and a conjugate diameter; but these lie outside the figure. The transverse is the distance between the origins of the opposite parabolas; and the conjugate is tangent to the vertex of the true or contemplated parabola, and is bisected by the transverse. The middle of the transverse is the *centre*, and its *projection* is the *axis* of the figure.

But, in the parabola, different abscissæ on its *axis*, reckoned from the origin, are to one another as the squares of their ordinates. The parabola, therefore, may be equated independent of the supposed diameters and abscissæ from the centre. Formulas will be given covering both methods.

In geometry, any double ordinate of a parabola is the *base* of a parabola, and its abscissa, reckoned from the origin, is the axis or height; thus a parabola is a plane figure having an area, a length of curve, &c.

Let t = transverse diameter.
c = conjugate diameter.
p = parameter.
f = distance from origin to focus.

Let x = abscissa from origin.
z = abscissa from centre.
y = ordinate.
$f : (\frac{1}{2}p)^2 :: x : y^2$.

$$t = \frac{c^2}{p} = \frac{c^2}{4f} = \frac{c^2 x}{y^2} = 2z - \frac{2y^2}{p} = \frac{c^2 z}{\frac{1}{2}c^2 + y^2}.$$

$$c = \sqrt{(tp)} = \sqrt{\frac{ty^2}{x}} = 2\sqrt{(tf)} = \sqrt{\frac{2y^2(z-x)}{x}} = \sqrt{(2zp - 2y^2)}.$$

$$p = \frac{c^2}{t} = \frac{y^2}{x} = 4f = \frac{\frac{1}{2}c^2 + y^2}{z} = \frac{y^2}{z - \frac{1}{2}t}.$$

$$f = \frac{c^2}{4t} = \frac{y^2}{4x} = \frac{1}{4}p = \frac{y^2}{4z - 2t}.$$

$$x = \frac{ty^2}{c^2} = \frac{y^2}{p} = \frac{y^2}{4f} = z - \tfrac{1}{2}t = \frac{y^2 z}{\tfrac{1}{2}c^2 + y^2}.$$

$$z = \frac{t(\tfrac{1}{2}c^2 + y^2)}{c^2} = \tfrac{1}{2}t + x = 2 + \frac{y^2}{p} = \frac{x(\tfrac{1}{2}c^2 + y^2)}{y^2}.$$

$$y = \sqrt{(px)} = 2\sqrt{(fx)} = \sqrt{\frac{c^2 x}{t}} = \sqrt{\left(\frac{c^2 z}{t} - \tfrac{1}{2}c^2\right)} =$$

$$\sqrt{(pz - \tfrac{1}{2}c^2)} = \sqrt{\frac{p(2z - t)}{2}} = \sqrt{(pz - 2ft)}.$$

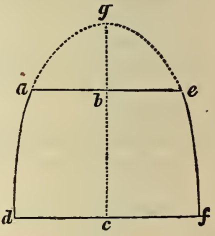
To find the Area of a Parabola.

$\Delta = 4yx \div 3 = 2bh \div 3$; b being the base and h the height.

To find the Area of a Zone of a Parabola.

$$\Delta = \frac{2a(b^3 - d^3)}{3(b^2 - d^2)} = \frac{2a}{3}\left(\frac{d^2}{b+d} + b\right);$$

b being the greater base, d the lesser, and a the distance between them.



To find the Altitude of a Parabola, b, d, and a being given.

$$h = b^2 a \div (b^2 - d^2).$$

To find the Length of the Curve of a Semi-parabola, or the Length of a Semi-parabola.

$$l = \sqrt{(x^2 + y^2 + \tfrac{1}{5}xy)}, \text{ nearly.}$$

NOTE.—The common expression for the approximate length of a semi-parabola, viz. $\sqrt{(1\tfrac{1}{3}x^2 + y^2)}$, considerably exceeds the true length, unless y is large, compared with x. When x is to y as 3 to 5, it agrees with the foregoing; and, when y is not less than 4 times greater than x, it is entitled to the preference.

HYPERBOLA.

A Hyperbola, like a parabola, is a single open curve of two branches, and has but one focus; but its eccentricity is greater than unity, and hyperbolas may have different eccentricities.

It is formed, when a plane intersects a conic surface in the direction of the base, at any angle to the base between that which would generate a triangle and that which would generate a parabola; or, in other words, at any angle short of 90° to the base, and greater than that of the side to the base, — at any angle to the axis less than that of the side and axis.

It is equated upon the principle of opposite hyperbolas having their vertices towards one another, as explained for the parabola.

In geometry, any double ordinate of a hyperbola is the *base* of a hyperbola, and the prolongation of the transverse diameter is the *axis*, or altitude; thus the hyperbola is a plane figure, having an area, a length of curve, &c.

Let t = transverse diameter.
c = conjugate diameter.
p = parameter.
f = distance from origin to focus.

Let x = abscissa from origin.
z = abscissa from centre.
y = ordinate.
ε = eccentricity.

$$\varepsilon^2 = 1 + \frac{c^2}{t^2} = 1 + \frac{p^2}{c^2} = 1 + \frac{p}{t} = \left(1 + \frac{2f}{t}\right)^2 = 1 + \frac{y^2}{tx + x^2}.$$

$$t = \left(\frac{y^2}{\varepsilon^2 - 1} - x^2\right) \div x = \frac{p}{\varepsilon^2 - 1} = \frac{c^2}{p} = \frac{cx[c + \sqrt{(c^2 + 4y^2)}]}{2y^2}$$

$$= 2(z - x) = \frac{c^2}{\sqrt{(\varepsilon^2 - 1)}}.$$

$$c = t\sqrt{(\varepsilon^2 - 1)} = \sqrt{(tp)} = \frac{ty}{\sqrt{(tx + x^2)}} = \frac{ty}{\sqrt{(z^2 - \frac{1}{4}t^2)}}.$$

$$p = t(\varepsilon^2 - 1) = \frac{y^2 - x^2(\varepsilon^2 - 1)}{x} = \frac{c^2}{t} = \frac{ty^2}{tx + x^2} = \frac{ty^2}{z^2 - \frac{1}{4}t}.$$

$$f = \frac{t\varepsilon - t}{2} = \frac{\sqrt{(t^2 + c^2)}}{2} - \frac{t}{2}.$$

$$x = \frac{t}{2}\sqrt{\left(1 + \frac{4y^2}{\varepsilon^2 - 1}\right)} - \frac{t}{2} = \frac{t\sqrt{(c^2 + 4y^2)}}{2c} - \frac{t}{2} = \frac{c\sqrt{(c^2 + 4y^2)}}{2p}$$

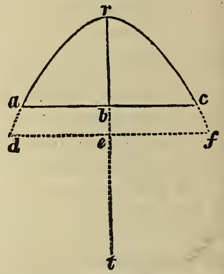
$$- \frac{2c}{p} = z - \tfrac{1}{2}t.$$

$$z = \frac{t}{2}\sqrt{\left(1 + \frac{4y^2}{\varepsilon^2 - 1}\right)} = \frac{t\sqrt{(c^2 + 4y^2)}}{2c} = \sqrt{\frac{t(tp + 4y^2)}{4p}} = \tfrac{1}{2}t - x.$$

$$y=(\varepsilon^2-1)(tx+x^2)=px+x(\varepsilon^2-1)=\frac{c\sqrt{(tx+x^2)}}{t}$$

$$=\sqrt{\frac{p(tx+x^2)}{t}}=\frac{c\sqrt{(4z^2-t^2)}}{2t}.$$

To find the Length of a Semi-hyperbola.



$$l=\sqrt{(x^2+y^2+\tfrac{4}{7}xy)} \text{ nearly.}$$

To find the Area of a Hyperbola.

$\Delta=\frac{4h}{3}\left(\frac{b}{4.75}+m\right)$ nearly; h being the altitude, b the base, and m the ordinate at one-third the altitude from the vertex.

NOTE.—The common expression for the approximate length of a hyperbola: viz., $2y\times\frac{15pt+x(21p+19t)}{15pt+x(21p+9t)}$, affords very incorrect results when x is great compared with y; but the empirical equation in the books, viz.:—

$$\Delta=\frac{4cx}{75t}\left[21\sqrt{\left(tx+\frac{5x^2}{7}\right)}+4\sqrt{(tx)}\right],$$

for the hyperbola, agrees very nearly with that offered above.

CYCLOID.

A CYCLOID is an elliptic arch, whose *span* is equal to the circumference of the generating circle, and *rise* equal to the diameter of that circle; or, in other words, it is a semi-ellipse by the transverse axis, when the axes are to each other as π to 2.

It is generated by a point in the circumference of a circle in the point of rest on a plane, when the circle makes one revolution from that point in a straight line.

Its eccentricity is constant, the square of which is expressed by

$$1-\frac{4}{\pi^2}=.594717+$$

Its foci are in the span of the arch, one on each side of the centre, and they are equally distant from that point: they are distant from their respective extremities of the curve, equal to

$$\frac{\pi d\,(1-\varepsilon)}{2}=\frac{\pi d-\sqrt{(\pi^2 d^2-4d^2)}}{2}.$$

Its parameter is a single ordinate, and is expressed by $\frac{2d}{\pi}$.

Its equation is expressed by $y=\frac{2\sqrt{x(\pi d-x)}}{\pi}$, which agrees with $y=\frac{c}{t}\sqrt{x(t-x)}$ for the ellipse, when the diameters are, one to the other, as π to 2.

An Epicycloid is a curve generated by a point in the circumference of a circle, which revolves about another circle, either on the convexity or concavity of its circumference.

The cycloid, geometrically considered, has a perimeter, an area, and a length of curve.

Let d, as before, equal the diameter of the generating circle; then $l=4.081d$, and $\Delta=\pi^2d^2\div 4=2.46741 25d^2$.

Note. — I have been thus particular in treating of the cycloid, because I differ in my view from some writers with regard to the law of the curve. It is generally stated by writers on the subject, and without further specifying the nature of the curve, that its length is equal to 4 times the diameter of the generating circle, and that the area is equal to 3 times the area of that circle; and they do not compute either by the rules they furnish as applicable to the ellipse. If the cycloid is not strictly elliptical, it is clear, without comment, that I am ignorant of its law, and that the foregoing is inapplicable. But to prove that the curve is not strictly elliptical, would be to prove — would it not? — that it is not generated by a point in the circumference of a circle rotating in a straight line on a plane.

To find the Distance of Objects at Sea, &c.

The curvature of the earth, at its mean radius of 3956 statute miles, or at 45° of latitude, is $[\sqrt{(3956^2+1^2)}-3956]\times 5280=0.66734075$ feet (8.0081 inches) in a single statute mile on the tangent, and is as the square of the distance or space between two levels. For a geographical mile on the tangent, therefore, it is $(6086\div 5280)^2\times 0.66734075=0.886043$ feet. The mean horizontal refraction on the water is about $\frac{1}{14}$ of the curvature for any given distance; thus the practical or apparent curvature is $(1-\frac{1}{14})^2\times .66734075=0.57541$ feet for a single statute mile on the tangent.

Prob. 1. — What is the tangent from the place of observation to the sea-horizon, the elevation being 30 feet?

$$\sqrt{(30\div .5754)}=7.22 \text{ miles.} \quad \textit{Ans.}$$

Prob. 2. — The distance from New York to Sandy Hook is 18 miles; at what elevation above the surface of the water, at either place, may the surface be seen at the other?

$$18^2\times 0.5754=186.43 \text{ feet.} \quad \textit{Ans.}$$

Prob. 3. — From an eminence 180 feet above the surface of the water at the place of observation, the flag of a ship is seen in the line of the sea-horizon, and the flag is known to be 60 feet above the surface where it is situated; required the distance (tangent) from the observer's eye to the flag.

$$\sqrt{\frac{180}{.5754}} + \sqrt{\frac{60}{.5754}} = 27.9 \text{ miles. } Ans.$$

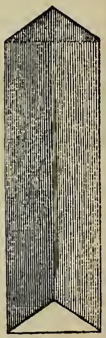
PROB. 4. — From an elevation 60 feet above the surface of the water at the place of observation, a vessel is seen in full view, and a portion of her canvas, supposed to be 20 feet above the surface where she lies, is seen in the line of the sea-horizon; what is the distance from the point of observation to the vessel?

$$\sqrt{\frac{60}{.5754}} - \sqrt{\frac{20}{.5754}} = 4.32 \text{ miles. } Ans.$$

STEREOMETRY, OR MENSURATION OF SOLIDS.

OF PARALLELOPIPEDS AND CUBES.

To find the lateral surface of a prism.



RULE. — Multiply the perimeter of the base by the altitude, and the product is the lateral surface. If the surface of the entire figure is required, add the areas of the ends to the lateral surface.

EXAMPLE. —The sides of a triangular prism are each $2\frac{1}{4}$ feet wide, and the length of either side is 16 feet; required its lateral surface.

$2\frac{1}{4} \times 3 = 6\frac{3}{4}$ = perimeter of base, and $6\frac{3}{4} \times 16 = 108$ square feet. *Ans.*

EXAMPLE. — A hexagonal prism has an altitude of 12 feet, two of its sides are 2 feet wide each, three are $1\frac{1}{2}$ feet wide each, and the remaining side is 9 inches; required the lateral surface of the prism.

$$2 \times 2 = 4$$
$$1.5 \times 3 = 4.5$$
$$.75 \times 1 = .75 = 9.25 \times 12 = 111 \text{ sq. feet. } Ans.$$

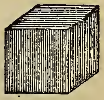
To find the solidity of a prism.

RULE. — Multiply the area of the base by the altitude, and the product is the solidity.

EXAMPLE. — The length of a triangular prism is 12 feet, and each side of its base is $2\frac{1}{2}$ feet; required its solidity.

$\sqrt{\frac{2.5}{2} + 2.5 \times \frac{2.5}{2}} = 2.165+ =$ sine of angle, or depth of base; or,
$2.5 \times 3 \times .28868^* = 2.165+ =$ height of triangle; and
$2.165 \times \frac{2.5}{2} = 2.706+ =$ area of base; or,
$2.5^2 \times .433012^* = 2.706325 =$ area of base; and
$2.706+ \times 12 = 32.475+$ cubic feet. *Ans.*

To find the solidity of a right prism or cube.

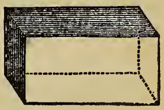


RULE. — Cube one of its edges.

EXAMPLE. — The length of a side of a right prism is 16 feet; required its solidity.

$16 \times 16 \times 16 = 4096$ cubic feet. *Ans.*

To find the solidity of a parallelopipedon.



RULE. — Multiply the length by the breadth, and that product by the height; the last product will be the solidity.

EXAMPLE. — A slab of marble is 8 feet long, 3 feet wide, and 6 inches thick; required its cubic contents.

$8 \times 3 \times .5 = 12$ feet. *Ans.*

OF PYRAMIDS.

To find the lateral surface of a regular pyramid.

RULE. — Multiply the perimeter of the base by half the slant height of the figure — half the length of a side — and the product is the surface of the sides. The surface of the entire figure is the surface of the sides with the area of the base added thereto.

EXAMPLE. — A triangular pyramid has a slant height of 60 feet. and each edge of its base is 20 feet; required its lateral surface.

$20 \times 3 \times \frac{60}{2} = 1800$ square feet. *Ans.*

To find the solidity of a pyramid.

RULE. — Multiply the area of the base by $\frac{1}{3}$ the perpendicular height, and the product is the solidity.

EXAMPLE. — A quadrilateral pyramid has a perpendicular height of 21 feet, and each side of its base is 8 feet; what are its cubic dimensions?

$8^2 \times \frac{21}{3} = 448$ feet. *Ans.*

* See TABLE of co-efficients, etc., relative to *Polygons.*

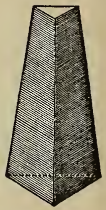
To find the lateral surface of a Frustum of a Pyramid, Frustum of a Cone, Prismoid, or Wedge.

RULE. — Multiply the sum of the perimeters of the bases by half the slant height.

EXAMPLE. — Each edge of one of the bases of the frustum of a hexagonal pyramid is 4 feet; each edge of the other base, 2 feet; and the slant height of the frustum is 24 feet: required its lateral surface.

$$4 \times 6 + \overline{2 \times 6} \times \tfrac{24}{2} = 432 \text{ square feet.} \quad \textit{Ans.}$$

To find the solidity of the frustum of a pyramid.



RULE. — To the square root of the product of the areas of the bases, add the areas of the bases, and multiply the sum by $\frac{1}{3}$ the perpendicular height of the frustum;* the product will be the solidity.

EXAMPLE. — The greater base of the frustum of a quadrilateral pyramid is 3 feet on each side, and the less base is 2 feet on each side, the perpendicular height of the frustum is 15 feet; what are its solid contents?

$3 \times 3 = 9$ feet = area of greater base.
$2 \times 2 = 4$ feet = area of less base.
$15 \div 3 = 5 = \frac{1}{3}$ height. Then,
$9 \times 4 = \sqrt{36} = 6 + 9 + 4 = 19 \times 5 = 95$ feet. *Ans.*

OF PRISMOIDS AND THE WEDGE.

To find the solidity of a prismoid.

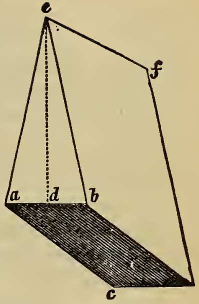
The RULE for finding the solidity of the frustum of a pyramid is equally applicable to the prismoid. Or,

RULE. — Add the areas of the ends, and four times the area of the mean between the ends, together, and multiply the sum by $\frac{1}{6}$ the perpendicular height; the product will be the capacity or solidity.

EXAMPLE. — The bottom of a rectangular cistern is 8 feet by 6 feet, the top is 4 by 3 feet, and the perpendicular depth is 12 feet; required its capacity.

8 . 6
4 . 3
$12 \,.\, 9 \div 2 = \overline{6 \times 4.5} = 27 \times 4 = 108 = 4$ area of mean; and
$6 \times 8 + \overline{4 \times 3} = 60 + 108 = 168 \times \frac{12}{6} = 336$ cubic feet. *Ans.*

* For another rule, see Mensuration of Prismoids.



To find the solidity of a wedge.

Rule. —Multiply the sum of the length of the edge and twice the length of the base, by the breadth of the base multiplied by the perpendicular depth or length of the wedge, and divide the product by 6; the quotient will be the solidity.

Example.—The length of the base of a wedge, *a c*, is 10 inches, the length of the edge, *e f*, is 8 inches, the breadth of the base, *a b*, is 4 inches, and the perpendicular depth, *d e*, is 12 inches; required the contents.

$8 + \overline{10 \times 2} = 28 \times 4 \times 12 = 1344 \div 6 = 224$ cubic in. *Ans.*

OF CYLINDERS.

To find the convex surface of a cylinder.

Rule.—Multiply the circumference by the length, and the product will be the convex surface. If the surface of the entire cylinder is required, add the areas of the ends to the convex surface.

To find the solidity of a cylinder.

Rule.— Multiply the area of an end by the length of the cylinder, and the product is the solidity.

Example.—The diameter of a cylinder is 6 feet, and its length is 8 feet; required its solidity or capacity.

$6 \times 6 = 36 \times .7854 = 28.2744 =$ area of end, and
$28.2744 \times 8 = 226\frac{2}{10}$— cubic feet. *Ans.*

To find the length of a helix, or spiral, wound round a cylinder.

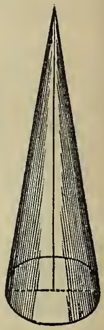
Rule.—Multiply the circumference of the cylinder by the number of revolutions the spiral makes around it, square the product, and thereto add the square of the length of the cylinder; the square root of the sum is the length of the spiral. The rule is applicable in finding the length of the thread of a screw, hand-rail to a winding staircase, &c., &c.

OF CONES.

To find the convex surface of a cone.

Rule. — Multiply the circumference of the base by half the slant height, and the product is the surface required. If the surface of the entire figure is required, add the area of the base to the convex surface.

Example. — The diameter of the base of a right cone is 8 feet, and the slant height is 18 feet; required the convex surface.



$8 \times 3.1416 = 25.1328 =$ circumference of base, and
$25.1328 \times \frac{18}{2} = 226\frac{2}{10}$ sq. feet. *Ans.*

To find the solidity of a cone.

Rule. — Multiply the area of the base by $\frac{1}{3}$ the perpendicular height, and the product is the solidity.

Example. — The diameter of the base of a right cone is 8 feet, and the perpendicular height of the cone is 15 feet; required its solidity.

$8^2 \times .7854 \times \frac{15}{3} = 251\frac{1}{3}$ cubic feet. *Ans.*

To find the solidity or capacity of the frustum of a cone.

Rule. — 1. To the square root of the product of the areas of the ends add the areas of the ends, and multiply the sum by $\frac{1}{3}$ the perpendicular height of the frustum; the product will be the solidity.

Or, — 2. Divide the difference of the cubes of the diameters of the ends by the difference of the diameters of the ends, and multiply the quotient by $\frac{1}{3}$ the perpendicular height of the frustum; this product multiplied by .7854 gives the solidity.

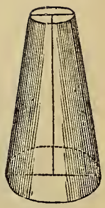
Or, — 3. To the product of the diameters of the ends add $\frac{1}{3}$ the square of the difference of the diameters, multiply the sum by .7854, and the product will be the mean area between the ends, which, multiplied by the perpendicular height of the frustum, gives the solidity.

Example. — The diameter of the larger end of a stick of round timber is 30 inches, that of the smaller end is 21 inches, and the length of the stick is 36 feet; required its contents.

30 inches $= 2\frac{1}{2}$ feet, and 21 inches $= 1\frac{3}{4}$ feet.

By Rule 2. $\left\{ \overline{2.5^3 \backsim 1.75^3} \div \overline{2.5 \backsim 1.75} = 13.6875 \times \frac{36}{3} \times .7854 = \right.$ 129 cubic feet. *Ans.*

By Rule 3. $\left\{ 2.5 \times 1.75 = 4.375 + .1875 = 4.5625 \times .7854 \times 36 = 129. \right.$ *Ans.*



EXAMPLE. — The interior diameter of the larger end of a circular cistern is 12 feet, that of the smaller end is 8 feet, and the perpendicular depth of the cistern is 14 feet; required its capacity in cubic feet.

$12 - 8 = 4 \times 4 = 16 \div 3 = 5.333 = \frac{1}{3}$ sq. of difference of ends; and

$\overline{12 \times 8} + 5.333 = 101.333 \times .7854 \times 14 = 1114.217.$ *Ans.*

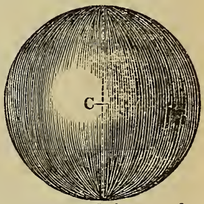
NOTE. — For an *Example* under Rule 1, see Mensuration of Pyramids. All rules which are applicable to the measuring of a cone, or frustum thereof, are also applicable to the pyramid, or its frustum; but, inasmuch as the areas of the ends of the two figures are not found with equal readiness, there will usually be a choice in the employment of rules.

OF SPHERES.

To find the surface of a sphere or globe.

RULE. — Multiply the diameter by the circumference; or, multiply the square of the diameter by 3.1416, and the product will be the surface.

To find the solidity of a sphere or globe.



RULE. — Multiply the superficies by $\frac{1}{6}$ of the diameter; or, multiply the square of the diameter by $\frac{1}{6}$ of the circumference; or, multiply the cube of the diameter by .5236, and the product is the solidity.

EXAMPLE. — Required the solidity of a cannon ball whose diameter is 9 inches.

$9 \times 9 \times 3.1416 = 254.46+$ sq. in., surface, or superficies, and

$254.46 \times \frac{9}{6} = 381.7$ cubic inches. *Ans.*

Or, $9 \times 9 \times 9 = 729 \times .5236 = 381.7.$ *Ans.*

To find the convex surface of a spherical segment or zone.

RULE. — Multiply the height by the circumference of the sphere.

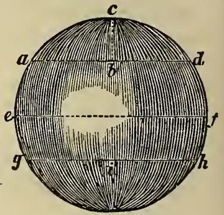
Or, $\Omega = 2\pi r v = \pi(\frac{1}{4}d^2 + v^2)$, for the segment; r being the radius of the sphere, d the diameter of the base of the segment, and v the height; and

$\Omega = 2\pi r h = \frac{1}{4}\pi(D^2 - d^2 + 4h^2 + 8hv)$, for the zone; D and d being the diameters of the bases, h the height of the zone, and v the height of a spherical segment whose base is equal to the less base of the zone.

To find the solidity of a spherical segment.

RULE. — 1. To the square of the height of the segment, add three times the square of half the base, and multiply the sum by the height, multiplied by .5236, and the product is the solidity.

Or, — 2. From three times the axis of the sphere, $e\ f$, subtract twice the height of the segment, and multiply the difference by the square of the height, multiplied by .5236, and the product will be the contents.



NOTE. — $\overline{\frac{1}{2}\,a\,d^2 + b\,c^2} \div b\,c = e\,f$, and $\overline{e\,f \times b\,c} - b\,c^2 = \sqrt{\overline{\frac{1}{2}\,a\,d^2}} = \frac{1}{2}\,a\,d$.

EXAMPLE. — The base, $a\ d$, of the segment $a\ d\ c$, is 18 inches, and the altitude, $b\ c$, is 8 inches; required the solidity of the segment.

$8^2 + \overline{9^2 \times 3} = 307 \times 8 \times .5236 = 1285.96$ cubic inches. *Ans*

To find the solidity of a spherical zone.

RULE. — Add the square of the radius of one base to the square of the radius of the other, and thereto add $\frac{1}{3}$ the square of the height; multiply their sum by the height, multiplied by 1.5708, and the product will be the contents.

EXAMPLE. — The base, $g\ h$, of the zone $g\ h\ d\ a$, is 4 feet, the base, $a\ d$, is 3 feet, and the height, $i\ b$, is $2\frac{1}{2}$ feet; the contents of the zone are required.

$4 \div 2 = 2$, radius of base $g\ h$.
$3 \div 2 = 1.5$ " " " $a\ d$.

$\overline{2.5^2} = 6.25 \div 3 = 2.0833 =$ $\frac{1}{3}$ square of height. Then,

$2^2 + \overline{1.5^2} + 2.083 = 8.333 \times 2.5 \times 1.5708 = 32.72+$ cub. feet. *Ans.*

To find the side of the greatest cube that can be cut from a given sphere.

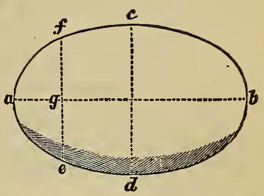
RULE. — Divide the square of the diameter of the sphere by 3, and the square root of the quotient is the side; or, multiply the diameter of the sphere by .57735, and the product is the side.

EXAMPLE. — The diameter of a globe is 15 inches; required the side of the greatest cube that may be cut from the globe.

$15 \times 15 = 225 \div 3 = \sqrt{75} = 8.66$ inches, or
$15 \times .57735 = 8.66$ inches. *Ans.*

OF SPHEROIDS.

To find the solidity of a spheroid.



RULE. — Multiply the square of the revolving axis by the fixed axis multiplied by .5236, and the product is the solidity.

EXAMPLE. — The fixed axis, *a b*, of the prolate spheroid *a c b d*, is 32 inches, and the revolving axis, *c d*, is 20 inches; required the solid contents.

$20^2 \times 32 \times .5236 = 6702.08$ cubic inches $=$
$6702.08 \div 1728 = 3.88-$ cubic feet. *Ans.*

To find the solidity of the segment of a spheroid.

RULE. — *When the base of the segment is parallel to the shorter axis of the spheroid.* — From three times the length of the longer axis, subtract twice the height of the segment, and multiply the difference by the square of the height, multiplied by the square of the shorter axis, multiplied by .5236, and divide the product by the square of the longer axis; the quotient will be the solid contents of the segment.

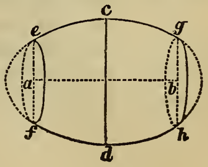
RULE. — *When the base of the segment is parallel to the longer axis of the spheroid.* — From three times the length of the shorter axis subtract twice the height of the segment, and multiply the difference by the square of the height multiplied by the longer axis, multiplied by .5236, and divide the product by the shorter axis; the quotient will be the solidity.

EXAMPLE. — The longer axis, *a b*, of the spheroid *a c b d*, being 32 inches, and the shorter, *c d*, 20 inches, what is the solidity of the segment *e f a*, whose base, *e f*, is 12 inches, and height, *g a*, 4 inches?

$32 \times 3 - 8 = 88 \times 16 \times 20^2 \times .5236 = 294891.52 \div 1024 = 287.98$ cub. inches. *Ans.*

To find the solidity of the middle frustum of a spheroid.

RULE. — *When the ends of the frustum are parallel to the revolving axis.* — To the square of the diameter of either end add twice the square of the revolving axis, and multiply the sum by the length of the frustum multiplied by .2618; the product will be the solidity; or the capacity, the frustum being a cask, and the measures taken of the interior.



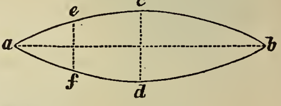
EXAMPLE. — The diameters, *e f* and *g h*, of the frustum *a d b c*, are 18 inches each, the revolving axis, *c d*, is 23 inches, and the length of the frustum, *a b*, is 20 inches ; required the cubic capacity of the frustum.

$23^2 \times 2 + 18^2 = 1382 \times 20 \times .2618 = 7236.152 \div 1728 = 4.187$ cubic feet. *Ans.*

OF SPINDLES AND CONOIDS.

To find the solidity of an elliptic spindle.

RULE. — To the square of twice the diameter of the spindle, at $\frac{1}{4}$ its length, add the square of its greatest diameter, and multiply the sum by the length, multiplied by .1309 ; the product will be the solidity, nearly.

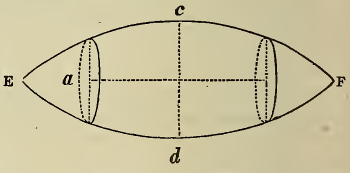


EXAMPLE. — The greatest diameter, *c d*, of the elliptic spindle *a c b d*, is 12 inches, the diameter at $\frac{1}{4}$ its length, *e f*, is $8\frac{1}{2}$ inches, and the length of the spindle, *a b*, is 40 inches ; required its solidity.

$8.5 \times 2 = 17$, and $\overline{17^2 + 12^2} \times 40 \times .1309 = 2267.188$ cubic. in *Ans.*

To find the solidity of a parabolic spindle.

RULE. — Multiply the square of the middle diameter by the length of the spindle multiplied by .41888, and the product is the solidity.



EXAMPLE. — The diameter, *c d*, at the middle of the parabolic spindle E *d* F *c*, is 15 inches, and the length of the spindle, E F, is 40 inches ; required its solidity.

$15^2 \times 40 \times .41888 = 3769.92$ cubic inches. *Ans.*

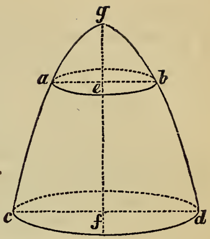
To find the solidity of the middle frustum of a parabolic spindle.

RULE. — To eight times the square of the greatest diameter, add three times the square of the least diameter, and four times the product of the two diameters, and multiply the sum by the length of the frustum, multiplied by .05236, and the product is the solidity.

EXAMPLE. — The greatest diameter, *c d*, of the frustum *a c b d*, is 28 inches, the least diameter (that of either end) is 20 inches, and the length of the frustum, *a b*, is 40 inches ; required the solidity.

$\overline{28^2 \times 8} + \overline{20^2 \times 3} + \overline{28 \times 20 \times 4} \times 40 \times .05236 = \frac{20340.8}{1728}$ cubic feet. *Ans.*

To find the solidity of a paraboloid, or parabolic conoid.



Rule. — Multiply the area of the base by half the altitude; or, multiply the square of the diameter of the base by the altitude multiplied by .3927, and the product will be the solidity.

Example. — The diameter of the base, *c d*, of the paraboloid, *c d g*, is 40 inches, and its height, *f g*, is 40 inches; required its solidity.

$40^2 \times .7854 \times 20 = 25132.8$ cubic inches. *Ans.*

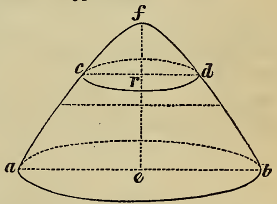
To find the solidity of a frustum of a paraboloid.

Rule. — Add the squares of the diameters of the two bases together, and multiply the sum by the distance between the bases, multiplied by .3927; the product will be the solidity.

Example. — The diameter of the base, *c d*, of the frustum, *c d b a*, is 40 inches, the diameter of the base, *a b*, is 22 inches, and the height of the frustum, *f e*, is 26 inches; required the solidity, or capacity.

$40^2 + 22^2 = 2084 \times 26 \times .3927 = 21278$ cubic inches. *Ans.*

To find the solidity of a hyperboloid or hyperbolic conoid.



Rule. — To the square of half the diameter of the base add the square of the diameter midway between the base and vertex, and multiply the sum by the distance between the base and vertex, multiplied by .5236; the product will be the solidity.

Example. — The diameter of the base, *a b*, of the hyperboloid, *a b f*, is 40 inches, the diameter midway between the base and vertex is 26 inches, and the altitude of the figure, *e f*, is 24 inches; required its solidity.

$20^2 + 26^2 = 1076 \times 24 \times .5236 = 13521.4+$ cubic in. *Ans.*

To find the solidity of a frustum of a hyperboloid.

Rule. — To the square of half the diameter of one end, add the square of half the diameter of the other end, and the square of the diameter midway between the two ends, and multiply the sum by the length of the frustum, multiplied by .5236; the product is the solidity.

Example. — The diameter of the base, *a b*, of the frustum, *a b d c*, is 40 inches, the diameter of the base, *c d*, is 17 inches, the diameter at the point equally distant from either base, is 29.1 inches, and the length of the frustum, *e r*, is 18 inches; required the solidity.

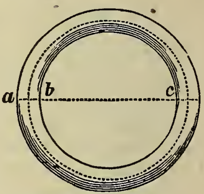
$20^2 + \overline{8.5}^2 + \overline{29.1}^2 = 1319 \times 18 \times .5236 = 12432$ cub. in. *Ans.*

Note. — The solidity or capacity of a frustum of any of the conic sections may be found by first finding the mean diameter, which, on being squared and multiplied by the length of the frustum decreased by being multiplied by .7854, gives the cubic contents. Instance, the last example: —

$40 - 17 = 23 \times .55^* = 12.65 + 17 = 29.65 =$ mean diameter, and

$\overline{29.65}^2 \times 18 \times .7854 = 12428$ cub. in. *Ans.*

To find the surface of a cylindrical ring.



Rule. — To the inner diameter of the ring add its thickness, and multiply the sum by the thickness, multiplied by 9.8696; the product is the surface.

Example. — The inner diameter, *b c*, of a cylindrical ring is 12 inches, and its thickness, *a b*, is three inches; required the superficies.

$12 + 3 = 15 \times 3 \times 9.8696 = 444$ sq. inches. *Ans.*

To find the solidity of a cylindrical ring.

Rule. — To the inner diameter of the ring add its thickness, and multiply the sum by the square of the thickness, multiplied by 2.4674; the product is the solidity.

Example. — The inner diameter, *b c*, is 12 inches, and the thickness, *a b*, is 3 inches; the solidity is required.

$\overline{12 + 3} \times 3^2 \times 2.4674 = 333.1$ cubic inches. *Ans.*

* For the co efficient multipliers, and rule in detail, see Gauging, *Rule* 6.

OF THE REGULAR BODIES.

The Regular Bodies are five in number, viz.:

The *Tetrahedron* or equilateral triangle; a solid bounded by four equilateral triangles.

The *Hexahedron* or cube; a solid bounded by six equal squares.

The *Octrahedron;* a solid bounded by eight equilateral triangles.

The *Dodecahedron;* a solid bounded by twelve regular and equal pentagons.

The *Icosahedron;* a solid bounded by twenty equilateral triangles.

The following Table *shows the superficies and solidity of each of the regular bodies, the linear edge of each being* 1.

No. of sides.	Names.	Superficies.	Solidities.
4	Tetrahedron.	1.73205	0.11785
6	Hexahedron.	6.00000	1.00000
8	Octahedron.	3.46410	0.47140
12	Dodecahedron.	20.64573	7.66312
20	Icosahedron.	8.66025	2.18169

To find the superficies of any of the regular bodies, by help of the foregoing table.

Rule. — Multiply the tabular number in the column of superficies by the square of the linear edge, and the product will be the surface.

Example. — The linear edge of a tetrahedron is 3 feet; required the superficies.

$$1.732 \times 3^2 = 15.588 \text{ square feet.} \quad \textit{Ans.}$$

To find the solidity of any of the regular bodies, by help of the foregoing table.

Rule. — Multiply the tabular number in the column of solidities by the cube of the linear edge, and the product will be the solidity or contents.

Example. — The linear edge of an octahedron is $2\frac{1}{2}$ feet; required the solidity.

$$.4714 \times 2.5 \times 2.5 \times 2.5 = 7.3656 \text{ cubic feet.} \quad \textit{Ans.}$$

PROMISCUOUS EXAMPLES IN GEOMETRY.

I. *To find the Diameter of a Circle, or the Side of a Square, whose Area shall bear a given ratio to the Area of a given Circle, or given Square.*

D = diameter of given circle, or side of given square.
d = diameter of required circle, or side of required square.
$a \div b$ = given ratio of area. $d = \sqrt{(aD^2 \div b)}$.

EXAMPLE. — An engraver has a drawing which may be circumscribed by a square whose sides are 8 inches, and he wishes to copy it at $\frac{2}{3}$ the size: required the sides of the square that will circumscribe the proposed copy. $\sqrt{[(2 \times 8^2) \div 3]} = 6.532$-in. *Ans.*

To find the Sides of a Rectangle, Rhombus, or Rhomboid, whose Area shall bear a given ratio to the Area of a given similar Rectangle, Rhombus, or Rhomboid.

L = length of given figure; H = breadth or perpendicular height of given figure; l = length of required figure; h = breadth or perpendicular height of required figure; $a \div b$ = given ratio of area.

$$l = \sqrt{(a L^2 \div b)}. \quad h = \sqrt{(a H^2 \div b)}$$

Or, $L : l :: H : h$, and $H : h :: L : l$.

EXAMPLE. — An engraver has a drawing which may be circumscribed by a rectangle whose length is 8, and breadth 6; and he wishes to copy it at half the size: required the length and breadth of a rectangle that will circumscribe the proposed copy.

$\sqrt{[(8 \times 8) \div 2]} = 5.65685$ = length of required rectangle. }
$8 \times 6 \div 2 \times 5.65685 = 4.24264$ = breadth required. } *Ans.*

II. A brace is to run 3 feet on the post of a building (from angle to extreme) and 4 feet on the plate. What must be the length of the brace (extreme from shoulder to shoulder), and at what angles must the shoulders of the tenons be cut?

$\sqrt{(4^2 + 3^2)} = 5$ feet, length of brace. *Ans.*

$3 \times 86.14 \div (5 + \frac{4}{2}) = 36°.91$, angle for head of brace. *Ans.*

$90° — 36°.91 = 53°.09$, angle for foot of brace. *Ans.*

III. The roof of a building 30 feet wide is to have a height equal to one-third the width of the building; required the length of the rafters, supposing the sides of the roof are to be equal, and the angles for their ends.

Half width of building $= 15$ feet; height of roof $= 10$ feet.

$\sqrt{(15^2 + 10^2)} = 18.03$ feet, length of rafters or slant height of roof. *Ans.*

$10 \times 86.14 \div (18.03 + 7.5) = 33°.74$, pitch of roof, or angle for foot of rafters. *Ans.*

$90° - 33°.74 = 56°.26$, angle for head of rafters if they are to be bevelled together; or $(90° - 56°.26) \times 2 = 67°.48$ (direct with that for the foot), if one is to rest upon the top of the other, or if they are to be halved together, or if an equilateral ridge-pole is to be used; in which latter case, the upper and lower angles of the pole must be $90° + 56°.26 - 33°.74 = 112°.52$ each, and the lateral angles $180° - 112°.52 = 67°.48$ each. But if the upper ends of the rafters are to be squared, by which the pole will have two right angles, the lateral angles of the pole must be 90° each, the upper angle 112°.52, as before, and the lower angle 67°.48. *Ans.*

IV. A king-post is to run 5.1 feet on the rafter, and 5 feet on the beam; required the length of the post, and the angles for its ends, the perpendicular h (dropped from the extremity of the run on the rafter to the beam) being 2.8 feet.

$b = \sqrt{(5.1^2 - 2.8^2)} = 4.26263$ feet, distance from foot of perpendicular to angle opposite post.

$p = \sqrt{[2.8^2 + (5 - b)^2]} = 2.89546$ feet, length of post. *Ans.*

$86.14h \div (5.1 + \frac{1}{2}b) = 33°.35$, angle opposite post, or pitch of roof; and $90° - 33°.35 + \dfrac{86.14 \times (5 - b)}{p + \frac{1}{2}h} = 71°.44$, angle for head of post. *Ans.*

$180° - 71°.44 - 33°.35 = 75°.21$, angle for foot of post (inverse to that for the head). *Ans.*

The foregoing is applicable to a roof of equal or unequal sides, and to hip-roofs generally.

NOTE.—Post perpendicular to rafter, acute angle of foot of post on outside.

Post perpendicular to beam, acute angle of head of post on outside.

Post inclining toward opposite angle, and inner vertical angle greater than 90°, acute angle of foot of post on outside; of head of post on inside.

Post inclining toward opposite angle, and inner vertical angle less than 90°, acute angle of foot of post on outside; of head, on outside.

Post declining from opposite angle, acute angle of foot of post on inside; of head, on outside.

V. Two perpendicular walls are standing on a plane 80 ft. apart; one is 20 feet high, and the other 30; at what distance on the plane, between them, from the base of the highest must a ladder be placed, so that, being inclined to either, it will reach the top; and what must be the length of the ladder?

$80^2 - 30^2 \backsim 20^2 = 5900 \div (80 \times 2) = 36\frac{7}{8}$ feet from base of highest wall. *Ans.*

$\surd(36\frac{7}{8}^2 + 30^2) = 47.54$ feet, length of ladder. *Ans.*

PROOF. $80 - 36\frac{7}{8} = 43\frac{1}{8}$ feet from base of lowest wall; and $\surd(43\frac{1}{8}^2 + 20^2) = 47.54$ feet, length of ladder, as before.

A pole 100 feet in length is standing on a plane; at what height must it be cut off, so that (the butt resting on the stump) the top will reach the ground 80 feet from the base of the stump?

$$\frac{100 - \frac{80^2}{100}}{2} = 18 \text{ feet. } \textit{Ans.}$$

$$\text{Or, } \frac{100^2 - 80^2}{100 \times 2} = 18 \text{ feet. } \textit{Ans.}$$

VI. In a Cone, $\frac{Dh}{D-d} = H$; $\frac{(D-d)H}{D} = h$; $\frac{Hd}{H-h} = D$; $\frac{(H-h)D}{H} = d$; D being the diameter of the base, H the altitude, d the diameter at any given altitude h above the base, and h the altitude above the base to any given diameter d, or H the slant length of the cone, and h the slant length of the frustum; also, $\frac{Dh}{D - \sqrt[3]{\frac{(S-s)D^3}{S}}} = H$; $D - \sqrt[3]{\frac{(S-s)D^3}{S}} \times \frac{H}{D} = h$; $\sqrt[3]{\frac{Sd^3}{S-s}} = D$; $\sqrt[3]{\frac{(S-s)D^3}{S}} = d$; S being the solidity of the cone, and s the solidity of the frustum.

It is required to cut from a cone, by a section parallel to the base, a frustum containing 16 cubic feet; the altitude (length of axis) of the cone is 14 feet, and the diameter of its base 4 feet; what must be the altitude of the frustum?

$$4 - \sqrt[3]{\frac{(58.6432 - 16) \times 64}{58.6432}} = .403 \times \frac{14}{4} = 1.41 \text{ feet. } \textit{Ans.}$$

VII In a frustum of a cone, $D - \frac{(D-d)h}{H} = m$; $\frac{(D-m)H}{D-d} = h$, $\frac{(D-d)h}{D-m} = H$; $\frac{(m-d)H}{H-h} + d = D$; $D - \frac{(D-m)H}{h} = d$; D being the diameter of the greater base, d the diameter of the less base, H the altitude of the frustum, h the altitude at any given diameter m above the greater base, and m the diameter at any given altitude h; also, $\left(\frac{DH}{D-d} - H\right)d^2.2618 + S = \frac{DH}{D-d}D^2.2618 = \frac{D^3H.2618}{D-d}$ = solidity of cone completed from the frustum, S being the solidity of the frustum; and $\frac{D^3H.2618}{D-d} : \frac{D^3H.2618}{D-d} - s :: D^3 : m^3$, or $\sqrt[3]{\left(D^3 - \frac{(D-d)s}{H.2618}\right)} = m$, the diameter of the less base of a frustum whose solidity is s, cut by a plane parallel to the bases, from the greater end of the frustum whose solidity is S; also, $\sqrt{\left(\frac{s}{h.2618} - \frac{3D^2}{4}\right)} - \frac{D}{2} = m$, $\sqrt{\left(\frac{s}{h.2618} - \frac{3m^2}{4}\right)} - \frac{m}{2} = D$, and $\frac{s}{(D^2 + m^2 + Dm).2618} = h$, the altitude of the frustum, the diameter of whose less base is m, and solidity s.

If the frustum whose solidity is s is to be taken from the smaller end of the given frustum, then $\sqrt[3]{D^3 \backsim \frac{(D-d) \times (S-s)}{H.2618}} = m$, S being the solidity of the given frustum; and $H - \frac{(D-m)H}{D-d} = h^1$, altitude of frustum whose solidity is s, taken from smaller end of given frustum.

$$\sqrt{A^2 - \left(\frac{D-d}{2}\right)^2} = H, \text{ and } \sqrt{H^2 + \left(\frac{D-d}{2}\right)^2} = A,$$ A being the slant length of the frustum.

A round stick of timber has diameter of greater end 3 feet, diameter of less end 2 feet, and length of axis 12 feet; at what distance on the axis, from the greater end, must I cut this stick, by a section parallel to the ends, in order that the frustum, cut from the greater end, may contain 25 cubic feet?

$$3 - \sqrt[3]{\left(3^3 - \frac{(3-2)25}{12 \times .2618}\right)} \times \frac{12}{3-2} = 3.955 \text{ feet.} \quad \textit{Ans.}$$

The interior dimensions of a vessel in the form of a frustum of a cone are, depth 12 inches, bottom diameter 10 inches, top diameter

6 inches; to what depth must the vessel be filled, it standing level upon its bottom, in order that it may contain 1½ wine gallons?

$$\sqrt[3]{\left(10^3 - \frac{(10-6) \times 231 \times 1.5}{12 \times .2618}\right)} = 8.2364 = \text{diameter at sur-}$$

face of contents, and

$$\frac{(10 - 8.2364) \times 12}{10 - 6} = 5.2908 \text{ inches.} \quad Ans.$$

VIII. The perpendicular depth (H) of a vessel in the form of a frustum of a cone is 7 feet, the bottom diameter (D) 5 feet, and the diameter of the open mouth, or top (d) 3 feet; the vessel is turned on edge, and filled with water till the level surface of the water just touches the uppermost point in the bottom and lowermost in the top; required the quantity of water in the vessel.

$$\frac{D^2 - \sqrt{(Dd)}d}{D - d} \times .2618DH = \text{content, then}$$

$$5 \times 5 - 3 \times \sqrt{(5 \times 3} = 13.381 \div (5-3) = 6.69 \times 7 \times 5 \times .2618$$
$$= 61.3 \text{ cubic feet.} \quad Ans.$$

The same vessel, and D the open mouth, other things as before,

$$\frac{D\sqrt{(Dd)} - d^2}{D - d} \times .2618dH = \text{contents} = 28.49 \text{ cubic feet.} \quad Ans.$$

IX. A vessel 12 feet high (H), and kept constantly full of water, has an opening in its side 4 feet (h) above the bottom, from which a jet is projected; to what distance on the plane, level with the bottom, is the jet projected?

$$\sqrt{(H - h)h} \times 2 = \text{projection, then}$$

$$\sqrt{(12 - 4) \times 4} = 5.657 \times 2 = 11.314 \text{ feet.} \quad Ans.$$

TRIGONOMETRY.

Trigonometry is a branch of Geometry. It treats of the relations of the sides and angles of triangles to each other. It enables us, having any three of the three sides and three angles of a triangle given, and one of them a side, to find the rest. It treats of the mensuration of the angles of triangles, therefore, and of the mensuration of the sides.

Trigonometry is of two kinds, *Rectilinear* and *Spherical.*

The former treats of right-lined triangles, and the latter of triangles formed by the intersections of three great circles upon the surface of a sphere.

Rectilinear trigonometry is often denominated *plane trigonometry*, and is divided into two parts, *rectangular* and *oblique-angular.*

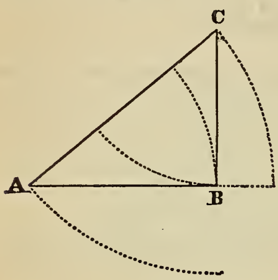
In rectilinear trigonometry, instead of making use of an arc of a circle as the measure of an angle, the *sine*, *tangent*, *secant*, or *cosine*, *cotangent*, *cosecant*, of that arc, is used; and the sides of plane triangles, therefore, take these names, interchangeably, according as one or another is made radius of the arc supposed, and according as one extremity or the other of the radius is made centre.

The circumference of every circle is divided into 360 equal parts, called degrees, (°); each degree into 60 equal parts, called minutes, (′); and each minute into 60 equal parts, called seconds, (″): and an angle at the centre of a circle, formed by any two sides of a triangle, is as many degrees, minutes and seconds, as there are degrees, minutes and seconds, in that portion of the circumference that the sides forming that angle embrace, or may be supposed to enclose.

And because the sides of triangles are *sines*, *tangents*, &c., of the arcs that measure the angles, they are also said to be *sines*, *tangents*, &c., of the angles that are measured by those arcs. (See Geometry — *definitions.*)

Proposition 1. — *In every right-angled triangle*, if the hypotenuse be made radius, one of the legs will be the sine of the angle whose vertex is made centre, and the other will be the cosine of the same angle, and either extremity of the hypotenuse, or the vertex of either acute-angle of a right-angled triangle may be made centre. Thus, if, in the diagram A B C, annexed, the hypotenuse A C be made radius, and A the centre, B C will be the sine of the angle at A, or of the angle A, and A B will be the cosine of the same angle. And if A C be made radius and C the centre, A B will be the sine of the angle C, and B C will be the cosine of the same angle.

Either leg, therefore, of a right-angled triangle that is the sine of one of the acute-angles, is also the cosine of the other acute-angle; and the legs of a right-angled triangle, the hypotenuse being radius, are sines of their opposite angles.



PROPOSITION 2. — *In every right-angled triangle*, if one of the legs be made radius, the other will be the tangent of the acute-angle whose vertex is made centre, and the hypotenuse will be the secant of the same angle, and either leg may be made radius. Thus, if A B be made radius, A will be centre, B C will be the tangent of the angle A, and the hypotenuse will be the secant of the same angle. And if B C be made radius, C will be centre, A B will be the tangent of the angle C, and A C will be the secant of C.

Either leg, therefore, of a right-angled triangle that is the tangent of one of the acute-angles, is also the cotangent of the other; and the hypotenuse being secant of the acute-angle whose vertex is made centre, is also cosecant of the other; one of the legs of a right-angled triangle being made radius, the other is the tangent of its opposite angle.

As four right-angles can be formed about the same point, therefore every right-angle is equal to a quadrant of the circle, or 90°.

The three angles of any triangle are equal to two right-angles, or 180°.

The two acute-angles of a right-angled triangle are equal to a right-angle.

When a particular angle of a triangle is referred to, and the three letters at the three angles of that triangle are used to express it, the letter at the angle referred to is placed in the middle; thus, both the angle and the triangle are indicated: in the expression, The angle A B C, the angle B, in the triangle A B C, is meant.

In the annexed table of natural sines, cosines and tangents, to given angles, the numbers, though not so marked, are considered as decimals to radius 1.

The natural sine of 90°, therefore, is 1; and the natural cosine of 90° is 0.

The tangent of 45° is 1, and so is the cotangent.

The tangent of 90° is infinite.

If two angles of a triangle are given, the other is said also to be given, for it is the difference between the sum of the two and 180°.

If one of the acute-angles of a right-angled triangle be given, all the angles of that triangle are said to be given, for the sum of the three angles of any triangle, minus the sum of any two, is equal the third and the right-angle is known for 90°.

In trigonometrical expressions and formulas, the following abbreviations and contractions are often made use of, viz.:

R, for tabular Radius, natural or logarithmic sine of 90°, or of the right-angle, (1.)*

sin A, (*or any other letter*,) for sine of the angle A, or whatever other letter.

cos A, for cosine of the angle A.

tan A, for tangent of the angle A.

sec A. for secant of the angle A.

cosec A, for cosecant of the angle A.

cot A, for cotangent of the angle A.

coversin, for coversed sine.

sin A comp B, for sine of the angle A, the complement of which angle is the angle B. That is, the angle A subtracted from 90°, the difference will be the angle B.

These expressions, such as sin A, tan B, &c., or their equivalents, when used as terms in the statement of a problem, or its solution, are to be taken as referring to the natural or tabular sine, tangent, &c., of the angle indicated, and not to the sides taking those names. Sometimes the contraction nat. or tab. is prefixed, (as nat cos B,) but usually it is omitted.

In any right-angled plane triangle, A B C —

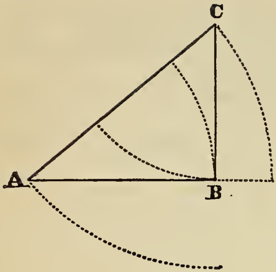
A being one of the acute angles, and C the other;

sin A = cos C	cos A = sin C.
tan A = cot C	cot A = tan C.
sec A = cosec C	cosec A = sec C.

And of the same angle, A or C, considered separately,

tan × cos = sin	cot × sin = cos
cos ÷ cot = sin	sin ÷ tan = cos
tan ÷ sec = sin	cot ÷ cosec = cos
R (1) ÷ cosec = sin	R ÷ sec = cos
sin × sec = tan	cosec × cos = cot
sin ÷ cos = tan	cos ÷ sin = cot
sec ÷ cosec = tan	cosec ÷ sec = cot
R ÷ cot = tan	R ÷ tan = cot
cosec × tan = sec	cot × sec = cosec
tan ÷ sin = sec	sec ÷ tan = cosec
cosec ÷ cot = sec	cot ÷ cos = cosec
R ÷ cos = sec	R ÷ sin = cosec

* The logarithmic radius or tabular sine of 90° is 10: the natural radius or tabular sine of 90° is 1. A table of logarithmic sines, tangents, &c., is called a table of artificial sines, tangents, &c.



In any right-angled plane triangle, A B C, (Fig.)

B C ÷ A C (tan ÷ sec) = sin A.
B C ÷ A B (sin ÷ cos) = tan A.
A C ÷ A B (R ÷ cos) = sec A.
A B ÷ A C (tan ÷ sec) = sin C.
A B ÷ B C (sin ÷ cos) = tan C.
A C ÷ B C (R ÷ cos) = sec C.

Or, if we denominate the longest side of the triangle *hypotenuse*, and of the other two sides make one *base* and the other *perpendicular;* then —

$\frac{\text{perp.}}{\text{hyp.}}$ = nat sin of angle opposite perp., or nat cos of angle opposite base.

$\frac{\text{base}}{\text{hyp.}}$ = nat sin of angle opposite base, or nat cos of angle opposite perp.

$\frac{\text{perp.}}{\text{base}}$ = nat tan of angle opposite perp., or nat cot of angle opposite base.

$\frac{\text{base}}{\text{perp.}}$ = nat tan of angle opposite base, or nat cot of angle opposite perp.

$\frac{\text{hyp.}}{\text{base}}$ = nat sec of angle opposite perp., or nat cosec of angle opposite base.

$\frac{\text{hyp.}}{\text{perp.}}$ = nat sec of angle opposite base, or nat cosec of angle opposite perp.

With reference to the sides of right-angled triangles, considered in connection with the natural sines, tangents, &c., of their opposite and included angles : —

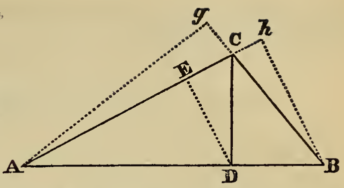
Preceding Fig.

A C × sin A = B C.	A C ÷ sec A = A B.
A B × tan A = B C.	A C ÷ cosec A = B C.
A B × sec A = A C.	A B ÷ cot A = B C.
B C × cosec A = A C.	A B ÷ cos A = A C.
A C × cos A = A B.	B C ÷ sin A = A C.
B C × cot A = A B.	B C ÷ tan A = A B.

In every plane triangle, — right-angled, acute-angled or obtuse-angled, — the natural sines of the angles are to each other as the opposite sides.

Thus, in the triangle A B C, annexed,

sin A : sin B : : B C : A C.
sin B : A C : : sin C : A B.
B C : A B : : sin A : sin C.



In every plane triangle, the sum of the squares of the sides adjacent any angle, minus the square of the side opposite that angle, divided by twice the product of the sides adjacent, equals the natural cosine of that angle.

Thus,

$$\frac{\overline{AB}^2 + \overline{AC}^2 - \overline{BC}^2}{AB \times AC \times 2} = \cos A.$$

$$\frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2\,AB \times BC} = \cos B.$$

$$\frac{\overline{AC}^2 + \overline{BC}^2 - \overline{AB}^2}{2\,AC \times BC} = \cos C.$$

Or, if the sides of the triangle be of unequal lengths, and we denominate them according to their relative lengths, as longest, mediate, shortest, we have the following more distinctive formulas:

$$\frac{\text{longest}^2 + \text{med}^2 - \text{short}^2}{\text{longest} \times \text{med} \times 2} = \text{nat cos smallest angle.}$$

$$\frac{\text{longest}^2 + \text{short}^2 - \text{med}^2}{\text{long} \times \text{short} \times 2} = \text{nat cos mediate angle.}$$

$$\frac{\text{med}^2 + \text{shortest}^2 - \text{longest}^2}{\text{mediate} \times \text{shortest} \times 2} = \text{nat cos greatest angle.}$$

$\sqrt{(1 - \sin^2)}$ of any angle = cos of that angle.

$1 - (2 \times \sin^2)$ of $\frac{1}{2}$ any angle = cos of that angle.

That is, A being the angle—

$$1 - 2 \sin^2 \text{ of } \tfrac{1}{2} A = \cos A.$$

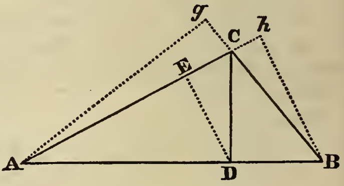
In every plane triangle, as the sum of any two sides is to their difference, so is the natural tangent of half the sum of the angles opposite those sides to the natural tangent of half their difference; and half the sum of the two angles, plus half their difference, equals the greater of the two. and half the sum of the two angles, minus half their difference equals the less, the greater angle being opposite the longer side.

Thus, in the oblique-angled triangle, A B C, (Fig.)

$$A\,C + B\,C : A\,C \backsim B\,C :: \tan\frac{A+B}{2} : \tan\frac{A \backsim B}{2}, \text{ and}$$

$$\frac{A+B}{2} + \frac{A \backsim B}{2} = B, \text{ and}$$

$$\frac{A+B}{2} - \frac{A \backsim B}{2} = A.$$



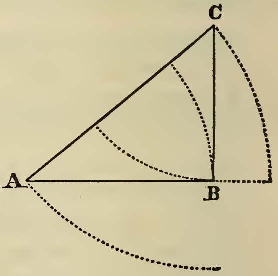
$A\,B + A\,C : A\,B - A\,C ::$ $\tan \frac{1}{2}(C+B) : \tan \frac{1}{2}(C-B)$, and

$\frac{1}{2}(C+B) + \frac{1}{2}(C-B) = C$, and $\frac{1}{2}(C+B) - \frac{1}{2}(C-B) = B$.

$A\,B + B\,C : A\,B - B\,C :: \tan \frac{1}{2}(C+A) : \tan \frac{1}{2}(C-A)$, and

$$\frac{C+A}{2} + \frac{C-A}{2} = C, \text{ and } \frac{C+A}{2} - \frac{C-A}{2} = A.$$

SOLUTIONS, — Right-angled Triangles.



A B C, the triangle,
A C, hypotenuse,
A B and B C, legs; B, the right angle.

The hypotenuse and angles given, to find the legs.

Suppose A C 51 feet, A 28° 10′, C (consequently) 90° — 28° 10′ = 61° 50′.

The sines of angles are to each other as the sides opposite those angles.

Turning, therefore, to the tables of natural sines, &c., we find, in the column of sines, against angle 28° 10′, sine .47204, consequently —

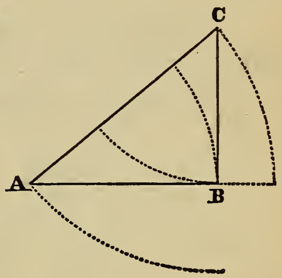
R		A C		sin A		B C
1	:	51	::	.47204	:	24.1 feet.

Having now A C and B C, side A B may be found by the rules in geometry; or, turning again to angle 28° 10′, in the tables, we find, against that angle, cosine .88158*; and as the cosine of one of the acute angles of a right-angled triangle is the sine of the other acute angle, consequently,

R . A C sin C A B.
1 : 51 : : .88158 : 45 feet.

The angles and one leg given, to find the hypotenuse and other leg.

As before, C = 61° 50′, and A = 90° — 61° 50′ = 28° 10′; A B = 45 feet.



sin C A B R A C
.88158 : 45 : : 1 : 51

sin C A B sin A B C
.88158 : 45 : : .47204 : 24.1.

Or, R A B tan A B C
1 : 45 : : .53545 : 24.1.

R A B sec A A C
1 : 45 : : 1.13433 : 51.

The hypotenuse and one leg given, to find the angles and other leg.

Let A C = 85 and B C = 58; then —

A C R B C sin A
85 : 1 : : 58 : .68235.

Turning now to the tables of nat. sines, &c., we find, in the column of sines, against the number 68235, or the number nearest thereto, angle 43°. We therefore have —

A C : R : : B C : sin A, 43°; and

90° — 43° = 47° = angle C.

R A C sin C A B
1 : 85 : : .73135 : 62.16.

The legs given, to find the angles and hypotenuse.

Let A B = 54.7, and B C = 32:

A B R B C tan A
54.7 : 1 : : 32 : : .58501.

Turning now to the tables of natural sines, tangents, &c., we find in the column of tangents 58501, or a number near thereto, and

* When an angle is greater than 45° the sine, tangent, &c., of its complement is used.

against that number we find 30° 20′; the angle A, therefore, is 30° 20′, and the angle C is

$$90° - 30° 20' = 59° 40'.$$

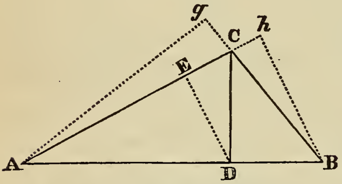
sin A		B C			R		A C
.50503	:	32	: :	1	:		63.36.

Or, sin 90° : B C : : sec 59° 40′ : A C
1 32 1.98008 63.36.

SOLUTIONS, — Oblique-angled triangles.

Let A B C be the triangle: (Fig. annexed.)

The angles and one side given, to find the other sides.



Suppose A 26° 40′, B 56°, C 180° — (26° 40′ + 56°) = 97° 20′, and A C 40 feet.

sin B : A C : : sin A : B C.

The angle B is greater than 45°; its sine, therefore, is the cosine of its complement, or the cosine of what it lacks of 90°. Its sine, therefore, is the cosine of 90° — 56° = 34°. Turning now to 34° in the tables, we find, against that angle, cos .82904; .82904, consequently, is the sine of 56°, and

sin B A C sin A B C
.82904 : 40 : : .44880 : 21.65 feet.

sin B : A C : : sin C : A B.

The angle C is not only greater than 45°, but it is greater than 90°. Its sine, therefore, is the cosine of the difference of its supplement and 90°. It is the cosine of the difference of 180° — 79° 20′ = 82° 40′ and 90° = 7° 20′. As in the preceding case, its sine, therefore, is the cosine of the difference between itself and 90°.

Turning now to angle 7° 20′, in the tables, we find its cosine .99182; therefore we have —

sin B A C sin C A B
.82904 : 40 : : .99182 : 47.85 feet.

Two sides and an angle opposite to one of them given, to find the other angles, and other side.

As before, A B 47.85, B C 21.65, C 97° 20′.

A B : sin C : : B C : sin A, 26° 40′; and

180° — (97° 20′ + 26° 40′) = 56° = B.

sin C : A B :: sin B : A C; or,

sin A : B C :: sin B : A C.

Two sides and their contained angle given, to find the other angles and side.

Suppose A B 47.85, B C 21.65, B 56°.

$$AB + BC : AB \backsim BC :: \tan \tfrac{1}{2}(A + C) : \tan \tfrac{1}{2}(A \backsim C).$$

$$\tan \tfrac{1}{2}(A + C) = \tan \frac{180° - B}{2} = \frac{R\ (1)}{\tan 90° \backsim \tfrac{1}{2}(180° - B).}$$

$\tan \tfrac{1}{2}(180° - B) = \cot 90° \backsim \tfrac{1}{2}(180° - B)$; therefore,

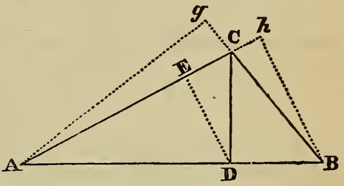
A B + B C		A B $\backsim$ B C		tan 62°		tan	
69.5	:	26.2	::	1.88073	:	.70899, 35° 20′;	and

$\frac{1}{2}(A + C)$		$\frac{1}{2}(A \backsim C)$	C		$\frac{1}{2}(A + C)$	$\frac{1}{2}(A \backsim C)$	A
62°	+	35.20	= 97° 20′	and	62°	— 35° 20′	= 26°40′.

sin A : B C : sin B : A C, or

sin C : A B :: sin B : A C.

The three sides given, to find the angles.



Let A B C be the triangle.
A B 47.85, A C 40, B C 21.65.

$$\overline{AB}^2 + \overline{AC}^2 - \overline{BC}^2 \div \overline{AB \times AC \times 2} = \cos A.$$
3420.9 — 3828 — .89365, 26° 40′.

B C :	sin A ::	A C	: sin B.
21.65	.44880	40	.82919 = cos comp B, = 90° — 34° = 56° = B.

$$180° - \overline{26°\ 40' + 56°} = 97°\ 20' = C.$$

Or, $\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2 \div \overline{AB \times BC \times 2} = \cos B$
1158.34 — 2071.9 — .55907 = sin comp B = 90° — 34° = 56° = B.

Or, $\tfrac{1}{2}(AB + AC + BC) \times BC \backsim \tfrac{1}{2}(BA + AC + BC) \div \overline{AB \times AC} = \cos^2 \tfrac{1}{2}A.$

$54.75 \times 33.1 \div 1914 = \sqrt{.94682} = .97305 = \cos \tfrac{1}{2} A$, 13° 20′; and $13°\ 20' \times 2 = 26°\ 40' = A.$

$$\frac{AB+AC+BC}{2} \times AC \backsim \frac{AB+AC+BC}{2} \div \overline{AB \times BC} = \cos^2 \frac{B}{2}.$$

$$\frac{AB+AC+BC}{2} \times AB \backsim \frac{AB+AC+BC}{2} \div \overline{AC \times BC} = \cos^2 \tfrac{1}{2} C.$$

In general, however, when the three sides of a triangle are given, and the angles are required, it is customary to drop a perpendicular upon the longest side of the triangle, or one that will fall within the figure, whereby the triangle is divided into two right-angled triangles; and then to find the angles, by the rules for right-angled triangles. In this case the sum of the two vertical angles will be equal the angle at the vertex. For rules for dropping the perpendicular, see GEOMETRY — *Triangles*.

To find the angles of a right-angled triangle approximately by means of the sides, or without the aid of trigonometrical tables.

Let r represent the hypotenuse, s the shortest side, c the longest side, A the smallest angle, or angle opposite the shortest side.

$$A = \frac{86.13961s}{r + \frac{1}{2}c + \frac{c - \frac{1}{2}s}{172.28}} \text{ very nearly.*}$$

EXAMPLE. — What is the smaller of the two acute angles of a right-angled triangle whose hypotenuse is 100, shortest side 48.735, and longest side 87.321 ?

$$\frac{86.13961 \times 48.735}{100 + 43.6605 + 0.3654} = 29^\circ.148, \text{ or } 29^\circ\ 8'\ 53'',$$

true angle $= 29^\circ\ 10'$. *Ans.*

$90^\circ - 29^\circ.148 = 60^\circ.853$ or $60^\circ\ 51'\ 7'' =$ greater acute angle.

* See CIRCLE, length of arc of, &c.

TABLE OF NATURAL SINES, COSINES, AND TANGENTS.

D.	M.	Sine.	Cosine.	Tangent.	D.	M.	Sine.	Cosine.	Tangent.
	1	00029	10000	00029	7		12187	99255	12278
	5	00145	10000	00145		10	12476	99219	12574
	10	00291	10000	00291		20	12764	99182	12869
	20	00582	99998	00582		30	13053	99144	13165
	30	00873	99996	00873		40	13341	99106	13461
	40	01164	99993	01164		50	13629	99067	13758
	50	01454	99989	01455	8		13917	99027	14054
1		01745	99985	01745		10	14205	98986	14351
	10	02036	99979	02036		20	14493	98944	14648
	20	02327	99973	02328		30	14781	98902	14945
	30	02618	99966	02619		40	15069	98858	15243
	40	02908	99958	02910		50	15356	98814	15540
	50	03199	99949	03201	9		15643	98769	15838
2		03490	99939	03492		10	15931	98723	16137
	10	03781	99929	03783		20	16218	98676	16435
	20	04071	99917	04075		30	16505	98629	16734
	30	04362	99905	04366		40	16792	98580	17033
	40	04653	99982	04658		50	17078	98531	17333
	50	04943	99878	04949	10		17365	98481	17633
3		05234	99863	05241		10	17651	98430	17933
	10	05524	99847	05533		20	17937	98378	18233
	20	05814	99831	05824		30	18224	98325	18534
	30	06105	99813	06116		40	18509	98272	18835
	40	06395	99795	06408		50	18795	98218	19136
	50	06685	99776	06700	11		19081	98163	19438
4		06976	99756	06993		10	19366	98107	19740
	10	07266	99736	07285		20	19652	98050	20042
	20	07556	99714	07578		30	19937	97992	20345
	30	07846	99692	07870		40	20222	97934	20648
	40	08136	99668	08163		50	20507	97875	20952
	50	08426	99644	08456	12		20791	97815	21256
5		08716	99619	08749		10	21076	97754	21560
	10	09005	99594	09042		20	21360	97692	21864
	20	09295	99567	09335		30	21644	97630	22169
	30	09585	99540	09629		40	21928	97566	22475
	40	09874	99511	09923		50	22212	97502	22781
	50	10164	99482	10216	13		22495	97437	23087
6		10453	99452	10510		10	22778	97371	23393
	10	10742	99421	10805		20	23062	97304	23700
	20	11031	99390	11099		30	23345	97237	24008
	30	11320	99357	11394		40	23627	97169	24316
	40	11609	99324	11688		50	23910	97100	24624
	50	11898	99290	11983					

D.	M.	Sine.	Cosine.	Tangent.	D.	M.	Sine.	Cosine.	Tangent.
14		24192	97030	24933	21	50	37191	92827	40065
	10	24474	96959	25242	22		37461	92718	40403
	20	24756	96887	25552		10	37730	92609	40741
	30	25038	96815	25862		20	37999	92499	41081
	40	25320	96742	26172		30	38268	92388	41421
	50	25601	96667	26483		40	38537	92276	41763
15		25882	96593	26795		50	38805	92164	42105
	10	26163	96517	27107	23		39073	92050	42447
	20	26443	96440	27419		10	39341	91936	42791
	30	26724	96363	27732		20	39608	91822	43136
	40	27004	96285	28046		30	39875	91706	43481
	50	27284	96206	28360		40	40141	91590	43828
16		27564	96126	28675		50	40408	91472	44175
	10	27843	96046	28990	24		40674	91355	44523
	20	28123	95964	29305		10	40939	91236	44872
	30	28402	95882	29621		20	41204	91116	45222
	40	28680	95799	29938		30	41469	90996	45573
	50	28959	95715	30255		40	41734	90875	45924
17		29237	95630	30573		50	41998	90753	46277
	10	29515	95545	30891	25		42262	90631	46631
	20	29793	95459	31210		10	42525	90507	46985
	30	30071	95372	31530		20	42788	90383	47341
	40	30348	95284	31850		30	43051	90259	47698
	50	30625	95195	32171		40	43313	90133	48055
18		30902	95106	32492		50	43575	90007	48414
	10	31178	95015	32814	26		43837	89879	48773
	20	31454	94924	33136		10	44098	89752	49134
	30	31730	94832	33460		20	44359	89623	49495
	40	32006	94740	33783		30	44620	89493	49858
	50	32282	94646	34108		40	44880	89363	50222
19		32557	94552	34433		50	45140	89232	50587
	10	32832	94457	34758	27		45399	89101	50953
	20	33106	94361	35085		10	45658	88968	51319
	30	33381	94264	35412		20	45917	88835	51688
	40	33655	94167	35740		30	46175	88701	52057
	50	33929	94068	36068		40	46433	88566	52427
20		34202	93969	36397		50	46690	88431	52798
	10	34475	93869	36727	28		46947	88295	53171
	20	34748	93769	37057		10	47204	88158	53545
	30	35021	93667	37388		20	47460	88020	53920
	40	35293	93565	37720		30	47716	87882	54296
	50	35565	93462	38053		40	47971	87743	54673
21		35837	93358	38386		50	48226	87603	55051
	10	36108	93253	38721	29		48481	87462	55431
	20	36379	93148	39055		10	48735	87321	55812
	30	36650	93042	39391		20	48989	87178	56194
	40	36921	92935	39727		30	49242	87036	56577

D.	M.	Sine.	Cosine.	Tangent.	D.	M.	Sine.	Cosine.	Tangent.
29	40	49495	86892	57962	37	30	60876	79335	76733
	50	49748	86748	57348		40	61107	79158	77196
30		50000	86603	57735		50	61337	78980	77661
	10	50252	86457	58124	38		61566	78801	78129
	20	50503	86310	58513		10	61795	78622	78598
	30	50754	86163	58904		20	62024	78442	79070
	40	51004	86015	59297		30	62251	78261	79544
	50	51254	85866	59691		40	62479	78079	80020
31		51504	85717	60086		50	62706	77897	80498
	10	51753	85567	60483	39		62932	77715	80978
	20	52002	85416	60881		10	63158	77531	81461
	30	52250	85264	61280		20	63383	77347	81946
	40	52498	85112	61681		30	63608	77162	82434
	50	52745	84959	62083		40	63832	76977	82923
32		52992	84805	62487		50	64056	76791	83415
	10	53238	84650	62892	40		64279	76604	83910
	20	53484	84495	63299		10	64501	76417	84407
	30	53730	84339	63707		20	64723	76229	84906
	40	53975	84182	64117		30	64945	76041	85408
	50	54220	84025	64528		40	65166	75851	85912
33		54464	83867	64941		50	65386	75661	86419
	10	54708	83708	65355	41		65606	75471	86929
	20	54951	83549	65771		10	65825	75280	87441
	30	55194	83389	66189		20	66044	75088	87955
	40	55436	83228	66608		30	66262	74896	88473
	50	55678	83066	67028		40	66480	74703	88992
34		55919	82904	67451		50	66697	74509	89515
	10	56160	82741	67875	42		66913	74314	90040
	20	56401	82577	68301		10	67129	74120	90569
	30	56641	82413	68728		20	67344	73924	91099
	40	56880	82248	69157		30	67559	73728	91633
	50	57119	82082	69588		40	67773	73531	92170
35		57358	81915	70021		50	67987	73333	92709
	10	57596	81748	70455	43		68200	73135	93252
	20	57833	81580	70891		10	68412	72937	93797
	30	58070	81412	71329		20	68624	72737	94345
	40	58307	81242	71769		30	68835	72537	94896
	50	58543	81072	72211		40	69046	72337	95451
36		58779	80902	72654		50	69256	72136	96008
	10	59014	80730	73100	44		69466	71934	96569
	20	59248	80558	73547		10	69675	71732	97133
	30	59482	80386	73996		20	69883	71529	97700
	40	59716	80212	74447		30	70091	71325	98270
	50	59949	80038	74900		40	70298	71121	98843
37		60182	79864	75355		50	70505	70916	99420
	10	60414	79688	75812	45		70711	70711	1.
	20	60645	79512	76272					

NOTE. --The foregoing TABLES present sines, cosines, and tangents, calculated for every degree and ten minutes of the quadrant. To furnish tables calculated for a smaller division of the circle than ten minutes, as for five minutes, or for one minute, would occupy too much space in this work. Besides, calculations to the sixth of a degree, for many practical purposes, are sufficiently minute. If, however, greater precision is desired, the tables furnish a very simple means whereby to obtain it, and to almost any extent short of scientific minuteness and accuracy.

Suppose, for instance, in the solution of a problem, the sine .64380 appears: now, on turning to the tables, the nearest sine we find to this is .64279, the sine of 40°; while the next nearest is .64501, the sine of 40° 10′; now, .64279 + .64501 = 1.28780 ÷ 2 = .64390 = sin 40° 5′; .64390, therefore, is the sine of an angle a trifle less than 40° 5′, but nearer to that angle than to any other of full minutes.

Again, suppose the sine .51433 appears; the nearest sine in the tables to this is .51504, which is the sine of 31°; and the next nearest is .51254, the sine of 30° 50′; .51504 + .51254 = 1.02758 ÷ 2 = .51379 = sin 30° 55′, and .51504 + .51379 = 1.02883 ÷ 2 = .51442 = sin 30° 57½′; .51433, therefore, is the sine of 30° 57′, very nearly.

The foregoing principles are also applicable to the cosines and tangents, and in the same manner.

The versed sine of any angle = 1 — cosine of that angle.
The coversed sine of any angle = 1 — sine of that angle.
The chord of any angle = sine of ½ of that angle × 2.

EXAMPLE. = The sides of a right-angled triangle are 3, 4 and 5 feet; required a side of the greatest square that may be cut from the triangle.

A right line that bisects the right angle and extends to the hypotenuse is equal to and becomes the diagonal of the required square, and the diagonal of any square multiplied by $\frac{1}{\sqrt{2}}$ = a side of that square; then 5 : 1 :: 3 : .6 = sin 36° 51′, and

Sin 90° + 36° 51′ — 45° = .9899 : 3 :: sin 90° — 36° 51′ = .8 : 2.4245 = diagonal, and 2.4245 × .70711 = 1.7144 = side of square. *Ans.*

Or sin 90° — 36° 51′(.8) : 2.4245 : sin 45°(.70711) : 2.143, and

$$(\overline{5 - 2.143}^2 - \overline{2.4245}^2) \div 4 \times 2 = .2855 + \frac{4}{2} = 2.2855, \text{ and}$$

$$\sqrt{(\overline{5 - 2.143}^2 - \overline{2.2855}^2)} = 1.714 \text{ feet, side of square. } \textit{Ans.}$$

RECAPITULATION. — When the given angle is greater than 45°, its sine is expressed by the cosine of the angle which is the difference of 90° and the given angle: thus the sine of 46° is the tabular cosine of 90—46 = cosine of 44°; and the sine of 62° 50′ is the cosine of 90 — 62° 50′ = tabular cosine of 27° 10′, &c.

To obtain the tangent of an angle that is greater than 45°, divide the tabular cosine of the angle which expresses the difference of 90° and the given angle by the tabular sine of that difference; thus the

$$\text{tangent of } 46° = \frac{\cos(90-46) = \cos 44°}{\sin 44°} = \frac{71934}{69466} = 1.0355, \text{ and}$$

the tangent of

$$58°\,40' = \frac{\cos(90° - 58°\,40') = \cos 31°\,20'}{\sin 31°\,20'} = \frac{85416}{52002} = 1.64255, \text{\&c.}$$

Number.	Square.	Cube.	Square Root.	Cube Root.
1	1	1	1.0000000	1.0000000
2	4	8	1.4142136	1.2599210
3	9	27	1.7320508	1.4422496
4	16	64	2.0000000	1.5874011
5	25	125	2.2360680	1.7099759
6	36	216	2.4494897	1.8171206
7	49	343	2.6457513	1.9129312
8	64	512	2.8284271	2.0000000
9	81	729	3.0000000	2.0800837
10	100	1000	3.1622777	2.1544347
11	121	1331	3.3166248	2.2239801
12	144	1728	3.4641016	2.2894286
13	169	2197	3.6055513	2.3513347
14	196	2744	3.7416574	2.4101422
15	225	3375	3.8729833	2.4662121
16	256	4096	4.0000000	2.5198421
17	289	4913	4.1231056	2.5712816
18	324	5832	4.2426407	2.6207414
19	361	6859	4.3588989	2.6684016
20	400	8000	4.4721360	2.7144177
21	441	9261	4.5825757	2.7589243
22	484	10648	4.6904158	2.8020393
23	529	12167	4.7958315	2.8438670
24	576	13824	4.8989795	2.8844991
25	625	15625	5.0000000	2.9240177
26	676	17576	5.0990195	2.9624960
27	729	19683	5.1961524	3.0000000
28	784	21952	5.2915026	3.0365889
29	841	24389	5.3851648	3.0723168
30	900	27000	5.4772256	3.1072325
31	961	29791	5.5677644	3.1413806
32	1024	32768	5.6568542	3.1748021
33	1089	35937	5.7445626	3.2075343
34	1156	39304	5.8309519	3.2396118
35	1225	42875	5.9160798	3.2710663
36	1296	46656	6.0000000	3.3019272
37	1369	50653	6.0827625	3.3322218
38	1444	54872	6.1644140	3.3619754
39	1521	59319	6.2449980	3.3912114
40	1600	64000	6.3245553	3.4199519
41	1681	68921	6.4031242	3.4482172
42	1764	74088	6.4807407	3.4760266

Number.	Square.	Cube.	Square Root.	Cube Root.
43	1849	79507	6.5574385	3.5033981
44	1936	85184	6.6332496	3.5303483
45	2025	91125	6.7082039	3.5568933
46	2116	97336	6.7823300	3.5830479
47	2209	103823	6.8556546	3.6088261
48	2304	110592	6.9282032	3.6342411
49	2401	117649	7.0000000	3.6593057
50	2500	125000	7.0710678	3.6840314
51	2601	132651	7.1414284	3.7084298
52	2704	140608	7.2111026	3.7325111
53	2809	148877	7.2801099	3.7562858
54	2916	157464	7.3484692	3.7797631
55	3025	166375	7.4161985	3.8029525
56	3136	175616	7.4833148	3.8258624
57	3249	185193	7.5498344	3.8485011
58	3364	195112	7.6157731	3.8708766
59	3481	205379	7.6811457	3.8929965
60	3600	216000	7.7459667	3.9148676
61	3721	226981	7.8102497	3.9304972
62	3844	238328	7.8740079	3.9578915
63	3969	250047	7.9372539	3.9790571
64	4096	262144	8.0000000	4.0000000
65	4225	274625	8.0622577	4.0207256
66	4356	287496	8.1240384	4.0412401
67	4489	300763	8.1853528	4.0615480
68	4624	314432	8.2462113	4.0816551
69	4761	328509	8.3066239	4.1015661
70	4900	343000	8.3666003	4.1212853
71	5041	357911	8.4261498	4.1408178
72	5184	373248	8.4852814	4.1601676
73	5329	389017	8.5440037	4.1793390
74	5476	405224	8.6023253	4.1983364
75	5625	421875	8.6602540	4.2171633
76	5776	438976	8.7177979	4.2358236
77	5929	456533	8.7749644	4.2543210
78	6084	474552	8.8317609	4.2726586
79	6241	493039	8.8881944	4.2908404
80	6400	512000	8.9442719	4.3088695
81	6561	531441	9.0000000	4.3267487
82	6724	551368	9.0553851	4.3444815
83	6889	571787	9.1104336	4.3620707
84	7056	592704	9.1651514	4.3795191

Number.	Square.	Cube.	Square Root.	Cube Root.
85	7225	614125	9.2195445	4.3968296
86	7396	636056	9.2736185	4.4140049
87	7569	658503	9.3273791	4.4310476
88	7744	681472	9.3808315	4.4479602
89	7921	704969	9.4339811	4.4647451
90	8100	729000	9.4868330	4.4814047
91	8281	753571	9.5393920	4.4979414
92	8464	778688	9.5916630	4.5143574
93	8649	804357	9.6436508	4.5306549
94	8836	830584	9.6953597	4.5468359
95	9025	857374	9.7467943	4.5629026
96	9216	884736	9.7979590	4.5788570
97	9409	912673	9.8488578	4.5947009
98	9604	941192	9.8994949	4.6104363
99	9801	970299	9.9498744	4.6260650
100	10000	1000000	10.0000000	4.6415888
101	10201	1030301	10.0498756	4.6570095
102	10404	1061208	10.0995049	4.6723287
103	10609	1092727	10.1488916	4.6875482
104	10816	1124864	10.1980390	4.7026694
105	11025	1157625	10.2469508	4.7176940
106	11236	1191016	10.2956301	4.7326235
107	11449	1225043	10.3440804	4.7474594
108	11664	1259712	10.3923048	4.7622032
109	11881	1295029	10.4403065	4.7768562
110	12100	1331000	10.4880885	4.7914199
111	12321	1367631	10.5356538	4.8058995
112	12544	1404928	10.5830052	4.8202845
113	12769	1442897	10.6301458	4.8345881
114	12996	1481544	10.6770783	4.8488076
115	13225	1520875	10.7238053	4.8629442
116	13456	1560896	10.7703296	4.8769990
117	13689	1601613	10.8166538	4.8909732
118	13924	1643032	10.8627805	4.9048681
119	14161	1685159	10.9087121	4.9186847
120	14400	1728000	10.9544512	4.9324242
121	14641	1771561	11.0000000	4.9460874
122	14884	1815848	11.0453610	4.9596757
123	15129	1860867	11.0905365	4.9731898
124	15376	1906624	11.1355287	4.9866310
125	15625	1953125	11.1803399	5.0000000
126	15876	2000376	11.2249722	5.0132979

Number.	Square.	Cube.	Square Root.	Cube Root.
127	16129	2048383	11.2694277	5.0265257
128	16384	2097152	11.3137085	5.0396842
129	16641	2146689	11.3578167	5.0527743
130	16900	2197000	11.4017543	5.0657970
131	17161	2248091	11.4455231	5.0787531
132	17424	2299968	11.4891253	5.0916434
133	17689	2352637	11.5325626	5.1044687
134	17956	2406104	11.5758369	5.1172299
135	18225	2460375	11.6189500	5.1299278
136	18496	2515457	11.6619038	5.1425632
137	18769	2571353	11.7046999	5.1551367
138	19044	2628072	11.7473444	5.1676493
139	19321	2685619	11.7898261	5.1801015
140	19600	2744000	11.8321596	5.1924941
141	19881	2803221	11.8743421	5.2048279
142	20164	2863288	11.9163753	5.2171034
143	20449	2924207	11.9582607	5.2293215
144	20736	2985984	12.0000000	5.2414828
145	21025	3048625	12.0415946	5.2535879
146	21316	3112136	12.0830460	5.2656374
147	21609	3176523	12.1243557	5.2776321
148	21904	3241792	12.1655251	5.2895725
149	22201	3307949	12.2065556	5.3014592
150	22500	3375000	12.2474487	5.3132928
151	22801	3442951	12.2882057	5.3250740
152	23104	3511008	12.3288280	5.3368033
153	23409	3581577	12.3693169	5.3484812
154	23716	3652264	12.4096736	5.3601084
155	24025	3723875	12.4498996	5.3716854
156	24336	3796416	12.4899960	5.3832126
157	24649	3869893	12.5299641	5.3946907
158	24964	3944312	12.5698051	5.4061202
159	25281	4019679	12.6095202	5.4175015
160	25600	4096000	12.6491106	5.4288352
161	25921	4173281	12.6885775	5.4401218
162	26244	4251528	12.7279221	5.4513618
163	26569	4330747	12.7671453	5.4625556
164	26896	4410944	12.8062485	5.4737037
165	27225	4492125	12.8452326	5.4848066
166	27556	4574296	12.8840987	5.4958647
167	27889	4657463	12.9228480	5.5068784
168	28224	4741632	12.9614814	5.5178484

Number.	Square.	Cube.	Square Root.	Cube Root.
169	28561	4826809	13.0000000	5.5287748
170	28900	4913000	13.0384048	5.5396583
171	29241	5000211	13.0766968	5.5504991
172	29584	5088448	13.1148770	5.5612978
173	29929	5177717	13.1529464	5.5720546
174	30276	5268024	13.1909060	5.5827702
175	30625	5359375	13.2287566	5.5934447
176	30976	5451776	13.2664992	5.6040787
177	31329	5545233	13.3041347	5.6146724
178	31684	5639752	13.3416641	5.6252263
179	32041	5735339	13.3790882	5.6357408
180	32400	5832000	13.4164079	5.6462162
181	32761	5929741	13.4536240	5.6566528
182	33124	6028568	13.4907376	5.6670511
183	33489	6128487	13.5277493	5.6774114
184	33856	6229504	13.5646600	5.6877340
185	34225	6331625	13.6014705	5.6980192
186	34596	6434856	13.6381817	5.7082675
187	34969	6539203	13.6747943	5.7184791
188	35344	6644672	13.7113092	5.7286543
189	35721	6751269	13.7477271	5.7387936
190	36100	6859000	13.7840488	5.7488971
191	36481	6967871	13.8202750	5.7589652
192	36864	7077888	13.8564065	5.7689982
193	37249	7189057	13.8924400	5.7789966
194	37636	7301384	13.9283883	5.7889604
195	38025	7414875	13.9642400	5.7988900
196	38416	7529536	14.0000000	5.8087857
197	38809	7645373	14.0356688	5.8186479
198	39204	7762392	14.0712473	5.8284867
199	39601	7880599	14.1067360	5.8382725
200	40000	8000000	14.1421356	5.8480355
201	40401	8120601	14.1774469	5.8577660
202	40804	8242408	14.2126704	5·8674673
203	41209	8365427	14.2478068	5.8771307
204	41616	8489664	14.2828569	5.8867653
205	42025	8615125	14.3178211	5.8963685
206	42436	8741816	14.3527001	5.9059406
207	42849	8869743	14.3874946	5.9154817
208	43264	8998912	14.4222051	5.9249921
209	43681	9129329	14.4568323	5.9344721
210	44100	9261000	14.4913767	5.9439220

Number.	Square.	Cube.	Square Root.	Cube Root.
211	44521	9393931	14.5258390	5.9533418
212	44944	9528128	14.5602198	5.9627320
213	45369	9663597	14.5945195	5.9720926
214	45796	9800344	14.6287388	5.9814241
215	46225	9938375	14.6628783	5.9907264
216	46656	10077696	14.6969385	6.0000000
217	47089	10218313	14.7309199	6.0092450
218	47524	10360232	14.7648231	6.0184617
219	47961	10503459	14.7986486	6.0276502
220	48400	10648000	14.8323970	6.0368107
221	48841	10793861	14.8660687	6.0459435
222	49284	10941048	14.8996644	6.0550489
223	49729	11089567	14.9331845	6.0641270
224	50176	11239424	14.9666295	6.0731779
225	50625	11390625	15.0000000	6.0822020
226	51076	11543176	15.0332964	6.0911994
227	51529	11697083	15.0665192	6.1001702
228	51984	11852352	15.0996689	6.1091147
229	52441	12008989	15.1327460	6.1180332
230	52900	12167000	15.1657509	6.1269257
231	53361	12326391	15.1986842	6.1357924
232	53824	12487168	15.2315462	6.1446337
233	54289	12649337	15.2643375	6.1534495
234	54756	12812904	15.2970585	6.1622401
235	55225	12977875	15.3297097	6.1710058
236	55696	13144256	15.3622915	6.1797466
237	56169	13312053	15.3948043	6.1884628
238	56644	13481272	15.4272486	6.1971544
239	57121	13651919	15.4596248	6.2058218
240	57600	13824000	15.4919334	6.2144650
241	58081	13997521	15.5241747	6.2230843
242	58564	14172488	15.5563492	6.2316797
243	59049	14348907	15.5884573	6.2402515
244	59536	14526784	15.6204994	6.2487998
245	60025	14706125	15.6524758	6.2573248
246	60516	14886936	15.6843871	6.2658266
247	61009	15069223	15.7162336	6.2743054
248	61504	15252992	15.7480157	6.2827613
249	62001	15438249	15.7797338	6.2911946
250	62500	15625000	15.8113883	6.2996053
251	63001	15813251	15.8429795	6.3079935
252	63504	16003008	15.8745079	6.3163596

Number.	Square.	Cube.	Square Root.	Cube Root.
253	64009	16194277	15.9059737	6.3247035
254	64516	16387064	15.9373775	6.3330256
255	65025	16581375	15.9687194	6.3413257
256	65536	16777216	16.0000000	6.3496042
257	66049	16974593	16.0312195	6.3578611
258	66564	17173512	16.0623784	6.3660968
259	67081	17373979	16.0934769	6.3743111
260	67600	17576000	16.1245155	6.3825043
261	68121	17779581	16.1554944	6.3906765
262	68644	17984728	16.1864141	6.3988279
263	69169	18191447	16.2172747	6.4069585
264	69696	18399744	16.2480768	6.4150687
265	70225	18609625	16.2788206	6.4231583
266	70756	18821096	16.3095064	6.4312276
267	71289	19034163	16.3401346	6.4392767
268	71824	19248832	16.3707055	6.4473057
269	72361	19465109	16.4012195	6.4553148
270	72900	19683000	16.4316767	6.4633041
271	73441	19902511	16.4620776	6.4712736
272	73984	20123648	16.4924225	6.4792236
273	74529	20346417	16.5227116	6.4871541
274	75076	20570824	16.5529454	6.4950653
275	75625	20796875	16.5831240	6.5029572
276	76176	21024576	16.6132477	6.5108300
277	76729	21253933	16.6433170	6.5186839
278	77284	21484952	16.6783320	6.5265189
279	77841	21717639	16.7032931	6.5343351
280	78400	21952000	16.7332005	6.5421326
281	78961	22188041	16.7630546	6.5499116
282	79524	22425768	16.7928556	6.5576722
283	80089	22665187	16.8226038	6.5654144
284	80656	22906304	16.8522995	6.5731385
285	81225	23149125	16.8819430	6.5808443
286	81796	23393656	16.9115345	6.5885323
287	82369	23639903	16.9410743	6.5962023
288	82944	23887872	16.9705627	6.6038545
289	83521	24137569	17.0000000	6.6114890
290	84100	24389000	17.0293864	6.6191060
291	84681	24642171	17.0587221	6.6267054
292	85264	24897088	17.0880075	6.6342874
293	85849	25153757	17.1172428	6.6418522
294	86436	25412184	17.1464282	6.6493998

Number.	Square.	Cube.	Square Root.	Cube Root.
295	87025	25672375	17.1755640	6.6569302
296	87616	25934336	17.2046505	6.6644437
297	88209	26198073	17.2336879	6.6719403
298	88804	26463592	17.2626765	6.6794200
299	89401	26730899	17.2916165	6.6868831
300	90000	27000000	17.3205081	6.6943295
301	90601	27270901	17.3493516	6.7017593
302	91204	27543608	17.3781472	6.7091729
303	91809	27818127	17.4068952	6.7165700
304	92416	28094464	17.4355958	6.7239508
305	93025	28372625	17.4642492	6.7313155
306	93636	28652616	17.4928557	6.7386643
307	94249	28934443	17.5214155	6.7459967
308	94864	29218112	17.5499288	6.7533134
309	95481	29503629	17.5783958	6.7606143
310	96100	29791000	17.6068169	6.7678995
311	96721	30080231	17.6351921	6.7751690
312	97344	30371328	17.6635217	6.7824229
313	97969	30664297	17.6918060	6.7896613
314	98596	30959144	17.7200451	6.7968844
315	99225	31255875	17.7482393	6.8040921
316	99856	31554496	17.7763888	6.8112847
317	100489	31855013	17.8044938	6.8184620
318	101124	32157432	17.8325545	6.8256242
319	101761	32461759	17.8605711	6.8327714
320	102400	32768000	17.8885438	6.8399037
321	103041	33076161	17.9164729	6.8470213
322	103684	33386248	17.9443584	6.8541240
323	104329	33698267	17.9722008	6.8612120
324	104976	34012224	18.0000000	6.8682855
325	105625	34328125	18.0277564	6.8753433
326	106276	34645976	18.0554701	6.8823888
327	106929	34965783	18.0831413	6.8894188
328	107584	35287552	18.1107703	6.8964345
329	108241	35611289	18.1383571	6.9034359
330	108900	35937000	18.1659021	6.9104232
331	109561	36264691	18.1934054	6.9173964
332	110224	36594368	18.2208672	6.9243556
333	110889	36926037	18.2482876	6.9313088
334	111556	37259704	18.2756669	6.9382321
335	112225	37595375	18.3030052	6.9451496
336	112896	37933056	18.3303028	6.9520533

Number.	Square.	Cube.	Square Root.	Cube Root.
337	113569	38272753	18.3575598	6.9589434
338	114244	38614472	18.3847763	6.9658198
339	114921	38958219	18.4119526	6.9726826
340	115600	39304000	18.4390889	6.9795321
341	116281	39651821	18.4661853	6.9863681
342	116964	40001688	18.4932420	6.9931906
343	117649	40353607	18.5202592	7.0000000
344	118336	40707584	18.5472370	7.0067962
345	119025	41063625	18.5741756	7.0135791
346	119716	41421736	18.6010752	7.0203490
347	120409	41781923	18.6279360	7.0271058
348	121104	42144192	18.6547581	7.0338497
349	121801	42508549	18.6815417	7.0405860
350	122500	42875000	18.7082869	7.0472987
351	123201	43243551	18.7349940	7.0540041
352	123904	43614208	18.7616630	7.0606967
353	124609	43986977	18.7882942	7.0673767
354	125316	44361864	18.8148877	7.0740440
355	126025	44738875	18.8414437	7.0806988
356	126736	45118016	18.8679623	7.0873411
357	127449	45499293	18.8944436	7.0939709
358	128164	45882712	18.9208879	7.1005885
359	128881	46268279	18.9472953	7.1071937
360	129600	46656000	18.9736660	7.1137866
361	130321	47045881	19.0000000	7.1203674
362	131044	47437928	19.0262976	7.1269360
363	131769	47832147	19.0525589	7.1334925
364	132496	48228544	19.0787840	7.1400370
365	133225	48627125	19.1049732	7.1465695
366	133956	49027896	19.1311265	7.1530901
367	134689	49430863	19.1572441	7.1595988
368	135424	49836032	19.1833261	7.1660957
369	136161	50243409	19.2093727	7.1725809
370	136900	50653000	19.2353841	7.1790544
371	137641	51064811	19.2613603	7.1855162
372	138384	51478848	19.2873015	7.1919663
373	139129	51895117	19.3132079	7.1984050
374	139876	52313624	19.3390796	7.2048322
375	140625	52734375	19.3649167	7.2112478
376	141376	53157376	19.3907194	7.2176522
377	142129	53582633	19.4164878	7.2240450
378	142884	54010152	19.4422221	7.2304268

Number.	Square.	Cube.	Square Root.	Cube Root.
379	143641	54439939	19.4679223	7.2367972
380	144400	54872000	19.4935887	7.2431565
381	145161	55306341	19.5192213	7.2495045
382	145924	55742968	19.5448203	7.2558415
383	146689	56181887	19.5703858	7.2621675
384	147456	56623104	19.5959179	7.2684824
385	148225	57066625	19.6214169	7.2747864
386	148996	57512456	19.6468827	7.2810794
387	149769	57960603	19.6723156	7.2873617
388	150544	58411072	19.6977156	7.2936330
389	151321	58863869	19.7230829	7.2998936
390	152100	59319000	19.7484177	7.3061436
391	152881	59776471	19.7737199	7.3123828
392	153664	60236288	19.7989899	7.3186114
393	154449	60698457	19.8242276	7.3248295
394	155236	61162984	19.8494332	7.3310369
395	156025	61629875	19.8746069	7.3372339
396	156816	62099136	19.8997487	7.3434205
397	157609	62570773	19.9248588	7.3495966
398	158404	63044792	19.9499373	7.3557624
399	159201	63521199	19.9749844	7.3619178
400	160000	64000000	20.0000000	7.3680630
401	160801	64481201	20.0249844	7.3741979
402	161604	64964808	20.0499377	7.3803227
403	162409	65450827	20.0748599	7.3864373
404	163216	65939264	20.0997512	7.3925418
405	164025	66430125	20.1246118	7.3986363
406	164836	66923416	20.1494417	7.4047206
407	165649	67419143	20.1742410	7.4107950
408	166464	67917312	20.1990099	7.4168595
409	167281	68417929	20.2237484	7.4229142
410	168100	68921000	20.2484567	7.4289589
411	168921	69426531	20.2731349	7.4349938
412	169744	69934528	20.2977831	7.4410189
413	170569	70444997	20.3224014	7.4470343
414	171396	70957944	20.3469899	7.4530399
415	172225	71473375	20.3715488	7.4590359
416	173056	71991296	20.3960781	7.4650223
417	173889	72511713	20.4205779	7.4709991
418	174724	73034632	20.4450483	7.4769664
419	175561	73560059	20.4694895	7.4829242
420	176400	74088000	20.4939015	7.4888724

Number.	Square.	Cube.	Square Root.	Cube Root.
421	177241	74618461	20.5182845	7.4948113
422	178084	75151448	20.5426386	7.5007406
423	178929	75686967	20.5669638	7.5066607
424	179776	76225024	20.5912603	7.5125715
425	180625	76765625	20.6155281	7.5184730
426	181476	77308776	20.6397674	7.5243652
427	182329	77854483	20.6639783	7.5302482
428	183184	78402752	20.6881609	7.5361221
429	184041	78953589	20.7123152	7.5419867
430	184900	79507000	20.7364414	7.5478423
431	185761	80062991	20.7605395	7.5536888
432	186624	80621568	20.7846097	7.5595263
433	187489	81182737	20.8086520	7.5653548
434	188356	81746504	20.8326667	7.5711743
435	189225	82312875	20.8566536	7.5769849
436	190096	82881856	20.8806130	7.5827865
437	190969	83453453	20.9045450	7.5885793
438	191844	84027672	20.9284495	7.5943633
439	192721	84604519	20.9523268	7.6001385
440	193600	85184000	20.9761770	7.6059049
441	194481	85766121	21.0000000	7.6116626
442	195364	86350888	21.0237960	7.6174116
443	196249	86938307	21.0475652	7.6231519
444	197136	87528384	21.0713075	7.6288837
445	198025	88121125	21.0950231	7.6346067
446	198916	88716536	21.1187121	7.6403213
447	199809	89314623	21.1423745	7.6460272
448	200704	89915392	21.1660105	7.6517247
449	201601	90518849	21.1896201	7.6574138
450	202500	91125000	21.2132034	7.6630943
451	203401	91733851	21.2367606	7.6687665
452	204304	92345408	21.2602916	7.6744303
453	205209	92959677	21.2837967	7.6800857
454	206116	93576664	21.3072758	7.6857328
455	207025	94196375	21.3307290	7.6913717
456	207936	94818816	21.3541565	7.6970023
457	208849	95443993	21.3775583	7.7026246
458	209764	96071912	21.4009346	7.7082388
459	210681	96702579	21.4242853	7.7138448
460	211600	97336000	21.4476106	7.7194426
461	212521	97972181	21.4709106	7.7250325
462	213444	98611128	21.4941853	7.7306141

Number.	Square.	Cube.	Square Root.	Cube Root.
463	214369	99252847	21.5174348	7.7361877
464	215296	99897344	21.5406592	7.7417532
465	216225	100544625	21.5638587	7.7473109
466	217156	101194696	21.5870331	7.7528606
467	218089	101847563	21.6101828	7.7584023
468	219024	102503232	21.6333077	7.7639361
469	219961	103161709	21.6564078	7.7694620
470	220900	103823000	21.6794834	7.7749801
471	221841	104487111	21.7025344	7.7804904
472	222784	105154048	21.7255610	7.7859928
473	223729	105823817	21.7485632	7.7914875
474	224676	106496424	21.7715411	7.7969745
475	225625	107171875	21.7944947	7.8024538
476	226576	107850176	21.8174242	7.8079254
477	227529	108531333	21.8403297	7.8133892
478	228484	109215352	21.8632111	7.8188456
479	229441	109902239	21.8860686	7.8242942
480	230400	110592000	21.9089023	7.8297353
481	231361	111284641	21.9317122	7.8351688
482	232324	111980168	21.9544984	7.8405949
483	233289	112678587	21.9772610	7.8460134
484	234256	113379904	22.0000000	7.8514244
485	235225	114084125	22.0227155	7.8568281
486	236196	114791256	22.0454077	7.8622242
487	237169	115501303	22.0680765	7.8676130
488	238144	116214272	22.0907220	7.8729944
489	239121	116930169	22.1133444	7.8783684
490	240100	117649000	22.1359436	7.8837352
491	241081	118370771	22.1585198	7.8890946
492	242064	119095488	22.1810730	7.8944468
493	243049	119823157	22.2036033	7.8997917
494	244036	120553784	22.2261108	7.9051294
495	245025	121287375	22.2485955	7.9104599
496	246016	122023936	22.2710575	7.9157832
497	247009	122763473	22.2934968	7.9210994
498	248004	123505992	22.3159136	7.9264085
499	249001	124251499	22.3383079	7.9317104
500	250000	125000000	22.3606798	7.9370053
501	251001	125751501	22.3830293	7.9422931
502	252004	126506008	22.4053565	7.9475739
503	253009	127263527	22.4276615	7.9528477
504	254016	128024064	22.4499443	7.9581144

Number.	Square.	Cube.	Square Root.	Cube Root.
505	255025	128787625	22.4722051	7.9633743
506	256036	129554216	22.4944438	7.9686271
507	257049	130323843	22.5166605	7.9738731
508	258064	131096512	22.5388553	7.9791122
509	259081	131872229	22.5610283	7.9843444
510	260100	132651000	22.5831796	7.9895697
511	261121	133432831	22.6053091	7.9947883
512	262144	134217728	22.6274170	8.0000000
513	263169	135005697	22.6495033	8.0052049
514	264196	135796744	22.6715681	8.0104032
515	265225	136590875	22.6936114	8.0155946
516	266256	137388096	22.7156334	8.0207794
517	267289	138188413	22.7376341	8.0259574
518	268324	138991832	22.7596134	8.0311287
519	269361	139798359	22.7815715	8.0362935
520	270400	140608000	22.8035085	8.0414515
521	271441	141420761	22.8254244	8.0466030
522	272484	142236648	22.8473193	8.0517479
523	273529	143055667	22.8691933	8.0568862
524	274576	143877824	22.8910463	8.0620180
525	275625	144703125	22.9128785	8.0671432
526	276676	145531576	22.9346899	8.0722620
527	277729	146363183	22.9564806	8.0773743
528	278784	147197952	22.9782506	8.0824800
529	279841	148035889	23.0000000	8.0875794
530	280900	148877000	23.0217289	8.0926723
531	281961	149721291	23.0434372	8.0977589
532	283024	150568768	23.0651252	8.1028390
533	284089	151419437	23.0867928	8.1079128
534	285156	152273304	23.1084400	8.1129803
535	286225	153130375	23.1300670	8.1180414
536	287296	153990656	23.1516738	8.1230962
537	288369	154854153	23.1732605	8.1281447
538	289444	155720872	23.1948270	8.1331870
539	290521	156590819	23.2163735	8.1382230
540	291600	157464000	23.2379001	8.1432529
541	292681	158340421	23.2594067	8.1482765
542	293764	159220088	23.2808935	8.1532939
543	294849	160103007	23.3023604	8.1583051
544	295936	160989184	23.3238076	8.1633102
545	297025	161878625	23.3452351	8.1683092
546	298116	162771336	23.3666429	8.1733020

Number.	Square.	Cube.	Square Root.	Cube Root.
547	299209	163667323	23.3880311	8.1782888
548	300304	164566592	23.4093998	8.1832695
549	301401	165469149	23.4307490	8.1882441
550	302500	166375000	23.4520788	8.1932127
551	303601	167284151	23.4733892	8.1981753
552	304704	168196608	23.4946802	8.2031319
553	305809	169112377	23.5159520	8.2080825
554	306916	170031464	23.5372046	8.2130271
555	308025	170953875	23.5584380	8.2179657
556	309136	171879616	23.5796522	8.2228985
557	310249	172808693	23.6008474	8.2278254
558	311364	173741112	23.6220236	8.2327463
559	312481	174676879	23.6431808	8.2376614
560	313600	175616000	23.6643191	8.2425706
561	314721	176558481	23.6854386	8.2474740
562	315844	177504328	23.7065392	8.2523715
563	316969	178453547	23.7276210	8.2572635
564	318096	179406144	23.7486842	8.2621492
565	319225	180362125	23.7697286	8.2670294
566	320356	181321496	23.7907545	8.2719039
567	321489	182284263	23.8117618	8.2767726
568	322624	183250432	23.8327506	8.2816255
569	323761	184220009	23.8537209	8.2864928
570	324900	185193000	23.8746728	8.2913444
571	326041	186169411	23.8956063	8.2961903
572	327184	187149248	23.9165215	8.3010304
573	328329	188132517	23.9374184	8.3058651
574	329476	189119224	23.9582971	8.3106941
575	330625	190109375	23.9791576	8.3155175
576	331776	191102976	24.0000000	8.3203353
577	332929	192100033	24.0208243	8.3251475
578	334084	193100552	24.0416306	8.3299542
579	335241	194104539	24.0624188	8.3347553
580	336400	195112000	24.0831891	8.3395509
581	337561	196122941	24.1039416	8.3443410
582	338724	197137368	22.1246762	8.3491256
583	339889	198155287	24.1453929	8.3539047
584	341056	199176704	24.1660919	8.3586784
585	342225	200201625	24.1867732	8.3634466
586	343396	201230056	24.2074369	8.3682095
587	344569	202262003	24.2280829	8.3729668
588	345744	203297472	24.2487113	8.3777188

Number.	Square.	Cube.	Square Root.	Cube Root.
589	346921	204336469	24.2693222	8.3824653
590	348100	205379000	24.2899156	8.3872065
591	349281	206425071	24.3104996	8.3919428
592	350464	207474688	24.3310501	8.3966729
593	351649	208527857	24.3515913	8.4013981
594	352836	209584584	24.3721152	8.4061180
595	354025	210644875	24.3926218	8.4108326
596	355216	211708736	24.4131112	8.4155419
597	356409	212776173	24.4335834	8.4202460
598	357604	213847192	24.4540385	8.4249448
599	358801	214921799	24.4744765	8.4296383
600	360000	216000000	24.4948974	8.4343267
601	361201	217081801	24.5153013	8.4390098
602	362404	218167208	24.5356883	8.4436877
603	363609	219256227	24.5560583	8.4483605
604	364816	220348864	24.5764115	8.4530280
605	366025	221445125	24.5967478	8.4576906
606	367236	222545016	24.6170673	8.4623479
607	368449	223648543	24.6373700	8.4670001
608	369664	224755712	24.6576560	8.4716471
609	370881	225866529	24.6779254	8.4762892
610	372100	226981000	24.6981781	8.4809261
611	373321	228099131	24.7184142	8.4855579
612	374544	229220928	24.7386338	8.4901848
613	375769	230346397	24.7588368	8.4948065
614	376996	231475544	24.7790234	8.4994233
615	378225	232608375	24.7991935	8.5040350
616	379456	233744896	24.8193473	8.5086417
617	380689	234885113	24.8394847	8.5132435
618	381924	236029032	24.8596058	8.5178403
619	383161	237176659	24.8797106	8.5224331
620	384400	238328000	24.8997992	8.5270189
621	385641	239483061	24.9198716	8.5316009
622	386884	240641848	24.9399278	8.5361780
623	388129	241804367	24.9599679	8.5407501
624	389376	242970624	24.9799920	8.5453173
625	390625	244140625	25.0000000	8.5498797
626	391876	245314376	25.0199920	8.5544372
627	393129	246491883	25.0399681	8.5589899
628	394384	247673152	25.0599282	8.5635377
629	395641	248858189	25.0798724	8.5680807
630	396900	250047000	25.0998008	8.5726189

Number.	Square.	Cube.	Square Root.	Cube Root.
631	398161	251239591	25.1197134	8.5771523
632	399424	252435968	25.1396102	8.5816809
633	400689	253636137	25.1594913	8.5862047
634	401956	254840104	25.1793566	8.5907238
635	403225	256047875	25.1992063	8.5952380
636	404496	257259456	25.2190404	8.5997476
637	405769	258474853	25.2388589	8.6042525
638	407044	259694072	25.2586619	8.6087526
639	408321	260917119	25.2784493	8.6132480
640	409600	262144000	25.2982213	8.6177388
641	410881	263374721	25.3179778	8.6222248
642	412164	264609288	25.3377189	8.6267063
643	413449	265847707	25.3574447	8.6311830
644	414736	267089984	25.3771551	8.6356551
645	416025	268336125	25.3968502	8.6401226
646	417316	269586136	25.4165302	8.6445855
647	418609	270840023	25.4361947	8.6490437
648	419904	272097792	25.4558441	8.6534974
649	421201	273359449	25.4754784	8.6579465
650	422500	274625000	25.4950976	8.6623911
651	423801	275894451	25.5147013	8.6668310
652	425104	277167808	25.5342907	8.6712665
653	426409	278445077	25.5538647	8.6756974
654	427716	279726264	25.5734237	8.6801237
655	429025	281011375	25.5929678	8.6845456
656	430336	282300416	25.6124969	8.6889630
657	431649	283593393	25.6320112	8.6933759
658	432964	284890312	25.6515107	8.6977843
659	434281	286191179	25.6709953	8.7021882
660	435600	287496000	25.6904652	8.7065877
661	436921	288804781	25.7099203	8.7109827
662	438244	290117528	25.7293607	8.7153734
663	439569	291434247	25.7487864	8.7197596
664	440896	292754944	25.7681975	8.7241414
665	442225	294079625	25.7875939	8.7285187
666	443556	295408296	25.8069758	8.7328918
667	444889	296740963	25.8263431	8.7372604
668	446224	298077632	25.8456960	8.7416246
669	447561	299418309	25.8650343	8.7459846
670	448900	300763000	25.8843582	8.7503401
671	450241	302111711	25.9036677	8.7546913
672	451584	303464448	25.9229628	8.7590383

Number.	Square.	Cube.	Square Root.	Cube Root.
673	452929	304821217	25.9422435	8.7633809
674	454276	306182024	25.9615100	8.7677192
675	455625	307546875	25.9807621	8.7720532
676	456976	308915776	26.0000000	8.7763830
677	458329	310288733	26.0192237	8.7807084
678	459684	311665752	26.0384331	8.7850296
679	461041	313046839	26.0576284	8.7893466
680	462400	314432000	26.0768096	8.7936593
681	463761	315821241	26.0959767	8.7979679
682	465124	317214568	26.1151297	8.8022721
683	466489	318611987	26.1342687	8.8065722
684	467856	320013504	26.1533937	8.8108681
685	469225	321419125	26.1725047	8.8151598
686	470596	322828856	26.1916017	8.8194474
687	471969	324242703	26.2106848	8.8237307
688	473344	325660672	26.2297541	8.8280099
689	474721	327082769	26.2488095	8.8322850
690	476100	328509000	26.2678511	8.8365559
691	477481	329939371	26.2868789	8.8408227
692	478864	331373888	26.3058929	8.8450854
693	480249	332812557	26.3248932	8.8493440
694	481636	334255384	26.3438797	8.8535985
695	483025	335702375	26.3628527	8.8578489
696	484416	337153536	26.3818119	8.8620952
697	485809	338608873	26.4007576	8.8663375
698	487204	340068392	26.4196896	8.8705757
699	488601	341532099	26.4386081	8.8748099
700	490000	343000000	26.4575131	8.8790400
701	491401	344472101	26.4764046	8.8832661
702	492804	345948408	26.4952826	8.8874882
703	494209	347428927	26.5141472	8.8917063
704	495616	348913664	26.5329983	8.8959204
705	497025	350402625	26.5518361	8.9001304
706	498436	351895816	26.5706605	8.9043366
707	499849	353393243	26.5894716	8.9085387
708	501264	354894912	26.6082694	8.9127369
709	502681	356400829	26.6270539	8.9169311
710	504100	357911000	26.6458252	8.9211214
711	505521	359425431	26.6645833	8.9253078
712	506944	360944128	26.6833281	8.9294902
713	508369	362467097	26.7020598	8.9336687
714	509796	363994344	26.7207784	8.9378433

Number.	Square.	Cube.	Square Root.	Cube Root.
715	511225	365525875	26.7394839	8.9420140
716	512656	367061696	26.7581763	8.9461809
717	514089	368601813	26.7768557	8.9503438
718	515524	370146232	26.7955220	8.9545029
719	516961	371694959	26.8141754	8.9586581
720	518400	373248000	26.8328157	8.9628095
721	519841	374805361	26.8514432	8.9669570
722	521284	376367048	26.8700577	8.9711007
723	522729	377933067	26.8886593	8.9752406
724	524176	379503424	26.9072481	8.9793766
725	525625	381078125	26.9258240	8.9835089
726	527076	382657176	26.9443872	8.9876373
727	528529	384240583	26.9629375	8.9917620
728	529984	385828352	26.9814751	8.9958899
729	531441	387420489	27.0000000	9.0000000
730	532900	389017000	27.0185122	9.0041134
731	534361	390617891	27.0370117	9.0082229
732	535824	392223168	27.0554985	9.0123288
733	537289	393832837	27.0739727	9.0164309
734	538756	395446904	27.0924344	9.0205293
735	540225	397065375	27.1108834	9.0246239
736	541696	398688256	27.1293199	9.0287149
737	543169	400315553	27.1477149	9.0328021
738	544644	401947272	27.1661554	9.0368857
739	546121	403583419	27.1845544	9.0409655
740	547600	405224000	27.2029140	9.0450419
741	549081	406869021	27.2213152	9.0491142
742	550564	408518488	27.2396769	9.0531831
743	552049	410172407	27.2580263	9.0572482
744	553536	411830784	27.2763634	9.0613098
745	555025	413493625	27.2946881	9.0653677
746	556516	415160936	27.3130006	9.0694220
747	558009	416832723	27.3313007	9.0734726
748	559504	418508992	27.3495887	9.0775197
749	561001	420189749	27.3678644	9.0815631
750	562500	421875000	27.3861279	9.0856030
751	564001	423564751	27.4043792	9.0896392
752	565504	425259008	27.4226184	9.0936719
753	567009	426957777	27.4408455	9.0977010
754	568516	428661064	27.4590604	9.1017265
755	570025	430368875	27.4772633	9.1057485
756	571536	432081216	27.4954542	9.1097669

Number.	Square.	Cube.	Square Root.	Cube Root.
757	573049	433798093	27.5136330	9.1137818
758	574564	435519512	27.5317998	9.1177931
759	576081	437245479	27.5499546	9.1218010
760	577600	438976000	27.5680975	9.1258053
761	579121	440711081	27.5862284	9.1298061
762	580644	442450728	27.6043475	9.1338034
763	582169	444194947	27.6224546	9.1377971
764	583696	445943744	27.6405499	9.1417875
765	585225	447697125	27.6586334	9.1457742
766	586756	449455096	27.6767050	9.1497576
767	588289	451217663	27.6947648	9.1537375
768	589824	452984832	27.7128129	9.1577139
769	591361	454756609	27.7308492	9.1616869
770	592900	456533000	27.7488739	9.1656565
771	594441	458314011	27.7668868	9.1696225
772	595984	460099648	27.7848880	9.1735852
773	597529	461889917	27.8028775	9.1775445
774	599076	463684824	27.8208555	9.1815003
775	600625	465484375	27.8388218	9.1854527
776	602176	467288576	27.8567766	9.1894018
777	603729	469097433	27.8747197	9.1933474
778	605284	470910952	27.8926514	9.1972897
779	606841	472729139	27.9105715	9,2012286
780	608400	474552000	27.9284801	9.2051641
781	609961	476379541	27.9463772	9.2090962
782	611524	478211768	27.9642629	9.2130250
783	613089	480048687	27.9821372	9.2169505
784	614656	481890304	28.0000000	9.2208726
785	616225	483736625	28.0178515	9.2247914
786	617796	485587656	28.0356915	9.2287068
787	619369	487443403	28.0535203	9.2326189
788	620944	489303872	28.0713377	9.2365277
789	622521	491169069	28.0891438	9.2404333
790	624100	493039000	28.1069386	9.2443355
791	625681	494913671	28.1247222	9.2482344
792	627264	496793088	28.1424946	9.2521300
793	628849	498677257	28.1602557	9.2560224
794	630436	500566184	28.1780056	9.2599114
795	632025	502459875	28.1957444	9.2637973
796	633616	504358336	28.2134720	9.2676798
797	635209	506261573	28.2311884	9.2715592
798	636804	508169592	28.2488938	9.2754352

Number.	Square.	Cube.	Square Root.	Cube Root.
799	638401	510082399	28.2665881	9.2793081
800	640000	512000000	28.2842712	9.2831777
801	641601	513922401	28.3019434	9.2870444
802	643204	515849608	28.3196045	9.2909072
803	644809	517781627	28.3372546	9.2947671
804	646416	519718464	28.3548938	9.2986239
805	648025	521660125	28.3725219	9.3024775
806	649636	523606616	28.3901391	9.3063278
807	651249	525557943	28.4077454	9.3101750
808	652864	527514112	28.4253408	9.3140190
809	654481	529475129	28.4429253	9.3178599
810	656100	531441000	28.4604989	9.3216975
811	657721	533411731	28.4780617	9.3255320
812	659344	535387328	28.4956137	9.3293634
813	660969	537367797	28.5131549	9.3331916
814	662596	539353144	28.5306852	9.3370167
815	664225	541343375	28.5482048	9.3408386
816	665856	543338496	28.5657137	9.3446575
817	667489	545338513	28.5832119	9.3484731
818	669124	547343432	28.6006993	9.3522857
819	670761	549353259	28.6181760	9.3560952
820	672400	551368000	28.6356421	9.3599016
821	674041	553387661	28.6530976	9.3637049
822	675684	555412248	28.6705424	9.3675051
823	677329	557441767	28.6879716	9.3713022
824	678976	559476224	28.7054002	9.3750963
825	680625	561515625	28.7228132	9.3788873
826	682276	563559976	28.7402157	9.3826752
827	683929	565609283	28.7576077	9.3864600
828	685584	567663552	28.7749891	9.3902419
829	687241	569722789	28.7923601	9.3940206
830	688900	571787000	28.8097206	9.3977964
831	690561	573856191	28.8270706	9.4015691
832	692224	575930368	28.8444102	9.4053387
833	693889	578009537	28.8617394	9.4091054
834	695556	580093704	28.8790582	9.4128690
835	697225	582182875	28.8963666	9.4166297
836	698896	584277056	28.9136646	9.4203873
837	700569	586376253	28.9309523	9.4241420
838	702244	588480472	28.9482297	9.4278936
839	703921	590589719	28.9654967	9.4316423
840	705600	592704000	28.9827535	9.4353880

Number.	Square.	Cube.	Square Root.	Cube Root.
841	707281	594823321	29.0000000	9.4391307
842	708964	596947688	29.0172363	9.4428704
843	710649	599077107	29.0344623	9.4466072
844	712336	601211584	29.0516781	9.4503410
845	714025	603351125	29.0688837	9.4540719
846	715716	605495736	29.0860791	9.4577999
847	717409	607645423	29.1032644	9.4615249
848	719104	609800192	29.1204396	9.4652470
849	720801	611960049	29.1376046	9.4689661
850	722500	614125000	29.1547595	9.4726824
851	724201	616295051	29.1719043	9.4763957
852	725904	618470208	29.1890390	9.4801061
853	727609	620650477	29.2061637	9.4838136
854	729316	622835864	29.2232784	9.4875182
855	731025	625026375	29.2403830	9.4912200
856	732736	627222016	29.2574777	9.4949188
857	734449	629422793	29.2745623	9.4986147
858	736164	631628712	29.2916370	9.5023078
859	737881	633839779	29.3087018	9.5059980
860	739600	636056000	29.3257566	9.5096854
861	741321	638277381	29.3428015	9.5133699
862	743044	640503928	29.3598365	9.5170515
863	744769	642735647	29.3768616	9.5207303
864	746496	644972544	29.3938769	9.5244063
865	748225	647214625	29.4108823	9.5280794
866	749956	649461896	29.4278779	9.5317497
867	751689	651714363	29.4448637	9.5354172
868	753424	653972032	29.4618397	9.5390818
869	755161	656234909	29.4788059	9.5427437
870	756900	658503000	29.4957624	9.5464027
871	758641	660776311	29.5127091	9.5500589
872	760384	663054848	29.5296461	9.5537123
873	762129	665338617	29.5465734	9.5573630
874	763876	667627624	29.5634910	9.5610108
875	765625	669921875	29.5803989	9.5646559
876	767376	672221376	29.5972972	9.5682982
877	769129	674526133	29.6141858	9.5719377
878	770884	676836152	29.6310648	9.5755745
879	772641	679151439	29.6479342	9.5792085
880	774400	681472000	29.6647939	9.5828397
881	776161	683797841	29.6816442	9.5864682
882	777924	686128968	29.6984848	9.5900937

Number.	Square.	Cube.	Square Root.	Cube Root.
883	779689	688465387	29.7153159	9.5937169
884	781456	690807104	29.7321375	9.5973373
885	783225	693154125	29.7489496	9.6009548
886	784996	695506456	29.7657521	9.6045696
887	786769	697864103	29.7825452	9.6081817
888	788544	700227072	29.7993289	9.6117911
889	790321	702595369	29.8161030	9.6153977
890	792100	704969000	29.8328678	9.6190017
891	793881	707347971	29.8496231	9.6226030
892	795664	707932288	29.8663690	9.6262016
893	797449	712121957	29.8831056	9.6297975
894	799236	714516984	29.8998328	9.6333907
895	801025	716917375	29.9165506	9.6369812
896	802816	719323136	29.9332591	9.6405690
897	804609	721734273	29.9499583	9.6441542
898	806404	724150792	29.9666481	9.6477367
899	808201	726572699	29.9833287	9.6513166
900	810000	729000000	30.0000000	9.6548938
901	811801	731432701	30.0166621	9.6584684
902	813604	733870808	30.0333148	9.6620403
903	815409	736314327	30.0499584	9.6656096
904	817216	738763264	30.0665928	9.6691762
905	819025	741217625	30.0832179	9.6727403
906	820836	743677416	30.0998339	9.6763017
907	822649	746142643	30.1164407	9.6798604
908	824464	748613312	30.1330383	9.6834166
909	826281	751089429	30.1496269	9.6869701
910	828100	753571000	30.1662063	9.6905211
911	829921	756058031	30.1827765	9.6940694
912	831744	758550528	30.1993377	9.6976151
913	833569	761048497	30.2158899	9.7011583
914	835396	763551944	30.2324329	9.7046989
915	837225	766060875	30.2489669	9.7082369
916	839056	768575296	30.2654919	9.7117723
917	840889	771095213	30.2820079	9.7153051
918	842724	773620632	30.2985148	9.7188354
919	844561	776151559	30.3150128	9.7223631
920	846400	778688000	30.3315018	9.7258883
921	848241	781229961	30.3479818	9.7294109
922	850084	783777448	30.3644529	9.7329309
923	851929	786330467	30.3809151	9.7364484
924	853776	788889024	30.3973683	9.7399634

Number.	Square.	Cube.	Square Root.	Cube Root.
925	855625	791453125	30.4138127	9.7434758
926	857476	794022776	30.4302481	9.7469857
927	859329	796597983	30.4466747	9.7504930
928	861184	799178752	30.4630924	9.7539979
929	863041	801765089	30.4795013	9.7575002
930	864900	804357000	30.4959014	9.7610001
931	866761	806954491	30.5122926	9.7644974
932	868624	809557568	30.5286750	9.7679922
933	870489	812166237	30.5450487	9.7714845
934	872356	814780504	30.5614136	9.7749743
935	874225	817400375	30.5777697	9.7784616
936	876096	820025856	30.5941171	9.7829466
937	877969	822656953	30.6104557	9.7854288
938	879844	825293672	30.6267857	9.7889087
939	881721	827936019	30.6431069	9.7923861
940	883600	830584000	30.6594194	9.7958611
941	885481	833237621	30.6757233	9.7993336
942	887364	835896888	30.6920185	9.8028036
943	889249	838561807	30.7083051	9.8062711
944	891136	841232384	30.7245830	9.8097362
945	893025	843908625	30.7408523	9.8131989
946	894916	846590536	30.7571130	9.8166591
947	896808	849278123	30.7733651	9.8201169
948	898704	851971392	30.7896086	9.8235723
949	900601	854670349	30.8058436	9.8270252
950	902500	857375000	30.8220700	9.8304757
951	904401	860085351	30.8382879	9.8339238
952	906304	862801408	30.8544972	9.8373695
953	908209	865523177	30.8706981	9.8408127
954	910116	868250664	30.8868904	9.8442536
955	912025	870983875	30.9030743	9.8476920
956	913936	873722816	30.9192477	9.8511280
957	915849	876467493	30.9354166	9.8545617
958	917764	879217912	30.9515751	9.8579929
959	919681	881974079	30.9677251	9.8614218
960	921600	884736000	30.9838668	9.8648483
961	923521	887503681	31.0000000	9.8682724
962	925444	890277128	31.0161248	9.8716941
963	927369	893056347	31.0322413	9.8751135
964	929296	895841344	31.0483494	9.8785305
965	931225	898632125	31.0644491	9.8819451
966	933156	901428696	31.0805405	9.8853574

Number.	Square.	Cube.	Square Root.	Cube Root.
967	935089	904231063	31.0966236	9.8887673
968	937024	907039232	31.1126984	9.8921749
969	938961	909853209	31.1287648	9.8955801
970	940900	912673000	31.1448230	9.8989830
971	942841	915498611	31.1608729	9.9023835
972	944784	918330048	31.1769145	9.9057817
973	946729	921167317	31.1929479	9.9091776
974	948676	924010424	31.2089731	9.9125712
975	950625	926859375	31.2249900	9.9159624
976	952576	929714176	31.2409987	9.9193513
977	954529	932574833	31.2569992	9.9227379
978	956484	935441352	31.2729915	9.9261222
979	958441	938313739	31.2889757	9.9295042
980	960400	941192000	31.3049517	9.9328839
981	962361	944076141	31.3209195	9.9362613
982	964324	946966168	31.3368792	9.9396363
983	966289	949862087	31.3528308	9.9430092
984	968256	952763904	31.3687743	9.9463797
985	970225	955671625	31.3847097	9.9497479
986	972196	958585256	31.4006369	9.9531138
987	974169	961504803	31.4165561	9.9564775
988	976144	964430272	31.4324673	9.9598389
989	978121	967361669	31.4483704	9.9631981
990	980100	970299000	31.4642654	9.9665549
991	982081	973242271	31.4801525	9.9699095
992	984064	976191488	31.4960315	9.9732619
993	986049	979146657	31.5119025	9.9766120
994	988036	982107784	31.5277655	9.9799599
995	990025	985074875	31.5436206	9.9833055
996	992016	988047936	31.5594677	9.9866488
997	994009	991026973	31.5753068	9.9899900
998	996004	994011992	31.5911380	9.9933289
999	998001	997002999	31.6069613	9.9966656
1000	1000000	1000000000	31.6227766	10.0000000
1001	1002001	1003003001	31.6385840	10.0033222
1002	1004004	1006012008	31.6543836	10.0066622
1003	1006009	1009027027	31.6701752	10.0099899
1004	1008016	1012048064	31.6859590	10.0133155
1005	1010025	1015075125	31.7017349	10.0166389
1006	1012036	1018108216	31.7175030	10.0199601
1007	1014049	1021147343	31.7332633	10.0232791
1008	1016064	1024192512	31.7490157	10.0265958

Number.	Square.	Cube.	Square Root.	Cube Root.
1009	1018081	1027243729	31.7647603	10.0299104
1010	1020100	1030301000	31.7804972	10.0332228
1011	1022121	1033364331	31.7962262	10.0365330
1012	1024144	1036433728	31.8119474	10.0398410
1013	1026169	1039509197	31.8276609	10.0431469
1014	1028196	1042590744	31.8433666	10.0464506
1015	1030225	1045678375	31.8590646	10.0497521
1016	1032256	1048772096	31.8747549	10.0530514
1017	1034289	1051871913	31.8904374	10.0563485
1018	1036324	1054977832	31.9061123	10.0596435
1019	1038361	1058089859	31.9217794	10.0629364
1020	1040400	1061208000	31.9374388	10.0662271
1021	1042441	1064332261	31.9530906	10.0695156
1022	1044484	1067462648	31.9687347	10.0728020
1023	1046529	1070599167	31.9843712	10.0760863
1024	1048576	1073741824	32.0000000	10.0793684
1025	1050625	1076890625	32.0156212	10.0826484
1026	1052676	1080045576	32.0312348	10.0859262
1027	1054729	1083206683	32.0468407	10.0892019
1028	1056784	1086373952	32.0624391	10.0924755
1029	1058841	1089547389	32.0780298	10.0957469
1030	1060900	1092727000	32.0936131	10.0990163
1031	1062961	1095912791	32.1091887	10.1022835
1032	1065024	1099104768	32.1247568	10.1055487
1033	1067089	1102302937	32.1403173	10.1088117
1034	1069156	1105507304	32.1558704	10.1120726
1035	1071225	1108717875	32.1714159	10.1153314
1036	1073296	1111934656	32.1869539	10.1185882
1037	1075369	1115157653	32.2024844	10.1218428
1038	1077444	1118386872	32.2180074	10.1250953
1039	1079521	1121622319	32.2335229	10.1283457
1040	1081600	1124864000	32.2490310	10.1315941
1041	1083681	1128111921	32.2645316	10.1348403
1042	1085764	1131366088	32.2800248	10.1380845
1043	1087849	1134626507	32.2955105	10.1413266
1044	1089936	1137893184	32.3109888	10.1445667
1045	1092025	1141166125	32.3264598	10.1478047
1046	1094116	1144445336	32.3419233	10.1510406
1047	1096209	1147730823	32.3573794	10.1542744
1048	1098304	1151022592	32.3728281	10.1575062
1049	1100401	1154320649	32.3882695	10.1607359
1050	1102500	1157625000	32.4037035	10.1639636

NOTE.—Since all numbers are the square roots of their squares, and the cube roots of their cubes, it follows that the numbers tabulated are the square roots of their respective squares, and the cube roots of their respective cubes. Moreover, one-half, one-third, one-fourth, &c., of the square root of any number is the square root of one-fourth, one-ninth, one-sixteenth, &c., of that number; thus one-tenth of the square root of 150 is the square root of $150 \div 100 = \sqrt{1\frac{1}{2}}$, and one-fourth part of the square root of 125.72, is the square root of $125.72 \div 16 = \sqrt{7.8575}$, &c. But the process of extracting the square roots of numbers by arithmetic is so simple, and the labor so trifling, that but little will be gained, generally, by resorting to the tables, especially when the square root of a mixed nnmber is required.

By the help of the Table, to find the Cube Root of a number that is not the exact Cube of any number tabulated, and that is not greater than the Cube of one more than the highest number tabulated.

$n =$ given number whose cube root is required.
$t =$ tabular cube nearest the given number.
$b =$ tabular cube root of t.
$r =$ cube root of given number, or cube root required.

$$r = \frac{(2n + t)\, b}{2t + n} \text{ nearly.}$$

NOTE.—This formula expresses the cube root correct to within about 1-100 of a unit, under its most unfavorable application, and often affords almost strict accuracy, even when the maximum error obtains, if the given number be large. When two tabular cubes are equally near the given number, or very nearly so, use the greater, by which a closer approximate will be obtained, and r will be plus by the amount of the error.

By help of the Table, to find the Square Root or Cube Root of a mixed number, whose integer, and the integer next higher, are tabulated.

$n =$ given mixed number whose root is required.
$R =$ root of the integer next higher than the integer of the given number.
$i =$ integer of the given number.
$s =$ root of the integer of the given number.
$r =$ root of the given mixed number, or root required.

$r = (R - s)(n - i) + s$, practically correct for ordinary purposes.

EXAMPLE.—Required the square root and cube root of 18.54.

$\sqrt{19} = 4.3588989$	$\sqrt[3]{19} = 2.6684016$
$\sqrt{18} = 4.2426407$	$\sqrt[3]{18} = 2.6207414$
.1162582	.0476602
.54	.54
4650328	1906408
5812910	2383010
.062779428	.025736508
$\sqrt{18} = 4.2426407$	$\sqrt[3]{18} = 2.6207414$
$\sqrt{18.54} = 4.305420128$. *Ans.*	$\sqrt[3]{18.54} = 2.646477908$. *Ans.*

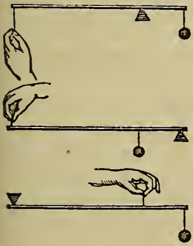
SECTION V.

MECHANICAL POWERS, CIRCULAR MOTION, &c.

The *Mechanical Powers* are the known elements of machinery. They are three in number, with some diversity of application. Strictly speaking, they are not POWERS, or sources of power; they simply convey applied force, and diffuse or concentrate it. In treating of them, the term *weight*, or *resistance*, is understood to be the force to be overcome, and the term *power*, the force applied to overcome or balance it. It is also to be understood that the deductions or conclusions arrived at are *theoretically* true; that is, that they are true upon the supposition that the whole power employed is expended to the end under consideration — that no friction or weight of machinery aids, or is to be overcome.

THE LEVER.

Lemma. — The power multiplied by its distance from the fulcrum equals the weight multiplied by its distance from the same point; and as the distance between the power and fulcrum is to the distance between the weight and fulcrum, so is the effect to the power.



Consequently, if we divide the weight by the power, we obtain a quotient equal the length of the longer arm of the lever, the length of the shorter arm being 1. And if we multiply the weight by its leverage, and divide the product by the power, we obtain a leverage for the power that will enable it to equipoise the weight. And if we multiply the power and its leverage together, and divide the product by the weight, we obtain for the weight the same result. So, too, if we divide the lever by the quotient obtained by dividing the weight by the power, to which quotient we have added 1, we obtain the relative position of the fulcrum, or the distance it must occupy from the opposing force. And, again, if we multiply the opposing force by its leverage, and divide the product by the leverage pertaining to the power, we obtain the requisite power to counterbalance the resistance.

EXAMPLE. — A weight of 1200 lbs., suspended 15 inches from the fulcrum, is to be raised by a power of 80 lbs.; at what distance from the fulcrum on the long arm of the lever must the power be applied, to accomplish that end?

$$80 : 1200 :: 1.25 : 18\tfrac{3}{4} \text{ feet.} \quad \textit{Ans.}$$

EXAMPLE. — The lever is 20 feet long, the opposing force 1200 lbs., and the available force 80 lbs.: at what distance from the former force must the fulcrum be placed, that the two forces may equipoise each other? $20 \times 80 \div (1200 + 80) = 1\frac{1}{4}$ feet; or

$$1200 \div 80 = 15, \text{ and } 20 \div \overline{15 + 1} = 1\tfrac{1}{4} \text{ feet. } \textit{Ans.}$$

EXAMPLE. — The longer arm of the lever is $18\frac{3}{4}$ feet, the shorter arm $1\frac{1}{4}$ feet, and the weight to be raised is 1200 lbs.; what power must be applied to raise it?

$$18.75 : 1.25 :: 1200 = 80 \text{ lbs. } \textit{Ans.}$$

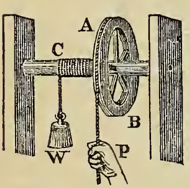
EXAMPLE. — A man, with a lever 5 feet in length, raised a weight of 2500 lbs. suspended across the lever 9 inches from the further end, which rested on a support; what force did the man exert?

$$5 : .75 :: 2500 = 375 \text{ lbs. } \textit{Ans.}$$

EXAMPLE. — A beam, 20 feet in length, supported at both ends and not elsewhere, bears a weight of 6000 lbs. placed 6 feet from one end; what is the pressure on each support?

20 : 14 :: 6000 : 4200 lbs. on the support nearest the weight. }
20 : 6 :: 6000 : 1800 lbs. on the support furthest from the w't. } *Ans.*

WHEEL AND AXLE.



The WHEEL and AXLE is a *revolving* lever. It partakes, in all respects, of the same principles as the preceding. The radius of the wheel is the longer arm of the lever, and the radius of the axle, the shorter. The fulcrum is the point of impact between them — at the circumference of the axle.

EXAMPLE. — The radius of the wheel is $2\frac{1}{2}$ feet, the radius of the axle is 9 inches, and the weight to be raised is 500 lbs.; the weight is attached to a rope wound round the axle; what power must be applied to the periphery of the wheel to raise it?

$$2.5 : .75 :: 500 : 150 \text{ lbs. } \textit{Ans.}$$

EXAMPLE. — The diameter of the wheel is 5 feet, the diameter of the axle or barrel, $1\frac{1}{2}$ feet, and the power is 150 lbs.; what weight may be raised?

$$1.5 : 5 :: 150 : 500 \text{ lbs. } \textit{Ans.}$$

EXAMPLE. — The power is 150 lbs., the resistance is 500 lbs., and the barrel has 9 inches radius; required the diameter of the wheel that will enable the power to equipoise the weight.

150 : 500 :: 9 : 30 in. radius, and $30 \times 2 = 60 \div 12 = 5$ feet. *Ans.*

EXAMPLE. — The length of the winch (crank) of a crane is 15 inches, the radius of the barrel around which the lifting chain coils is 3 inches, the pinion has 8 teeth, and the wheel 68; required the weight that a force of 30 lbs. applied to the winch will raise.

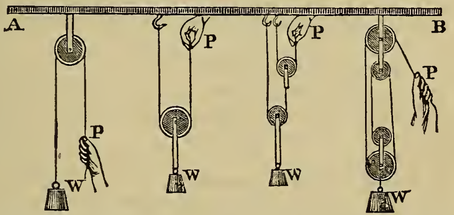
$68 \div 8 = 8\frac{1}{2}$ ($8\frac{1}{2}$ to 1) velocity of pinion to wheel, and
$15 \times 8.5 \div 3 = 42.5$ lbs. exertive force, or force to 1 of applied power — gained at the expense of space, and
42.5 lbs. $\times$ 30 lbs. (applied power) $=$ 1275 lbs. effective power. *Ans.*

Example. — The exertive force, or effect to power, of a crane, is to be as $42\frac{1}{2}$ to 1, the radius of the wheel to that of the pinion as $8\frac{1}{2}$ to 1, and the throw of the winch — its length — the radius of the circle which it describes — is to be $1\frac{1}{4}$ feet; what must be the diameter of the barrel?

$8.5 \times 1.25 = 10.625 \div 42.5 = .25 \times 2 = .5$ ft. or 6 in. *Ans.*

Note. — By additional wheels and pinions, as in the system of pulleys or block and tackle, which see, the exertive force of a crane may be increased to almost any conceivable extent; but always, as with the block and tackle, and as shown in the above example, at a relative expense to space.

THE PULLEY.



A *single pulley*, fixed and turning on its own axis, affords no mechanical advantage. It serves but to change the direction of the power.

In the common system of pulleys, or block and tackle, the advantage is as the number of ropes engaged in supporting the lower or rising block, to 1 of applied force.

Rule. — 1. Divide the given weight by the number of cords leading to, from, or attached to, the lower block, and the quotient is the requisite power to produce an equilibrium.

Rule. — 2. Multiply the given power by the number of cords leading to, from, or attached to the lower block, and the product is the weight that may be raised.

Rule. — 3. Divide the weight to be raised by the power to be applied, and the quotient is the requisite number of cords that must connect with the lower block.

Example. — The lower, running, or rising block has 5 sheaves or pulleys; the fixed or stationary has 4; and the weight to be raised is 2250 lbs. What force must be applied to raise it? Necessarily the end of the rope is attached to the lower block, therefore 9 ropes are attached to or connected with it; hence —

$$2250 \div 9 = 250 \text{ lbs. } \textit{Ans.}$$

NOTE. — In the *Spanish burton*, having two movable pulleys and two separate ropes, the effect is to the power as 5 to 1. In a system of 4 movable pulleys and 4 separate ropes, it is as 16 to 1. And in a system having 4 movable and 4 fixed pulleys, and 4 separate ropes, it is as 81 to 1.

INCLINED PLANE.

Lemma. — The product of the length of the plane and power is equal to the product of the height of the plane and weight.

The velocity, therefore, or force, or momentum, with which a body descends an inclined plane, impelled by its own gravity, is to that with which the same body would descend perpendicularly through space, as the height of the plane to its length, or as the size of its angle of inclination to radius. And the space the body describes upon the plane, in any given time, compared with that which it would describe falling freely, in the same time, is as its velocity upon the plane to that of perpendicular descent. And, the spaces being the same, the times will be inversely in that proportion.

The deductions, therefore, are —

1. That the product of the weight and height of plane, divided by the length of plane, gives the requisite power to sustain or balance the weight.
2. That the product of the power and length of plane, divided by the height of plane, (which reverses the former process,) gives the weight or resistance that the power will overcome.
3. That — (the times being equal) — the velocity attained, or force acquired, or space described, by a body falling freely from rest, multiplied by the height of plane, affords a product which, divided by the length of plane, gives the velocity attained, or force acquired, or space described, by a body moving down the plane, impelled by its own gravity.
4. That the product of the weight and base of plane, divided by the length of plane, gives the pressure on the plane.

To find the base of the plane.

RULE. — From the square of the length of the plane, subtract the square of the height, and the square root of the difference is the base.

To find the height of the plane.

RULE. — From the square of the length of the plane subtract the square of the base, and the square root of the difference is the height, *To find the length of the plane.*

Rule. — Add the square of the base and the square of the height together, and find the square root of the sum, which will be the length sought.

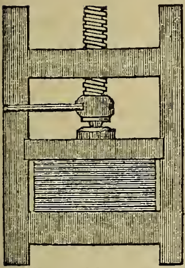
WEDGE.

The wedge is a *double* inclined plane. Its principles are the same, and they are wholly covered by the preceding.

The power multiplied by the length of a side, equals the resistance multiplied by half the breadth of the head.

When, therefore, both sides of the substance to be cleft are movable, the product of the resistance and half the breadth of the head, divided by the length of the side of the wedge, gives the requisite force to be applied. And when only one side of the substance is movable, the product of the resistance and breadth of the head, divided by the length of a side of the wedge, gives the power required.

SCREW.



If we take the figure of an inclined plane — a right-angled triangle say, cut from paper — and unite the extremities of the base, we have the figure of the screw, in principle; and the principle of the screw is that of an inclined plane curved to a cylinder; and the *screw* is not a mechanical power, any more than the *wedge*, or *wheel and axle*. It is the plane that is an element of machinery, and not the *curve* or the cylinder around which the plane is placed. And it appears that the *screw*, the *inclined plane*, and the *right-angled triangle*, are mathematically the same.

Thus, if we would find the length of the thread of a screw by the circumference and pitch, we are to find it as we would find the length of the inclined plane by the base and height, or the hypotenuse of a triangle by the base and perpendicular, and so, in like manner, for the other lines of the figure.

The *pitch* of the screw or *rise* of the thread in a revolution corresponds to the height of the plane or perpendicular of the triangle. The circumference of the screw corresponds to the base of the plane or base of the triangle. And the length of the thread making one revolution around the cylinder — the *working* circumference of the screw — corresponds to the length of the plane or hypotenuse of the triangle. The mechanical advantage of a screw is as the length of the plane to the size of its angle of inclination.

The ordinary *screw*, therefore, — the piece of mechanism, — is this plane repeated along a cylinder a greater or less number of times, whereby the spiral thread alluded to, now the sectional thread of the instrument, becomes the *continuous* thread, helix or spiral, that extends, by construction, at a uniform angle to the cylinder's axis, throughout its length. And it is to be borne in mind, always, that the pitch of a screw is the distance, parallel to the axis, between any two consecutive windings of the *same* apparent continuous thread, measured on its face, from centre to centre.

If a screw have more than one apparent continuous thread, therefore, — and screws are often constructed with two, and sometimes more — the pitch of such screw is still the same as it would be were but one such thread employed, or as it would be were all but one removed. Twelve rafters to the side of the roof of a building can sustain more *pressure* than four, and in the ratio of three to one, but the *pitch* of such roof is neither greater nor less in consequence of the *number* of rafters employed.

The screw multiplies the extent of the action of the inclined plane, it will be perceived, as many times as the plane is repeated along its cylinder, whereby great advantage of mechanical *application* is obtained; but the *mechanical advantage* of the screw, it is apparent is still with the plane, and not in the number of times the plane is used. The POWER is no more with the latter than it is with the *curve* of the plane (which is another advantage of mechanical application) instead of being in the plane itself.

And there is still another advantage pertaining to the screw, or to this mode of employing the inclined plane, viz., that the length of the plane or working circumference of the screw may be increased, in effect, to almost or quite any desired extent, by the employment of a simple bar or rod to turn the screw; whereby, the *power* and *pitch* remaining the same, the mechanical advantage is enhanced as much as the working circumference is increased; that is, it is made greater as much as the circumference of a circle, the bar being the radius thereof.

The *power*, *force*, or *mechanical advantage* of a screw, as we have said, is that of the inclined plane employed, and is as its length to the size of its angle of inclination, or it is as the *working* circumference of the screw to the *pitch*.

If we let P represent power,

L	"	length of lever,
l	"	length of inclined plane,
W	"	weight or pressure,
p	"	pitch of screw or angle of inclination,
r	"	radius of screw,
C	"	circumference described by power,
x	"	effect of power at circumference of screw.

Then we have —

$l : p :: W : P$	$W : l :: P : p$	$L : r :: x : P.$
$l : W :: p : P$	$r : L :: P : x$	$P : W :: p : C$
$P : W :: p : l$	$P : x :: r : L$	$C : p :: W : P$

EXAMPLE. — The circumference of a screw is 12 inches, its pitch $1\frac{1}{4}$ inches, and the power is 30 lbs.; what weight may be raised?

$$\sqrt{(12^2 + 1\tfrac{1}{4}^2)} = 12.065, \text{ working circumference of screw, and}$$

$$1.25 : 12.065 :: 30 : 289.56 \text{ lbs. } \textit{Ans.}$$

And if a bar 14 feet in length (rectilinear distance from the point on the bar at which the power is applied to the circumference of the screw) be employed to turn the screw, the power remaining the same, what weight may be raised?

$14 \times 12 \times 2 \times 3.1416 = 1056$ inches, circumference of circle, bar as radius; and

$1056 + 12.065 = 1068.065$ inches, circumference described by power; and

$1.25 : 1068.065 :: 30 : 25633$ lbs. *Ans.*

Or, $12.065 : 289.56 :: 1068.065 : 25633$ lbs. *Ans.*

The foregoing exhibits the method of finding the strict theoretical force or mechanical advantage of the screw. But for most practical purposes, more especially if we take into account the fact that the actual force in consequence of the friction is only about two-thirds that of the theoretical, the rectilinear distance from the point on the bar at which the power is applied, to the centre of the screw, may be taken as the radius of the circumference described by the power, or as the effective or working circumference of the screw, instead of the true working circumference as found above; and this is more especially true if the pitch of the screw be but slight or inconsiderable. Thus, in the aforementioned screw of 12 inches circumference and $1\frac{1}{4}$ inches pitch, the difference between the actual circumference and the working is only .065 of an inch, which, when the effect is of some magnitude, is of no particular account. The example next below is illustrative, and is given in proof of this position.

EXAMPLE. — The pitch of a screw is $1\frac{1}{4}$ inches, the power 30 lbs., and the rectilinear distance from the centre of the screw to the point on the bar at which the power is applied is 169.91 inches; required the weight that may be raised, supposing this rectilinear distance to be the radius of the circle described by the power.

$169.91 \times 2 \times 3.1416 = 1067.57$ inches, circumference by assumed radius; and

$1.25 : 1067.57 :: 30 : 25621$ lbs. *Ans.* And showing an error of only 12 lbs. in 25633 lbs., consequent upon having employed the assumed radius instead of the real, and that, too, under a pitch so unfavorable as the one supposed.

When a hollow screw revolves upon one of less diameter and pitch, the effect is the same as that of a single screw whose pitch is equal to the difference of the pitches of the two screws.

Thus, if a hollow screw of $\frac{1}{6}$ of an inch pitch revolves upon one of $\frac{1}{8}$ of an inch pitch, the power to the weight is as $\frac{1}{6} \backsim \frac{1}{8} = \frac{1}{24}$; that is, the power being 1, the weight will be 24.

A screw of this description and with these pitches, therefore, if turned with a bar 6 inches in length, (distance from the power to the centre of the screw,) will, in order to produce an equilibrium, require a power to the weight as 1 to $24 \times 2 \times 3.1416 \times 6 = 905$.

In a complex machine, composed of the screw and wheel and axle, the relations of the weight and power are as under: —

Let R represent radius of wheel.
r " radius of axle.
p " pitch of screw.
C " circumference described by power.
x " effect of power on the wheel.

Then —

$P \times C = x \times p$	$P \times C \times R = W \times p \times r$
$W \times r = R \times x$	$P : W :: p \times r : C \times R$
$P \times C \times x \times R = x \times W \times r \times p$	$p \times r : C \times R :: P : W$

And, if intermediate movers are inserted, (wheels and pinions, or drums and pulleys,) the same principles still apply.

Example. — The length of the crank (lever) which turns an endless screw, is 24 inches, and the pitch of the screw is $\frac{3}{4}$ of an inch; it turns a wheel of 30 inches radius, which turns a pinion of 7 inches radius, which turns a wheel of 22 inches radius, which turns an axle, around which the lifting chain winds, of three inches radius; what weight will a power of 50 lbs. applied to the crank raise?

$$\overset{p}{.75} \times \overset{r}{7} \times \overset{r}{3} : \overset{C}{150.8} \times \overset{R}{30} \times \overset{R}{22} :: \overset{P}{50} : \overset{W}{315962} \text{ lbs. } Ans.$$

Note. — It is clear, we may substitute the diameters of the wheels and pinions for their radii, if we prefer; or we may work by the number of teeth in each, in which latter case, the circumference of the axle, in the foregoing, would come in to be employed.

If the screw, acting upon the periphery of the wheel, have more than one thread, the real, or obvious pitch, spoken of above, must be taken, increased an equal number of times.

LATERAL OR TRANSVERSE STRENGTH OF BODIES.

The *transverse strength* of a body is its power to resist force or weight acting upon it in a direction perpendicular to its length.

Against each particular denomination of material in the following TABLE is placed the weight (mean of various experiments) required to break a solid, uniform bar, One Foot in Length and One Inch Square, of that material, the bar being fixed at one end and the weight suspended from the other, the action of the weight direct with the bar's sides.

The Woods of American growth, and seasoned.

Materials.	Breaking Weight in lbs.	Greatest Deflection in Inches.	Materials.	Breaking Weight in lbs.	Greatest Deflection in Inches.
Hickory,	270	8.	Pitch Pine,	225	
White Ash,	234	2.5	Yellow Pine,	150	1.70
White Oak,	220	9.	White Pine,	138	1.40
Chestnut,	170	1.7	Cast Iron,	684	0.63
Elm,	142		Wrought Iron,	1012	

About 600 lbs. suspended from the end of a square bar of wrought iron, of dimensions and fixed as supposed, causes the bar to deflect about one inch, at which it takes a permanent set, or bend.

As the weight written against any particular denomination of material, in the foregoing table, is the weight required to break that material, under the length, lateral figure, and condition supposed, it follows that the same weight may be taken as the constant or coefficient in determining the weight required to break the same denomination of material, under different lengths, lateral figures, relative conditions, &c.

C = tabular constant, or initial weight, above.
l = length of bar or beam in feet.
b = breadth of rectangular bar in inches.
v = vertical dimensions, or depth of rectangular bar in inches.
W = breaking weight, or ultimate transverse strength of bar under investigation.

1. *When the bar is fixed at one end, and the weight suspended from the other; the weight of the bar not being taken into account.*

$$l : v^2 \times b :: C : W.$$

2. *When the beam is supported* (not fixed) *at both ends, and the weight in the middle.*

$$l : bv^2 :: 4C : W.$$

3. *Fixed at both ends, and the load in the middle.*

$$l : bv^2 :: 6C : W.$$

4. *Fixed at one end, and the load distributed uniformly over its whole length.*

$$l : v^2b :: 2C : W.$$

5. *Supported at both ends, and load distributed uniformly over whole length.*

$$l : v^2b :: 8C : W.$$

6. *Supported at both ends, and load at the distance* m *from one end.*

$$m \times (l - m) : v^2bl :: C : W.$$

Or, $2(l - m) \times 2(l - n) \div l = l^1 =$ effective length, and

$$l^1 : bv^2 :: 4C : W.$$

NOTE. — bv^2 in a square beam = a side of the square beam cubed.

EXAMPLE. — A beam of white Oak, 3 feet in length, 4 inches deep, and 2 inches in breadth, is fixed at one end; what weight is required to break the beam, the weight being suspended from the other end?

$3 : 4^2 \times 2 :: 220 :: 2346\frac{2}{3}$ lbs. *Ans.*

EXAMPLE. — The same beam, same manner of support, &c., as the foregoing, but the greater cross-section of the beam placed horizontally; what weight is required to break it?

$3 : 2^2 \times 4 :: 220 : 1173\frac{1}{3}$ lbs. *Ans.*

EXAMPLE. — A beam of cast iron, 8 feet in length and 6 inches square, is supported at both ends; what is its ultimate transverse strength, it being loaded in the middle?

$8 : 6^2 \times 6$, *i. e.*, $8 : 6^3 :: 684 \times 4 : 73872$ lbs. *Ans.*

A beam fixed at both ends, other things being equal, will bear one half more than when merely supported at both ends.

The length of a beam supported at each end, or by two supports, is the distance from one support to the other.

Round beams, supported in the middle and loaded at each end, have the same sustaining power as when supported at each end and loaded in the middle; and the same is true for rectangular beams, the action of the load, in both cases, being, in the same manner, direct with the beam's central plane.

An equilateral triangular beam, supported in the middle, and loaded at each end, has the same sustaining power as when supported at each end and loaded in the middle, if the beam be inverted.

When a beam is partly loaded at any given locality of its length, to find what weight, acting upon it at any other given locality, must be added to break it.

Let E = breaking weight at the locality of the given partial load.
F = breaking weight at the locality of the required partial load.

Let a = given partial load.
x = required partial load ; then

$$E : F :: E - a : x.$$

With regard to required depths, breadths, &c.

Let S = effective coëfficient in all cases ; that is, = C, or any multiple of C, demanded by the conditions, as set forth in Prob. 1, 2, 3, 4 5, or 6, foregoing ; then

$l : v^2 b :: S : W.$	$S : l :: W : bv^2.$
$W : S :: bv^2 : l.$	$Sv^2 : W :: l : b.$
$bv^2 : W :: l : S.$	$Sb : W :: l : v^2.$

$\sqrt[3]{\left(\frac{Wlv^1}{Sb^1}\right)} = v$, and $\frac{vb^1}{v^1} = b$; $\frac{v^1}{b^1}$ being the ratio fixed upon for the depth to the breadth.

Example. — What must be the *depth* of a pitch-pine beam, resting on two supporters 20 feet apart, that its ultimate transverse strength may be 24000 pounds, the beam being 4 inches in breadth, and the load resting uniformly along its whole length ?

By referring to the table of initial weights, we find the prime coefficient for pitch pine to be 225 lbs., and by problem 5 we find that 8 times that quantity, or 1800 lbs., is the effective coëfficient for the case in hand. Hence

$$1800 \times 4 : 24000 :: 20 : \sqrt{66\tfrac{2}{3}} = 8.165 \text{ inches. } \quad \textit{Ans.}$$

Comparative transverse strength of figures, or of beams, &c., of different figures and positions; both members of the couplet supposed to be of the same material, same length, in the same manner loaded, and in the same way supported.

□, a square beam, the weight acting direct with the sides.

◇, a square beam, the weight acting direct with the diagonal.

△, an equilateral triangular beam, the weight acting direct with the perpendicular, and tending to convex a side.

▽, an equilateral triangular beam, the weight acting direct with the perpendicular, and tending to convex an angle.

○, a round beam, or solid cylinder.

a = side of square beam ; s = side of equilateral triangular beam ; h = perpendicular of triangle ; d = diameter of round beam ; t = diagonal of square ; A = area of transverse section.

The maximum transverse strength of an equilateral triangular beam is obtained when the load tends to convex a side directly, and the minimum when it tends to convex an angle directly.

The maximum transverse strength of a square beam is afforded when the action of the load is direct with the sides, and the minimum when it is direct with the diagonal of the cross section.

In an equilateral triangle, $\frac{4A}{\sqrt{3}} = s^2$; and $s^2 - \frac{1}{4}s^2$ or $\frac{3A}{\sqrt{3}} = h^2$; therefore $A = \frac{1}{2}hs$.

In a square, $\sqrt{2A} = t$, the diagonal.

In a circle, $\frac{4A}{\pi} = d^2$; $\therefore \frac{2\sqrt{A}}{\sqrt{\pi}} = d$.

Comparative Transverse Strengths, Sectional Areas equal.

Formulas of the absolute transverse strengths of the respective members. — Relative transverse strengths of the couplets, reduced to the unit of comparison.

$$\frac{\square}{\diamond} = \frac{a^3}{2a^2(t-a)} = \frac{A\sqrt{A}}{2A(\sqrt{2a}-\sqrt{A})} = \frac{1}{2(\sqrt{2}-1)} = \frac{1}{.828427}$$

$$\frac{\square}{\bigcirc} = \frac{a^3}{\frac{3}{16}\pi d^3} = \frac{A\sqrt{A}}{\frac{3}{2}(A\sqrt{A} \div \sqrt{\pi})} = \frac{1}{3 \div 2\sqrt{\pi}} = \frac{1}{.846284}$$

$$\frac{\triangle}{\square} = \frac{\frac{5}{8}h^3}{a^3} = \frac{15A\sqrt{A} \div 8\sqrt[4]{3}}{A\sqrt{A}} = \frac{15 \div 8\sqrt[4]{3}}{1} = \frac{1.424692}{1}$$

$$\frac{\square}{\triangledown} = \frac{a^3}{\frac{5}{16}h^2 s} = \frac{A\sqrt{A}}{15A\sqrt{A} \div 8\sqrt{3}\sqrt[4]{3}} = \frac{1}{15 \div 8\sqrt{3}\sqrt[4]{3}} = \frac{1}{.822546}$$

$$\frac{\triangle}{\triangledown} = \frac{\frac{5}{8}h^3}{\frac{5}{16}h^2 s} = \frac{15A\sqrt{A} \div 8\sqrt[4]{3}}{15A\sqrt{A} \div 8\sqrt{3}\sqrt[4]{3}} = \frac{\sqrt{3}}{1} = \frac{1.732051}{1}$$

$$\frac{\triangle}{\bigcirc} = \frac{\frac{5}{8}h^3}{\frac{3}{16}\pi d^3} = \frac{15A\sqrt{A} \div 8\sqrt[4]{3}}{\frac{3}{2}(A\sqrt{A} \div \sqrt{\pi})} = \frac{5\sqrt{\pi} \div 4\sqrt[4]{3}}{1} = \frac{1.683467}{1}$$

$$\frac{\bigcirc}{\triangledown} = \frac{\frac{3}{16}\pi d^3}{\frac{5}{16}h^2 s} = \frac{\frac{3}{2}(A\sqrt{A} \div \sqrt{\pi})}{15A\sqrt{A} \div 8\sqrt{3}\sqrt[4]{3}} = \frac{4\sqrt{3}\sqrt[4]{3} \div 5\sqrt{\pi}}{1} = \frac{1.028859}{1}$$

$$\frac{\diamond}{\triangledown} = \frac{2a^2(t-a)}{\frac{5}{16}h^2 s} = \frac{2A(\sqrt{2A}-\sqrt{A})}{15A\sqrt{A} \div 8\sqrt{3}\sqrt[4]{3}} = \frac{16\sqrt{3}\sqrt[4]{3}(\sqrt{2}-1) \div 15}{1} = \frac{1.0071495}{1}.$$

Comparative Transverse Strengths, side of Square Bar, side of Equilateral Triangular Bar, and diameter of Round Bar, equal.

$$\frac{\square}{\bigcirc} = \frac{16}{3\pi} = \frac{1.697653}{1}. \quad \text{Inv.} = \frac{1}{.589049} \qquad \frac{\bigcirc}{\triangle} = \frac{4\pi}{5\sqrt{3}} = \frac{1.45104}{1}. \quad \text{Inv.} = \frac{1}{.689161}$$

$$\frac{\square}{\triangle} = \frac{64}{15\sqrt{3}} = \frac{2.463364}{1}. \quad \text{“} = \frac{1}{.405949} \qquad \frac{\bigcirc}{\triangledown} = \frac{4\pi}{5} = \frac{2.513274}{1}. \quad \text{“} = \frac{1}{.397887}$$

EXAMPLE. — Required the transverse strength, or breaking weight, of a solid cylinder of cast iron 3 inches in diameter and 4 feet in length, one end being fixed, and the weight suspended from the other.

$$W = \frac{C\frac{3}{16}\pi d^3}{l} = \frac{684 \times 3 \times 3.1416 \times 3^3}{4 \times 16} = 2719.637 \text{ lbs. } Ans.$$

Also, $W = \frac{Sd^3}{l}$, S being $C \times .589049$, the relative strength of a round beam to that of a square beam, the diameter of the one being equal to a side of the other, page 282.

EXAMPLE. — What is the transverse strength of an equilateral triangular beam of white oak, 12 feet in length and 6 inches to a side, the beam resting on two supports, angle up, and the load suspended from the middle?

$$W = \frac{5h^3 4C}{8l} = \frac{5 \times 140.296 \times 4 \times 220}{8 \times 12} = 6430.24 \text{ lbs. } Ans.$$

Also, $W = \frac{4Ss^3}{l}$, S being $C \times .405949$, the relative strength of an equilateral triangular beam, angle up, to that of a square beam of the same width of side.

HOLLOW CYLINDERS.

The lateral strength of a hollow cylinder, within certain limits of expansion, is to that of a solid cylinder of the same material, quantity of matter, and length, other things being equal, as the difference of the cubes of the diameters to the cube of the square root of the difference of the squares of the diameters, nearly.

D = diameter of hollow cylinder.

o = diameter of bore, or diameter of interior.

m = diameter of the hollow cylinder if converted into a solid cylinder without changing its length.

d = diameter of a solid cylinder, other things being equal, that has a transverse strength equal to that of the hollow cylinder.

W, S, l, as in the preceding.

$D^2 = m^2 + o^2$; $d^3 = D^3 - o^3$; $m^2 = D^2 - o^2$; $o^2 = D^2 - m^2$.

EXAMPLE. — What is the transverse strength of a hollow cylinder of cast iron; the diameter being 8 inches, interior diameter 6 inches, and length 12 feet; it being supported at both ends, and the load suspended from the middle?

$$W = \frac{4S(D^3 - o^3)}{l} = \frac{402.91(512 - 216)4}{12} = 39754 \text{ lbs. } Ans.$$

EXAMPLE. — What would be the diameter of the above described cylinder if it were converted into a solid cylinder of the same length?

$$\sqrt{(8^2 - 6^2)} = 5.2915 \text{ inches. } Ans.$$

By which it appears that the lateral strength of a solid cylinder 5.29 inches in diameter is doubled if expanded into a hollow cylinder of the same length and 8 inches in diameter; or $\frac{8^3 - 6^3}{[\sqrt{(8^2 - 6^2)}]^3} = \frac{296}{148.162} = 1.998.$

NOTE.—In practice, with a view to safety, a material should not be relied on as having a permanent lateral strength exceeding one-third its ultimate lateral strength.

RESULTS OF EXPERIMENTS BY MAJOR WADE.

Square Bars, sectional area 1 inch.

MATERIALS.	Specific gravity.	Tensile force.	Transverse strength.	Crushing weight.	Hardness.
Cast steel	7.727			198944	
"	8.953	128000	1916	391985	
Wrought iron....	7.704	38027	542	40000	10.45
"	7.858	74592		127720	12.14
Cast iron........	6.9	9000	416	84529	4.57
"	7.4	45970	958	174120	33.51
Bronze	7.978	17698			4.57
"	8.953	56786			5.94

TORTIONAL STRENGTH.

Solid Cylinders, length 1 foot, diameter 1 inch.

MATERIALS.	Specific gravity.	At ½ degree.	Ultimate.	Specific gravity.	At ½ degree.	Ultimate.
Cast steel	7.727		5511			
Wrought iron.....	7.704	970	1296	7.858	1320	1836
Cast iron	6.9	1006	1660	7.4	2500	3060
Bronze	7.978	620	1687	8.953	833	1020

The elasticity of wrought iron is equal to a tensile force of about 21000 lbs. per square inch of transverse section, and that of cast iron is balanced by a tensile force of about 5000 lbs. per square inch of cross section.

To find the ultimate resistance to internal pressure, or bursting, of hollow tubes, cylinders, steam boilers, &c.

C = cohesive or tensile force per square inch of the material, foregoing table, or table page 74.
l = length of the tube, or shell, in inches.
d = diameter of the tube, or vessel, in inches.
t = thickness of the plate, or side of the tube, in inches.
W = ultimate resistance, or bursting pressure, in lbs., per square inch.

RESULTS OF EXPERIMENTS ON WELDED WROUGHT IRON TUBES BY W. FAIRBAIRN.

Ends secured to head-plates.

Resistance to external pressure.				Resistance to internal pressure.			
l	d	t	W	l	d	t	W
12	6	.043	475	30	6	.043	65.
24	6	.043	235	59	6	.043	32.
30	6	.043	230	30	12	.043	22.
60	12	.043	110	60	12	.043	12.5

Although these experiments, at first sight, seem to warrant the conclusion that the *length* of a tube, or shell, is properly an element to be employed in computing its resistance to internal pressure, or bursting, yet the length, within ordinary practical limits, can only, in a very slight degree, affect the tensile force of the material; and this position is very nearly sustained by the last three of the experiments alluded to. The length, for all ordinary purposes of calculation, may be rejected.

EXAMPLE. — The diameter of the wrought-iron shell of a steam boiler is 36 inches, and the thickness of the plate $\frac{1}{4}$ inch; what is the ultimate resistance of the shell per square inch to internal pressure, or bursting?

$$W = \frac{4Ct}{\sqrt{(\pi d^2)}}, \text{ or by the common rule } W = \frac{C \times 2t}{d} \text{ then}$$

$$\frac{55000 \times 4 \times .25}{\sqrt{(3.1416 \times 36^2)}} = 862 \text{ lbs. } Ans.$$

NOTE.—This rule, it should be borne in mind, gives the ultimate resistance to internal pressure of the *plate* or material (supposing the initial force C to be correctly taken), and makes no allowance for weakness due to the manner of building. In the case of a single-riveted tube, or steam-boiler, it is customary to take one-half the ultimate resistance of the plate as the ultimate strength of the structure; and a double-riveted is one-third stronger than a single-riveted. The cohesive or tensile strength of wrought-iron boiler-plates ranges from 42000 to 62000 lbs. per square inch. These remarks are also applicable in the following proposition.

To find the ultimate resistance to external pressure, or collapsing, of tubes, flues, hollow cylinders, &c.

$W = \frac{C\sqrt{C \times t^2}}{\pi l d}$ nearly, C being the crushing force per square inch of the material, foregoing table, or table page 289.

EXAMPLE. — What is the resistance to collapsing of a wrought-iron tube, the thickness being $\frac{1}{4}$ inch, diameter $7\frac{1}{2}$ inches, and length 100 inches, assuming the initial crushing force of the material to be 83500 lbs.?

$$\frac{83500 \times \sqrt{83500 \times .25^2}}{3.1416 \times 7.5 \times 100} = \frac{7680312 \times .25^2}{100 \times 7.5} = 640 \text{ lbs. } Ans.$$

w = weight with which the beam is loaded, in pounds.
a = tabular or initial deflection, page 279.
A = actual deflection in inches.
W, C, l, b, and v, as before.

1. *When the beam is fixed at one end, and loaded at the other; the weight of the beam not being taken into account.*

Deflection varies as $\frac{wl^3}{bv^3}$; and $\frac{wl^3a}{Cbv^3} = A$.

Ratio of Load to Breaking Weight, or Deflection to Greatest Deflection, as $\frac{wl}{Cbv^2}$. $\frac{Cbv^2}{l} = W$. $\frac{Cbv^2}{l} - w$ = Difference of load and breaking weight.

2. *Fixed at one end, and load distributed uniformly over whole length.*

Deflection varies as $\frac{wl^3}{bv^3}$; and $\frac{3wl^3a}{8Cbv^3} = A$.

Ratio of Load to Breaking Weight, &c., as $\frac{wl}{2Cbv^2}$.

3. *Supported at both ends, and loaded in the middle.*

Deflection varies as $\frac{wl^3}{bv^3}$; and $\frac{wal^3}{32Cbv^3} = A$.

Ratio of Load to Breaking Weight, &c., as $\frac{wl}{4Cbv^2}$.

4. *Supported at both ends, and loaded uniformly along the whole length.*

Deflection varies as $\frac{wl^3}{bv^3}$; and $\frac{5}{8} \times \frac{wal^3}{32Cbv^3} = A$.

Ratio of Load to Breaking Weight, &c., as $\frac{wl}{8Cbv^2}$.

The deflection of a round beam is to that of a square beam, the diameter of the former being equal to a side of the latter, and other things equal, as the transverse area of the square beam to $\frac{3}{4}$ the transverse area of the round beam; as $\frac{4s^2}{3d^2.7854}$; as $\frac{1.7}{1}$, nearly.

The deflection of a hollow cylinder is to that of a solid cylinder of the same material, quantity of matter and length, other things being equal, as the diameter of the latter to the greater diameter of the former; that is, their deflections vary inversely as their strengths; and the deflections of hollow cylinders, one with another, other things being equal, are inversely as their exterior diameters nearly.

RESISTANCE TO TORSION.

THE following TABLE shows the weight or force in pounds (mean of experiments), required to twist asunder bars One Inch Square and One Inch in Diameter of the materials named, the Weight acting upon the bar at One Inch from the bar's axis.

Materials.	Sq. Bar. lbs.	Rd. Bar. lbs.
Cast Steel,	28560	17620
Blistered Steel,	24380	15040
American Wrought Iron,	14780	9120
Swedish Wrought Iron,	13870	8560
Cast Iron,	13780	8500
Yellow Brass,	6860	4230
Cast Copper,	6280	3875
Oak,	6160	3800
Fir,	6690	4130

American Wrought Iron twists and takes a permanent set — square bar with 9640 lbs., round bar with 5950 lbs. applied. Swedish Wrought Iron twists and takes a permanent set — square bar with 9790 lbs., round, with 6040 lbs. applied.

C = tabular weight above, special for the case.

W = breaking weight in pounds.

r = leverage of applied force or W's radius of action, in inches, — (the perpendicular distance in inches from the axis of the shaft to the point on the lever or crank, where the motive power is directly applied.)

d = side of square shaft or diameter of round shaft, in inches.

$$W = \frac{Cd^3}{r}. \qquad C = \frac{Wr}{d^3}.$$

$$r = \frac{Cd^3}{W}. \qquad d^3 = \frac{Wr}{C}.$$

EXAMPLE. — Required the weight or force necessary to twist asunder a bar of square rolled iron, 2 inches to the side, the force acting upon the bar through a lever or crank thirty inches in length.

$14780 \times 8 \div 30 = 3941$ lbs. *Ans.*

EXAMPLE. — What must be the diameter of a cast-iron cylinder in order that it may have a torsional strength equal to 20000 pounds acting upon it through a leverage of 13 feet?

$$20000 \times 156 \div 8500 = \sqrt[3]{367.06} = 7.16 \text{ inches.} \quad \textit{Ans.}$$

For Practical Purposes, with a view to safety, $\frac{Wr}{\frac{1}{3}C} = d^3$.

The above cylinder, therefore, for practical purposes, other things remaining unchanged, should have a diameter of

$$20000 \times 156 \times 3 \div 8500 = \sqrt[3]{1101.18} = 10.32 \text{ inches.}$$

To find the number of degrees torsion that a given weight, with a given radius of action, will occasion in a solid cast-iron shaft of given length and diameter.

$\frac{wlr}{660\ d^4} = N$; $\frac{wlr}{660\ N} = d^4$, &c.; w being the *weight* or applied force in pounds; l, the length of the shaft in feet; r, w's radius of action in inches; N, the number of degrees torsion.

EXAMPLE. — What must be the diameter of a solid cast-iron shaft, in order that if 1200 pounds force be applied to it, through a leverage of 22 inches, to set it in motion, the torsion shall be but 4 degrees in its entire length, its length being 20 feet?

$$\sqrt[4]{(1200 \times 20 \times 22 \div 660 \times 4)} = 3.76 \text{ inches.} \quad \textit{Ans.}$$

NOTE. — For Practical Rules applicable to Revolving Shafts, Journals or Gudgeons, see JOURNALS OF SHAFTS, page 320.

HOLLOW CYLINDERS.

The United States *Ordinance Manual* furnishes the following formula, deduced by Lieutenant Rodman from the results of experiments by Major Wade, for finding the torsional strength of hollow cylinders, substituting the foregoing symbols to avoid definitions, namely,

$W = C \times \frac{D^4 - o^4}{Dr}$; D being the external, and o the internal diameter of the cylinder; W, C and r, as before.

Of course this formula does not admit of resolution so as to find D; $\frac{Wr}{C} = \frac{D^4 - o^4}{D} = d^3$; it is not mathematical, therefore, in any degree, to the transverse area of the matter in cylinders; $\left(D^3 - \frac{Wr}{C}\right) \times D = o^4$.

Basing our calculations upon the results of these experiments, or upon the foregoing formula for strength, it may be shown that if we take from the centre of a solid cylinder (by boring or otherwise), a cylinder equal in diameter to $\frac{1}{3}$ the diameter of the cylinder from which we take it, the hollow cylinder left will retain .987654321— of the torsional strength it possessed before any of its material was abstracted; and that if we take, in like manner, from a solid cylinder, a cylinder equal in diameter to $\frac{1}{2}$ the diameter of the cylinder from which we take it, the cylinder thus abstracted from will still retain $\frac{15}{16}$ of the torsional strength it possessed when solid; also, that if we take from the centre of a solid cylinder $\frac{1}{2}$ its material $= \frac{7071}{10000}$ its diameter, we shall diminish its torsional strength but 25 per cent.

RESISTANCE TO LONGITUDINAL COMPRESSION.

Solid bodies of the same material and length resist longitudinal pressure one with another, as are their transverse areas one with another, respectively; but their transverse areas remaining the same, their power of resistance is slightly diminished by an increase of length; and when the length of a body exceeds by about four to six times the square root of its transverse area (as a general thing for most substances), it cannot be crushed by longitudinal compression; but, being left free, will bend and break transversely.

The following table shows the force in pounds required to crush bars or blocks, in the direction of their lengths, of the materials named, the transverse area of each piece being one inch, and the length of each not more than three times nor less than once and a half the square root of its transverse area. The woods supposed to be seasoned, and the length of each bar of wood, in all cases, in the direction of its fibres longitudinally.

Materials.	Pounds.	Materials.	Pounds.
Ash, Walnut,	6650	White Spruce,	5950
Elm, Live Oak,	6840	White Pine,	5780
Beech,	6960	Good Red Brick,	800
Birch,	7980	Seneca Sandstone,	10700
Chestnut,	5350	Freestone,	3050
Hickory,	8925	Quincy Granite,	15300
White Oak,	6100	Cast Iron,	129000
Maple, Yellow Pine,	8150	Wrought Iron,	83500

Note.—The elasticity of cast iron is barely overcome by a pressure of 5000 lbs. to the square inch, and it will bear a compression of $\frac{1}{888}$ before crumbling. Wrought iron will bear a compression of $\frac{1}{863}$ without permanent alteration.

C = tabular weight, foregoing.
l = length of column in feet.
s = side of square column in inches.
d = diameter of round column in inches.
b = breadth of rectangular column in inches.
t = thickness of rectangular column in inches.
W = reliable or practical resistance.

$$C \times \text{transverse area} = \text{crushing weight}.$$

For ordinary practical purposes.

Solid Square Column.

$$4 : s^2 :: C : W. \quad s^2 : W :: 4 : C. \quad C : W :: 4 : s^2.$$

Solid Rectangular Column.

$$4 : bt :: C : W. \quad bt : W :: 4 ; C. \quad C : W :: 4 : bt.$$
$$Cb : W :: 4 : t. \quad Ct : W :: 4 : b.$$

Solid Cylindrical Column.

$$5.1 : d^2 :: C : W. \quad C : W :: 5.1 : d^2.$$

Example. — Required the reliable power of a White Oak post to resist longitudinal pressure, the breadth being six inches and the thickness 5 inches.

$$6100 \times 6 \times 5 \div 4 = 45750 \text{ lbs.} \quad \textit{Ans.}$$

Example. — Required the reliable or practical resistance to longitudinal compression, of an Elm shaft, 6 inches in diameter.

$$6840 \times 6^2 \div 5.1 = 48284 \text{ lbs.} \quad \textit{Ans.}$$

In keeping with the mass of matter, and the liability of flexure removed; any length of column. — Practical or reliable resistance.

Materials.	Solid Square Column.	Solid Cylindrical Column.
Wrought Iron,	$\frac{17800ls^3}{4s^2+.16l^2} = W.$	$\frac{11125d^4}{4d^2+.16l^2} = W.$
Cast Iron,	$\frac{15300ls^3}{4s^2+.18l^2} = W.$	$\frac{9562d^4}{4d^2+.18l^2} = W.$
White Oak,	$\frac{3960ls^3}{4s^2+.5l^2} = W.$	$\frac{2475d^4}{4d^2+.5l^2} = W.$

OF CENTRES OF SURFACES AND CENTRES OF GRAVITY.

SURFACES.

To find the centre of an equilateral triangle.

Side × .57735 = distance from the vertex of each angle.
Side × .28868 = distance from the middle of each side.

In any triangle, if a line be drawn from the vertex of either angle to the middle of the opposite side, it will pass through the centre of the triangle, and the centre will be on that line at $\frac{2}{3}$ its length from the vertex, or $\frac{1}{3}$ its length from its junction with the side. It is at that point, therefore, where any two such lines cross each other.

To find the centre of a square.

Side × .5 = distance from the middle of each side.
Side × .707107 = distance from the vertex of each angle.

To find the centre of a regular pentagon.

Side × .688188 = distance from the middle of each side.
Side × .850644 = distance from the vertex of each angle.

To find the centre of a regular hexagon.

Side × .86604 = distance from the middle of each side
Side × 1 = distance from the vertex of each angle.

To find the centre of a regular octagon.

Side × 1.2071 = distance from the middle of each side.

Side × 1.3065 = distance from the vertex of each angle.

To find the centre of a rectangle.

$\frac{1}{2}\sqrt{(A^2 + a^2)}$, or $\sqrt{(\frac{1}{2}A)^2 + (\frac{1}{2}a)^2}$ = distance from the vertex of each angle.

$\frac{1}{2}$ the length of A from the middle of a, and
$\frac{1}{2}$ the length of a from the middle of A, A being one of the longer sides, and a one of the shorter.

To find the centre of a rhombus.

Side × .5 = distance from the middle of each side.
From the angles; on a diagonal at half its length.

To find the centre of a rhomboid.

From the middle of A, distant $\frac{1}{2}$ a, and from the middle of a, distant $\frac{1}{2}$ A, to the same point; A being one of the longer sides, and a one of the shorter. From the angles, same as in the rhombus.

To find the centre of a trapezoid.

$\frac{b}{3} \times \frac{A + 2a}{A + a}$ = distance from the middle of A, on the line b; A being the longer of the two parallel sides, a the shorter, and b a line passing from the middle of A to the middle of a.

To find the centre of a trapezium.

Draw a diagonal and find the centre of each of the triangles thereby formed, and pass a line from one centre to the other. Then draw the other diagonal, and find the centre of each of the triangles thereby formed, and connect those centres together by passing a line from one to the other. The point at which the lines cross each other is the centre of the figure. And this principle is applicable to all quadrilaterals.

In a *circle* —

$$\text{Circumference} \times .159155 = \text{radius}.$$

To find the centre of a semicircle.

$\frac{\text{Radius}}{2.3562}$ = distance on bisecting radius from the middle of the chord or centre of the circle.

$\sqrt{\left(\text{Radius}^2 + \left(\frac{\text{radius}}{\frac{3}{4}\pi}\right)^2\right)}$ = distance from both extremities of the arc; π being the ratio of circumference to diameter, that is, $\pi = 3.1416$.

Radius $\times$ .42441 = distance on bisecting radius from the middle of the chord.

Radius $\div$ 1.737354, or radius $\times$ .575588 = distance on bisecting radius from the vertex.

To find the centre of the segment of a circle.

$\frac{\text{Chord of the segment}^3}{12 \times \text{area of segment}}$ = distance from the centre of the circle, on the versed sine.

$\sqrt{}$ ($\frac{1}{2}$ chord of segment^2 + distance of centre of segment from middle of chord^2) = distance of centre from both extremities of the arc.

Radius of circle — distance from centre of circle = distance from vertex.

To find the centre of a sector of a circle.

$$\frac{\frac{2}{3}\text{ chord of arc}\times\text{radius of circle}}{\text{length of arc}}=\text{distance from centre of circle,}$$

on radius perpendicular to the chord, and bisecting the sector.

$\sqrt{(\frac{1}{2}\text{ chord of arc}^2+(\text{radius}-\text{versed sine}-\text{distance from centre of circle})^2)}=$ distance from both extremities of the arc.

Radius — distance from centre of circle = distance from vertex.

To find the centre of a parabola.

On its axis at $\frac{2}{5}$ its length from the middle of the base.

$\sqrt{(\frac{1}{2}\text{ base}^2+\text{distance from base}^2)}=$ distance from both extremities of the curve.

Axis — distance from base = distance from vertex.

In an *Ellipse*, the centre is at a right section of the figure, as in the parallelogram.

In a *Zone*, the centre is at a right section of the figure, as in a trapezium.

SOLIDS.

The centre of gravity of a *Cube* is its geometrical centre, and the same is true for a *Right Prism*, *Parallelopiped*, *Cylinder*, *Sphere*, *Spheroid*, *Ellipsoid*, and any *Spindle*. It is also true for the *Middle Frustum* of any spindle, the *Middle Frustum* of a spheroid, the two *Equal Frustums* of a paraboloid, and the two *Equal Frustums* of a cone.

In a *Cone* and *Pyramid*, the centre of gravity is in the axis, at $\frac{1}{4}$ the length of the axis from the base.

In a *Paraboloid*, the centre of gravity is in the axis, at $\frac{1}{3}$ the length of the axis from the base.

To find the centre of gravity of a frustum of a cone or frustum of a regular pyramid.

$$\frac{(D+d)^2+2\,D^2}{(D+d)^2-D\,d}\times\frac{\text{height}}{4}=\text{distance on the axis from the centre}$$

of the less base; D being the diameter of the greater base, and *d* the diameter of the less, in a frustum of a cone; or, D being a side of the greater base, and *d* a side of the less, in a frustum of a regular pyramid.

To find the centre of gravity of a prismoid.

$$\frac{(\sqrt{A}+\sqrt{a})^2+2\,A}{(\sqrt{A}+\sqrt{a})^2-\sqrt{A}\times\sqrt{a}}\times\frac{\text{height}}{4}=\text{distance on the axis}$$

from the less base; A being the area of the greater base, and a the area of the less. And this rule is applicable to the frustum of a pyramid, of any number of equal sides, and to the frustum of a cone.

To find the centre of gravity of a frustum of a paraboloid.

$$\frac{D^2\times 2+d^2}{D^2+d^2}\times\frac{\text{height}}{3}=\text{distance on the axis from the less base;}$$

D being the diameter of the greater base, and d the diameter of the less.

To find the centre of gravity of a spherical segment.

$$\frac{3.1416\times\text{height of segment}^2\times(\text{radius of sphere}-\frac{1}{2}\text{ height of segment})^2}{\text{solid contents of segment}}=\text{dis-}$$

tance from the centre of the sphere, on the axis of the segment; the solid contents and lines being in the same denomination of measure.

Radius of sphere — distance from centre of sphere to centre of gravity of segment = distance on the axis of the segment from the segment's vertex.

NOTE.— $\frac{\text{Radius of base of segment}^2+\text{height of segment}^2}{\text{height of segment}\times 2}=\text{radius of sphere.}$

To find the centre of gravity of a spherical sector.

$$\frac{(\text{Radius}-\frac{1}{2}\text{ versed sine})\times 3}{4}=\text{distance on bisecting radius from centre of sphere.}$$

To find the centre of gravity of a system of bodies.

$$\frac{a\times b+a'\times b'+a''\times b'',\ \&c.}{a+a'+a'',\ \&c.}=\text{centre required;}$$

a, a', a'', &c., being the solid contents or weights, and b, b', b'', &c., the distances of their respective centres of gravity from the given plane

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OF CENTRES OF OSCILLATION AND PERCUSSION.

The centre of oscillation applies to bodies fixed at one end and vibrating in space; and the centre of percussion applies to bodies revolving around a fixed axis. The two centres, in the same body, are at the same point.

The centre of oscillation or percussion is that point in a body, under one or the other of the supposed motions, in which the whole motion, tendency to motion and force in the body, may be supposed collected. It is that point, therefore, in the body under motion, that would strike any obstacle with the greatest effect, or, if met by a staying force, would serve to stop the motion and tendency to motion, of the whole mass, at the same instant.

In *Pendulums*, or rods of uniform diameter and density, having one end fixed, and a ball or weight appended to the other:

$$\text{Weight of rod} \times \tfrac{1}{2} \text{ length of rod} = \text{momentum of rod.}$$

$$\text{Weight of ball} \times (\text{length of rod} + \text{radius of ball}) = \text{momentum of ball.}$$

$$\frac{\text{Weight of rod} \times \text{length of rod}^2}{3} = \text{force of rod.}$$

$$\text{Weight of ball} \times (\text{length of rod} + \text{radius of ball})^2 = \text{force of ball.}$$

$$\frac{\text{Force of rod} + \text{force of ball}}{\text{momentum of rod} + \text{momentum of ball}} = \text{distance from the point of suspension or}$$

axis of motion, to the centre of oscillation or percussion.

And in a rod of uniform diameter and density, having one end fixed, and two or more balls or weights attached:

$$\text{Weight of rod} \times \tfrac{1}{2} \text{ length of rod} = \text{momentum of rod.}$$

$$\text{Weight of 1st ball} \times \text{distance from axis of motion} = \text{momentum of 1st ball.}$$

$$\text{Weight of 2d ball} \times \text{distance from axis of motion} = \text{momentum of 2d ball.}$$

$$\frac{\text{Weight of rod} \times \text{length of rod}^2}{3} = \text{force of rod.}$$

$$\text{Weight of 1st ball} \times \text{distance from axis of motion}^2 = \text{force of 1st ball.}$$

$$\text{Weight of 2d ball} \times \text{distance from axis of motion}^2 = \text{force of 2d ball.}$$

$$\frac{\text{Force of rod} + \text{sum of the forces of the balls}}{\text{momentum of rod} + \text{momenta of the balls}} = \text{distance from the axis of motion to}$$

the centre of oscillation or percussion.

By another Rule.—Suspend the body freely by a fixed point, and cause it to vibrate in small arcs, and note the number of vibrations it makes per minute. Then—

Number of vibrations per minute^2 : 60^2 : : length of pendulum that vibrates seconds in the respective locality : distance from point of suspension to centre of oscillation.

Example.—The rod is 10 feet in length, and weighs 21 pounds; the weight of a ball, having its centre at the lower extremity of the rod, is 16 pounds, and that of another, affixed to the rod 4 feet from the point of suspension, is 9 pounds; required the centre of oscillation in the system.

$\frac{\frac{1}{3}(21 \times 10^2) + (9 \times 4^2) + (16 \times 10^2)}{21 \times 5 + (9 \times 4) + (16 \times 10)} = 8.12$ feet from the point of suspension. *Ans.*

Example. — If, on suspending the body supposed in the last example by its unloaded end, it is found to vibrate 38 times in a minute, what is the distance from the point of suspension to the centre of oscillation?

$$38^2 : 60^2 :: 3.25846 : 8.12 \text{ feet. } Ans.$$

In a *Right Line*, *Cylinder* or *Equilateral Rectangular Prism*, one end being in the axis of motion, the centre of oscillation or percussion is distant from that end $\frac{2}{3}$ the length of the line, cylinder or prism.

The axis of motion being in the vertex of the figure, and the figure moving *flatwise*, the centre of oscillation or percussion is distant from the vertex,

In an isosceles triangle, $\frac{2}{3}$ its height;
In a circle, $\frac{5}{4}$ its radius;
In a parabola, $\frac{5}{7}$ its height.

But the axis of motion being in the vertex of the figure, and the figure moving *sidewise*, the centre of oscillation or percussion is distant from the vertex,

In a circle, $\frac{3}{4}$ its diameter;
In a sector of a circle, $\frac{3 \text{ arc} \times \text{radius}}{4 \text{ chord}}$;
In a parabola, $\frac{5}{7}$ height $+ \frac{1}{3}$ parameter;
In a cone, $\frac{4}{5}$ axis $+ \frac{\text{radius of base}^2}{5 \text{ axis}}$;
In a sphere, $\frac{\text{radius}^2 \times 2}{\text{radius} \times 5} +$ radius;

In a rectangle, suspended by one angle, $\frac{2}{3}$ the diagonal;

In a parabola, suspended by the middle of its base, $\frac{4}{7}$ axis $+ \frac{1}{2}$ parameter;

In a sphere, suspended by a thread, $\frac{\text{radius}^2 \times 2}{5 \times (\text{radius} + \text{length of thread})}$ + radius + length of thread.

Example. — Required the length of a rod of uniform diameter and density, that, being suspended by one end, will vibrate seconds in the latitude of New York.

$$2 : 3 :: 39.10153 : 58.6523 \text{ inches. } Ans$$

EXAMPLE. — A ball of 4 inches radius is suspended by a string 24 inches in length; required the centre of oscillation or percussion in the system.

$$\frac{4^2 \times 2}{(4+24) \times 5} = \frac{32}{140} = .2286 + 4 + 24 = 28.2286 \text{ inches from}$$

the point of suspension. *Ans.*

EXAMPLE. — Required the centre of oscillation in a parabola whose height is 10 inches and base 8 inches, supposing the parabola to be vibrating sidewise.

$$\frac{10 \times 5}{7} + \frac{4^2}{10 \times 3} = 7.676 \text{ inches from vertex. } \textit{Ans.}$$

The centre of oscillation in a sphere suspended by a point in its circumference being given, to find the radius of the sphere.

$h - \frac{2h}{7} =$ radius; h being the distance from the point of suspension to centre of oscillation.

EXAMPLE. — Required the radius of a sphere, that, being suspended by a point in its surface, will vibrate seconds at 23° of latitude.

$$39.01206 - \frac{39.01206 \times 2}{7} = 27.865 + \text{ inches. } \textit{Ans.}$$

The radius of the ball or bob of a pendulum being given, to find the length of the rod or string by which that ball must be suspended, in order that the pendulum may vibrate seconds at a given locality.

$P - \left(\frac{r^2 \times 2}{P \times 5}\right) + r = l$; P being the length of a pendulum that vibrates seconds in the respective locality, r the radius of the ball, and l the length of the string or rod; the last supposed to be without weight.

EXAMPLE. — The radius of a ball being 4 inches, required the length of a thread, (supposed without weight,) whereby to form a pendulum with that ball, that will vibrate seconds in the latitude of New York.

$$39.10153 - \left(\frac{4^2 \times 2}{39.10153 \times 5} + 4\right) = 34.937 \text{ inches. } \textit{Ans.}$$

CENTRE OF GYRATION.

The centre of gyration is that point in a revolving body or system of bodies, in which, if the whole quantity of matter were collected, the angular velocity would be the same; that is, the momentum or quantity of motion in the revolving mass is centred at this point.

The centre of gyration is at a greater distance from the axis of motion than the centre of gravity, and at a less distance from that axis than the centre of percussion. Its distance from that axis is a mean proportional between the two, in the same mass.

To find the centre of gyration in a body or system of bodies.

Rule 1. — Multiply the distance of the centre of percussion from the axis of motion, by the distance of the centre of gravity from the axis of motion, and the square root of the product will be the distance of the centre of gyration from the axis of motion.

Rule 2. — Multiply the weight of the several particles by the squares of their distances from the axis of motion, and divide the sum of the products by the sum of the weights; the square root of the quotient will be the distance of the centre of gyration from the axis of motion.

Example. — On a lever (supposed without weight) revolving about one end, there are placed — one weight of 4 lbs., at 3 feet from the axis of motion; one of 6 lbs., at 4 feet; and one of 5 lbs., at 5 feet from that axis; required the centre of gyration in the system.

$$\sqrt{\left(\frac{4 \times 3^2 + (6 \times 4^2) + (5 \times 5^2)}{4 + 6 + 5}\right)} = 4.14$$ feet from the axis of motion. *Ans.*

A weight of 15 lbs., therefore, placed 4.14 feet from the axis of motion, and revolving in the same time, would have the same impetus or momentum as the three weights in their respective places.

Example. — A lever 6 feet in length, and weighing 14 lbs., is revolving about one end; at 4 feet from the axis of motion, on the lever, is placed a weight of 5 lbs., and at 6 feet from that axis, another of 8 lbs.; required the centre of gyration in the system.

$$\sqrt{\left(\frac{14 \times \frac{6}{3}^2 + (5 \times 4^2) + (8 \times 6^2)}{14 + 5 + 8}\right)} = 4.455$$ feet from the axis of motion. *Ans.*

Example. — Required the centre of gravity and centre of percussion in the last mentioned system.

$$\frac{14 \times \frac{6}{2} + (5 \times 4) + (8 \times 6)}{14 + 5 + 8} = 4.074, \text{ centre of gravity. } \textit{Ans.}$$

$$\frac{14 \times \frac{6^2}{3} + (5 \times 4^2) + (8 \times 6^2)}{14 \times \frac{6}{2} + (5 \times 4) + (8 \times 6)} = 4.873, \text{ centre of percus. } \textit{Ans.}$$

Example. — The centre of gravity in a body, or system of bodies, being 4 feet from the axis of motion, and the centre of percussion 6 feet; at what distance from that axis is the centre of gyration?

$$\sqrt{(4 \times 6)} = 4.899 \text{ feet. } \textit{Ans.}$$

The centre of gyration is distant from the axis of motion: —

In a straight, uniform rod, revolving about one end; the length of the rod $\times$ .57735, or $\sqrt{\left(\frac{\text{length of rod}^2}{3}\right)}$.

In a circular plate, revolving on its centre, or a cylinder, revolving about its axis, or a wheel of uniform thickness, revolving about its axis; the radius $\times$ .7071.

In a circular plate, revolving about one of its diameters as an axis; the radius $\times$ .5.

In a thin, hollow sphere, revolving about one of its diameters as an axis; the radius $\times$ .8164.

In a cone, revolving about its axis; the radius of the base $\times$ .5477.

In a four-sided equilateral pyramid, revolving about its apex; the axis $\times$ .866.

In a paraboloid, revolving about its axis; the radius of the base $\times$ .57735.

In a sphere, revolving about one of its diameters as an axis; the radius $\times$ .6325.

In a straight lever, the arms being R and r, $= \sqrt{\frac{R^3 + r^3}{(R - r) \times 3}}$.

In wheels in general revolving about their axes, $=$ $\sqrt{\left(\frac{R \times r^2 \times 2 + A \times l^2 \times 2}{(R + A) \times 2}\right)}$; R being the weight of the rim, r the radius of the wheel, l the length of the arms, and A the weight of the arms.

In a water-wheel in operation, $=$ $\sqrt{\left(\frac{R \times r^2 \times 2 + A \times l^2 \times 2 + W \times r^2}{(R + A + W) \times 2}\right)}$, nearly; W being the weight of the water on loaded arch.

THE ENERGY of a revolving body =

$$\frac{\text{Velocity of centre of gyration in feet per second}^2}{64\frac{1}{3}} \times \text{weight of}$$

revolving body.

The energy of a revolving body is its weight multiplied into the height due to the velocity with which the centre of gyration moves in its circle. The height in feet, due to a velocity in feet per second $= (\text{velocity} \div 8.02)^2$, or $\text{velocity}^2 \div 64\frac{1}{3}$.

EXAMPLE. — A grindstone two feet in diameter and weighing 140 lbs., makes 80 revolutions per minute; required the energy with which the stone moves, or the mechanical power that must be communicated, to give it that motion.

$\left(\frac{2 \times .7071 \times 3.1416 \times 80}{60}\right)^2 \div 64\frac{1}{3} = .5456$, energy, the weight being 1, and $.5456 \times 140 = 76.38$ lbs. *Ans.*

CENTRAL FORCES.

THE CENTRAL FORCES are the *centrifugal force* and *centripetal force;* these forces are opposed to each other. The former is the force with which a body, moving in a curve, tends to fly off from the axis of its motion, and the latter is the force that maintains the body in its curvilinear path.

To find the centrifugal force of a body.

$$\frac{\text{Revolutions per minute}^2 \times \text{diameter of centre of gyration's circle in feet}}{5865} = \text{centrifu-}$$

gal force, the weight being 1.

$\left(\frac{\text{Velocity of centre of gyration in feet per second}}{4.01}\right)^2 \div$ twice the distance of the centre of gyration from centre of motion = centrifugal force, the weight being 1.

If we let

v represent velocity of centre of gyration in feet per second,
r " radius, in feet, of circle described by centre of gyration,
w " weight of body,
c " centrifugal force; then —

$$\frac{v^2 \times w}{r \times 32\frac{1}{6}} = c. \qquad \frac{c \times 32\frac{1}{6} \times r}{v^2} = w.$$

$$\frac{v^2 \times w}{c \times 32\frac{1}{6}} = r. \qquad \sqrt{\left(\frac{c \times 32\frac{1}{6} \times r}{w}\right)} = v.$$

EXAMPLE. — Required the centrifugal force of a fly-wheel whose centre of gyration is 8 feet from the axis of motion, and which centre moves with a velocity of 34 feet per second.

$$\left(\frac{34}{4.01}\right)^2 \div (8 \times 2) = 4.49 \text{ times the weight of the wheel.} \quad \textit{Ans.}$$

In fly-wheels in general,

$$\frac{\text{Diameter of wheel in feet} \times .6136}{\text{time in seconds of one revolution}^2} \times \text{weight of rim} = \text{centrifugal force, nearly.}$$

FLY-WHEELS.

FLY-WHEELS are used both as regulators of force and magazines of power; when, more especially for the former purpose, they should be placed as near as practicable to the prime mover; and when, more especially for the latter, as near as practicable to the working point.

To find the weight of the rim of a fly-wheel proper for an engine of a given horse-power.

$$\frac{\text{Horse-power of engine} \times 1368}{\text{diameter of wheel in feet} \times \text{revolutions per minute}} = \text{requisite weight of rim in 100 pounds.}$$

EXAMPLE. — What should be the weight of the rim of the fly-wheel of a 40 horse-power engine, the wheel being 16 feet in diameter, and making 42 revolutions per minute?

$$\frac{40 \times 1368}{16 \times 42} = 81.43 \times 100 = 8143 \text{ lbs.} \quad \textit{Ans.}$$

THE GOVERNOR.

THE GOVERNOR is a regulator of force merely, and acts upon the principle of central forces. It makes half as many revolutions in any given time, as a pendulum, the length of which is equal to the perpendicular distance between the point at which the arms are suspended and the plane in which the centre of oscillation in the system moves, makes in the same time.

If we let

l = perpendicular distance referred to, in inches,

r = number of revolutions made by the system per minute,

Then,

$$\left(\frac{187.6}{r}\right)^2 = l, \text{ and } \frac{187.6}{\sqrt{l}} = r.$$

EXAMPLE. — A governor makes 40 revolutions a minute; what is the perpendicular distance from the point at which the arms are suspended, to the plane in which the centre of oscillation moves?

$187.6 \div 40 = 4.69 \times 4.69 = 22$ inches. *Ans.*

$\sqrt{l} \times .31986 =$ time in seconds of each revolution.

FORCE OF GRAVITY.

THE FORCE OF GRAVITY is greatest at the earth's surface, and decreases from the surface to the centre as the distance from the surface increases; from the surface upwards it decreases as the square of the distance from the centre, in semi-diameters of the earth.

The semi-diameter is usually taken at 4000 miles.

A body, therefore, weighing at the earth's surface 500 lbs., would, were it removed 1000 miles from the surface towards the the centre, weigh

$$4000 : 500 :: 3000 : 375 \text{ lbs.}$$

And were the same body placed 1000 miles above the earth's surface, it would be

$(4000 + 1000) \div 4000 = 1\frac{1}{4}$ semi-diameters from the centre, and at that locality would weigh —

$$500 \div 1.25^2 = 320 \text{ lbs.}$$

And in order that the same body may weigh but 250 lbs., it must be elevated above the surface

$$\sqrt{(500 \div 250)} = 1.4142 \text{ semi-diameters from the centre, or}$$

$$(4000 \times 1.4142) - 4000 = 1657 \text{ miles.}$$

NOTE. — The force of gravity at any locality, as measured by the descent of a body falling freely through space, is equal in feet per second to the length of a pendulum in feet that vibrates seconds at that locality $\times$ 4.93483, for the first second of the body's descent.

Thus, the force of gravity of a body for the first second of its descent, at the level of the sea, at the latitude of 41°, is $3.25846 \times 4.93483 = 16.08$ feet.

To find the time which a body will be in falling from rest, through any space, the space fallen through being given, or the maximum velocity attained in falling being given.

$$\sqrt{\left(\frac{\text{Space fallen through in feet}}{16\frac{1}{12}}\right)} = \text{time of falling in seconds.}$$

$$\frac{\text{Maximum velocity attained in feet per second}}{32\frac{1}{6}} = \text{time of falling in seconds.}$$

$$\frac{\text{Space fallen through in feet} \times 2}{\text{maximum velocity attained in feet per second}} = \text{time of falling in seconds.}$$

To find the maximum velocity attained by a body falling freely from rest, through space, the space fallen through being given, or the time of descending being given.

$\sqrt{}$(Space fallen through in feet $\times 16\frac{1}{12}$) $\times 2 =$ maximum velocity attained in feet per second.

Time of falling in seconds $\times 32\frac{1}{6} =$ maximum velocity attained in feet per second.

$$\frac{\text{Space fallen through in feet} \times 2}{\text{time of falling in seconds}} = \text{maximum velocity attained in feet per second.}$$

To find the space fallen through by a body falling freely from rest, the time of descending being given, or the maximum velocity attained in falling being given.

Time of falling in seconds$^2 \times 16\frac{1}{12} =$ space fallen through in feet.

$$\frac{\text{Maximum velocity attained in feet per second}^2}{64\frac{1}{3}} = \text{space fallen through in feet.}$$

Maximum velocity attained in feet per second $\times \frac{1}{2}$ time of falling in seconds $=$ space fallen through in feet.

To find the height of a stream projected vertically from a pipe.

$$\frac{\text{Quantity in cub. ft. discharged per minute} \times 2.4}{\text{area of discharging orifice in inches}} = \text{maximum velocity in feet per}$$
second, or velocity in feet per second when the stream leaves the pipe.

$$\left(\frac{\text{Maximum velocity of stream in feet per second}}{8.02}\right)^2 = \text{height in feet due to that ve-}$$
locity, or height in feet to which the stream will ascend.

EXAMPLE. — From a fire-engine there was discharged through a pipe $\frac{3}{4}$ of an inch in diameter, 15 cubic feet of water in 1 minute, the pipe being directed vertically; what height did the stream attain?

$$\left(\frac{15 \times 1728}{.75 \times .75 \times .7854 \times 12 \times 60 \times 8.02}\right)^2 = 103.22 \text{ feet.} \quad \textit{Ans.}$$

To find the force or power requisite to produce the foregoing result.

$$\frac{\text{Height of column of water equal to pressure of atmosphere (33.87 feet)}}{\text{pressure of atmosphere on 1 square inch of surface (14.7 lbs.)}} = \text{height of}$$
column of water of 1 inch transverse area, weighing 1 lb. $=$ 2.305 feet; then.

$$\frac{\text{Pounds of water discharged per second} \times 2.305}{\text{area of discharging orifice in inches}} = \text{maximum velocity in feet per second, and}$$

Maximum velocity in feet per second $\times$ weight in lbs. discharged per second $=$ momentum in lbs., or force in lbs. required.

$\sqrt{(\text{Height of projection in feet} \times 16\tfrac{1}{12}) \times 2}$ = maximum velocity in feet per second.

Example. — Required the constant force requisite to project a continuous stream of water perpendicularly to the height of 103.22 feet, through a pipe $\frac{3}{4}$ of an inch in diameter.

$$\frac{\sqrt{(103.22 \times 16\tfrac{1}{12}) \times 2}}{2.305 \div .75^2 \times .7854} \times \sqrt{(103.22 \times 16\tfrac{1}{12}) \times 2} = 1275 \text{ lbs. } Ans.$$

$$\text{Or, } \frac{15 \times 62.5 \times 2.305}{60 \times .75^2 \times .7854} \times \frac{15 \times 62.5}{60} = 1275 \text{ lbs.}$$

Note. — The force requisite to project a body vertically to any given height is equal to the force or momentum with which that body will return to the place from whence projected. The time occupied in ascending is equal to that occupied in descending.

OF PENDULUMS.

Pendulums of the same length vibrate faster the farther they are removed from the equator, either north or south.

The length of a pendulum to vibrate seconds, or 60 times in a minute, at the level of the sea, at the temperature of 60° F., at the latitude of Trinidad, is 39.01879 inches; at Jamaica, 39.03508; at New York, 39.10153; at Bordeaux, 39.11282; at Paris, 39.12843; at London, 39.1393; at Edinburgh, 39.1554; and at Greenland, 39.20328 inches.

To find the length of a pendulum that will make a given number of vibrations in a given time.

Rule. — As the number of vibrations required in one minute is to 60 vibrations, so is the square root of the length of a pendulum that makes 60 vibrations in a minute at the respective locality, to the square root of the length of the pendulum required.

Example. — Required the length of a pendulum that will vibrate half seconds at the level of the sea, at the latitude of New York.

$$120 : 60 :: \sqrt{39.10153} : 3.126561, \text{ and}$$
$$3.126561 \times 3.126561 = 9.77538 \text{ inches. } Ans.$$

To find the number of vibrations that a pendulum of a given length will make in a given time.

Rule. — As the square root of the length of the given pendulum is to the square root of the length of a pendulum that vibrates seconds in the respective locality, so is 60 vibrations to the number of vibrations it will make per minute.

Example. — Required the number of vibrations that a pendulum 24 inches in length will make per minute, at the level of the sea, at the latitude of New York.

$$\sqrt{24} : \sqrt{39.10153} :: 60 : 76.585. \quad \textit{Ans.}$$

To find the length of a pendulum that will vibrate as many times in a minute as there are inches in that pendulum's length.

Rule. — Multiply the square root of the length of a pendulum that vibrates seconds in the respective locality, by 60, find the cube root of the product, and the square of the last root will be the length sought.

Example. — What must be the length of a pendulum, in order that its length in inches and the number of its vibrations per minute, at the latitude of New York, may be equal?

$$\sqrt{(39.10153)} \times 60 = 375.1873, \text{ and}$$
$$\sqrt[3]{375.1873} = 7.2125, \text{ and}$$
$$7.2125^2 = 52.02 \text{ inches.} \quad \textit{Ans.}$$

Note. — The length of a pendulum is the distance from the point of suspension to the centre of oscillation. See Centres of Oscillation and Percussion.

SCREW-CUTTING IN A LATHE.

Screws to any degree of fineness not exceeding about ⅛ inch pitch, that is, having not more than about 8 threads in an inch, may be cut in a lathe by means of a wheel on the end of the leading screw, and another on the end of the lathe spindle, with a *carrier*, or intermediate wheel, called a *stud-wheel*, to connect them. And all that is necessary in this kind of training, in order to obtain a screw of any given or predetermined pitch, is, that the wheel on the end of the leading screw bear the same ratio to that on the end of the lathe spindle, that the pitch of the screw employed bears to that of the screw intended.

The carrier, whatever be the number of its teeth, and the same would be true if two or more carriers, of whatever size, were employed, in no way affects the ratio of velocity between the wheels alluded to. It receives the motion directly from one of them, and conveys it directly to the other, and is introduced to obviate the necessity of employing larger wheels. It is both a *driven* and *driver* in the train, and, consequently, in calculating the gearing to produce a screw of a given or required pitch, needs not to be taken into account.

But screws of a greater degree of fineness than about 8 threads in an inch, more especially if the pitch of the screw attached to the lathe be large, are more conveniently cut by help of two intermediate wheels, of unequal sizes, both upon the same shaft, called a *stud-wheel and pinion;* whereby the requisite velocity of the spindle may be more effectively obtained, and with a nearer approach to uniformity in size of the wheels employed. When this last mode of training is resorted to, all the wheels in the train are influential in determining the pitch the contemplated screw will take; and the equation will be as the product of the drivers to the product of the driven; that is —

If we let a = pitch of leading screw,
a' = No. of teeth in stud pinion,
a'' = No. of teeth in spindle-wheel, } *drivers*,
b = pitch of contemplated screw,
b' = No. of teeth in stud-wheel,
b'' = No. of teeth in leading screw-wheel, } *driven*,

We shall have, $a \times a' \times a'' = b \times b' \times b''$.

And, in the train first supposed, whether with or without carriers, —

$$a \times a'' = b \times b'';$$

And the product of either full side of the equation, divided by the product of either two terms in the opposite, will give the remaining term, or term sought.

EXAMPLE. — a is $\frac{3}{8}$ inch, a'' has 24 teeth, and b is to equal $\frac{1}{4}$ inch; required the requisite number of teeth in b''.

$$\tfrac{3}{8} \times 24 = 9, \text{ and } 9 \div \tfrac{1}{4} = 36 \text{ teeth. } \textit{Ans.}$$

Or, $(.375 \times 24) \div .25 = 36$; or, $.25 : 24 :: .375 : 36$. *Ans.*

EXAMPLE. — a'' has 60 teeth, a' has 20, and $a = \frac{1}{2}$ inch; b'' has 120 teeth, and b is fixed upon at $\frac{1}{16}$ inch; required the number of teeth requisite for b'.

$$(\tfrac{1}{2} \times 20 \times 60) \div (120 \times \tfrac{1}{16}) = 80 \text{ teeth in } b'. \textit{ Ans.}$$

$$\text{Or, } \overline{.5 \times 20 \times 60} \div \overline{120 \times .0625} = 80. \textit{ Ans.}$$

But it is more convenient, frequently, in solving problems of this nature, to substitute the number of threads in a given length of the screws, for their pitches, respectively.

Thus, if we let $a =$ No. of threads in an inch of leading screw,
$b =$ No. of threads in an in. of contemplated screw,

We shall have, as an equation for the foregoing train —

$$a \times b' \times b'' = b \times a' \times a''.$$

That is, the product of the teeth in the driving-wheels multiplied by the number of threads in an inch of the contemplated screw, is equal to the product of the teeth in the driven wheels multiplied by the number of threads in an inch of the leading screw.

Example. — a' has 20 teeth, a'' has 60, and b is to have 16 threads in an inch; a has 2 threads in an inch, and b'' has 120 teeth; required the number of teeth requisite for b'.

$$\frac{20 \times 60 \times 16}{120 \times 2} = 80. \quad Ans.$$

Example. — The leading screw has 2 threads in an inch, the leading screw-wheel has 120 teeth, the stud-wheel 80 teeth, and the spindle-wheel 60 teeth; required the number of teeth that must be in the stud pinion, in order that the contemplated screw may have 16 threads in an inch.

$$\frac{\cancel{2} \times \overset{2}{\cancel{120}} \times 80}{\cancel{60} \times \underset{\underset{4}{\cancel{8}}}{\cancel{16}}} = \frac{80}{4} = 20 \text{ teeth in } a'. \quad Ans.$$

The following table exhibits the *change wheels* proper for cutting screws of various pitches from 1 inch to $\frac{1}{36}$ inch, or from one thread in an inch to 36 threads in an inch, the leading screw having a pitch of $\frac{1}{2}$ inch.

Note. — If the leading screw have 3 threads in an inch, the following table is made applicable by changing either of the driven wheels for one having $\frac{1}{3}$ less number of teeth. 3 : 2 : : tabular driven : driven required, when the leading screw has $\frac{1}{3}$ inch pitch, and 4 : 2 : : tabular driven : driven required, when the leading screw has $\frac{1}{4}$ inch pitch, &c. Or 2 : 3 : : either of the tabular drivers : driver required in its stead, when the leading screw has 3 threads in an inch, and 2 : 4 : : either of the tabular drivers : driver required in its stead, when the leading screw has 4 threads in an inch, &c.

Pitch of leading screw × No. teeth in stud-wheel × No. teeth in screw-wheel = pitch of contemplated screw × No. teeth in stud-pinion × No. teeth in spindle-wheel, in all instances.

The tabular wheels may be taken in any proportion, greater or less.

TABLE

Of Change Wheels for Screw-cutting; the leading screw being of ½ inch pitch, or containing 2 threads in an inch.

Number of threads in inch of contemplated screw.	Number of teeth in		Number of threads in inch of contemplated screw.	Number of teeth in				Number of threads in inch of contemplated screw.	Number of teeth in			
	Lathe spindle-wheel.	Leading screw-wheel.		Lathe spindle-wheel.	Stud wheel.	Stud pinion.	Leading screw-whell.		Lathe spindle-wheel.	Stud wheel.	Stud pinion.	Leading screw-wheel.
1	80	40	8¼	40	55	20	60	19	50	95	20	100
1¼	80	50	8½	90	85	20	90	19½	80	120	20	130
1½	80	60	8¾	60	70	20	75	20	60	100	20	120
1¾	80	70	9½	90	90	20	95	20¼	40	90	20	90
2	80	80	9¾	40	60	20	65	21	80	120	20	140
2¼	80	90	10	60	75	20	80	22	60	110	20	120
2½	80	100	10½	50	70	20	75	22½	80	120	20	150
2¾	80	110	11	60	55	20	120	22¾	80	130	20	140
3	80	120	12	90	90	20	120	23¾	40	95	20	100
3¼	80	130	12¾	60	85	20	90	24	65	120	20	130
3½	80	140	13	90	90	20	130	25	60	100	20	150
3¾	80	150	13½	60	90	20	90	25½	30	85	20	90
4	40	80	13¾	80	100	20	110	26	70	130	20	140
4¼	40	85	14	90	90	20	140	27	40	90	20	120
4½	40	90	14¼	60	90	20	95	27½	40	100	20	110
4¾	40	95	15	90	90	20	150	28	75	140	20	150
5	40	100	16	60	80	20	120	28½	30	90	20	95
5½	40	110	16¼	80	100	20	130	30	70	140	20	150
6	40	120	16½	80	110	20	120	32	30	80	20	120
6½	40	130	17	45	85	20	90	33	40	110	20	120
7	40	140	17½	80	100	20	140	34	30	85	20	120
7½	40	150	18	40	60	20	120	35	60	140	20	150
8	30	120	18¾	80	100	20	150	36	30	90	20	120

OF STEAM AND THE STEAM ENGINE.

It has been found by experiment that the force requisite to overcome the friction of a locomotive engine and attendant machinery, the engine being without load, is equal to a pressure of about 1.15 lbs. effective on each square inch of the cylinder's cross sectional area, or equal to 1 lb. effective for the engine, and 0.15 lb. effective for the attendant machinery; this item, however, for practical purposes, is usually taken at $1\frac{1}{2}$ lbs.; by effective pressure is meant a pressure over and above that of the atmosphere, or over and above 14.7 lbs. on each square inch of surface.

If we let

d = diameter of cylinder in inches,

s = stroke of piston in feet,

r = revolutions per minute,

p = mean effective pressure in lbs. per square inch, as shown by the indicator,

$a = s \times r \times 2$ = velocity of piston in feet per minute,

$h = 1\frac{1}{2}$ lbs. = effective pressure on each square inch of cylinder's cross sectional area, requisite to overcome friction, then,

To find the effective power or force of a steam engine.

$$\frac{d^2 \times (p - h) \times a}{42017} = \text{effective force in horse-power.}$$

Example. — What is the effective force of a steam engine, the diameter of the cylinder (d) being 36 inches, the stroke of the piston (s) 7 feet, the effective pressure (p) 30 lbs., and making $17\frac{1}{2}$ revolutions (r) per minute?

$36^2 \times (30 - 1.5) \times 7 \times 17.5 \times 2 \div 42017 = 215.37$ horse-power. *Ans.*

To find the nominal power of a low-pressure or condensing engine.

$$\frac{d^2 \times a}{6000} = \text{nominal force in horse-power.}$$

To find the nominal power of a high-pressure, or non-condensing engine.

$$\frac{d^2 \times p \times a}{120{,}000} = \text{nominal force in horse-power. Or,}$$

$$\frac{d^2 \times p \times \sqrt[3]{s}}{940} = \text{nominal force in horse-power, the piston moving}$$

at the ordinary speed of 128 times the cube root of the stroke.

To find the pressure of the steam on each square inch of the boiler's surface.

Let P = pressure in lbs. on safety valve,

p = pressure in lbs. on each square inch of surface,

W = *weight* or resistance in lbs., as indicated by the spring balance,

w = sum of the weights of the lever, safety valve, and balancing weight, in lbs.,

s = length of lever in inches from its axis of motion to a point vertical to the centre of the valve,

l = length of lever in inches from its axis of motion to W, or to the point on the lever at which the spring balance is attached,

a = area of safety valve in square inches.

Then,

$$s : l :: W : P - w; \text{ and } \frac{P - w + w}{a} = p.$$

Example. — The length of the lever from its axis of motion to a point over the centre of the safety valve (s) is 3 inches, its length from the axis of motion to the point at which the *weight* or spring balance is attached (l) 24 inches, the *weight*, or pressure, as indicated by the spring balance (W) 40 lbs., the sum of the weights of the lever, valve, &c., (w) 6 lbs., and the area of the valve 6½ square inches; required the pressure of the steam per square inch.

$$3 : 24 :: 40 : 320, \text{ and}$$

$$320 + 6 = 326 \div 6.5 = 50.15 \text{ lbs. } \textit{Ans.}$$

$$W : P - w :: s : l. \quad l : s :: P - w : W.$$

$$P - w : W :: l : s.$$

To find the volume of steam compared with the volume of water.

Let F = elastic force of steam in pounds per square inch,

V = volume of steam compared with volume of water.

Then,

$$\frac{24250}{F} + 65 = V, \text{ and } \frac{24250}{V - 65} = F,$$ nearly; the volume of water in every instance, being 1.

Example. — The elastic force of the steam is 40 lbs. to each square inch of the surface of its bulk; what space does it occupy, compared with the space it would occupy if it were condensed to water?

24250 ÷ 40 = 606 + 65 = 671; that is, when the elastic force of the steam is 40 pounds to each square inch of the surface of its bulk, the volume of the steam, compared with its volume in water, is as 671 to 1.

Note. — The preceding formulas are not strictly correct for all densities of steam, but give the mean of a general range.

The volume of steam compared with the volume of water as 1, has been found, by experiment, when the elastic force is 14.7 lbs. = 1694; 20 lbs. = 1280; 25 lbs. = 1044; 30 lbs. = 883; 35 lbs. = 767; 40 lbs. = 679; 45 lbs. = 610; 50 lbs. = 554; 55 lbs. = 508; 60 lbs. = 470; 65 lbs. = 437; 70 lbs. = 408; 75 lbs. = 383; 80 lbs. = 362; 90 lbs. = 325; 100 lbs. = 295; 150 lbs. = 205; 200 lbs. = 158.

To find the temperature of the steam, its elastic force being known.

Rule. — Multiply the 6th root of the elastic force in inches of mercury by 177, and subtract 100 from the product; the remainder will be the temperature in degrees, Fahrenheit nearly.

Or, if we let

a = elastic force in inches of mercury,

b = temperature of steam in degrees, F.

Then,

$$\sqrt[6]{a} \times 177 - 100 = b, \text{ and } \left(\frac{b + 100}{177}\right)^6 = a.$$

Example. — The elastic force of the steam is 150 inches of mercury, (150 ÷ 2.04 = 73½ pounds to each square inch of the surface of its bulk). ; what is the temperature?

$$\sqrt[6]{150} = 2.305 \times 177 = 408 - 100 = 308°. \quad \textit{Ans.}$$

Note. — The pressure of the steam in pounds per square inch × 2.04 = pressure in inches of mercury.

The temperature due to a pressure of 14.7 lbs. per square inch = 212°; 20 lbs. = 228°; 25 lbs. = 241°; 30 lbs. = 252°; 35 lbs. = 261°; 40 lbs. = 269°; 45 lbs. = 276°; 50 lbs. = 283°; 55 lbs. = 289°; 60 lbs. = 296°; 65 lbs. = 301°; 70 lbs. = 306°; 75 lbs. = 311°; 80 lbs. = 316°; 85 lbs. = 320°; 90 lbs. = 324°; 100 lbs. = 332°; 150 lbs. = 363°; 200 lbs. = 387°.

To find the quantity of water required for steam per minute by an engine in motion.

Rule. — Multiply the velocity of the piston in feet per minute, by the square of the cylinder's diameter in feet multiplied by 0.7854, and divide the product by the volume of steam compared with the volume of water, due to the pressure exerted.

Example. — The diameter of the cylinder is 2½ feet, the velocity of the piston 245 feet per minute, and the constant pressure exerted by the steam 60 pounds to the square inch; what quantity of water must be converted into steam per minute?

$$2.5^2 \times .7854 \times 245 \div 470 = 2.56 \text{ cubic feet.} \quad \textit{Ans.}$$

Note. — For a single-acting engine half the quantity indicated by the above rule is required.

To find the quantity of steam required to raise a given quantity of water of a given temperature to a required temperature.

For the present purpose, we may assume that the sum of the latent and sensible heat of vapor of water is constant, and = 1178°. Therefore, the latent heat of vapor of water at 32° = 1178 — 32

$= 1146$; at $100° = 1178 - 100 = 1078°$; at $212° = 1178 - 212 = 966°$, &c.

The latent heat of water at 32° of sensible heat $= 140°$.

If we let

$a =$ temperature of steam employed,
$b =$ temperature of water to be raised,
$c =$ temperature to which the water is to be raised,
$d =$ volume of steam compared with the volume in water due to the temperature of the steam,
$w =$ quantity of water in cubic feet to be heated;

Then

$\frac{c - b}{966 + a - c} =$ quantity of water (say in cubic feet) that must be converted into steam having a temperature a, required to raise 1 cubic foot of water from b to c.

$\frac{(c - b) \times d}{966 + a - c} =$ quantity of steam in cubic feet, at temperature a, required to raise 1 cubic foot of water from b to c.

$\frac{w \times (c - b)}{966 + a - c} =$ quantity of water in cubic feet that must be converted into steam, and have a temperature at a, required to raise the given quantity of water from b to c.

$\frac{w \times (c - b) \times d}{966 + a - c} =$ quantity of steam in cubic feet, at temperature a, required to raise the given number of cubic feet of water from b to c.

Example. — What quantity of water in steam at 212° will raise 100 cubic feet of water from 60° to 200°?

$\frac{100 \times 140}{966 + 212 - 200} = 14.31$ cubic feet. *Ans.* Making $14.31 \times 1694 = 24241$ cubic feet of steam at 212 degrees.

To find the quantity of water at a given temperature required to reduce a given quantity of steam to a given temperature.

Let

$a =$ temperature of steam to be reduced,
$c =$ temperature to which the steam is to be reduced,
$b =$ temperature of the water injected.

Then

$\frac{966 + a - c}{c - b} =$ number of cubic inches of the water required to reduce 1 cubic foot of the steam from a to c nearly.

To find the Mean Force of Steam acting Expansively, ana the Advantage Gained.

N.	C.	N.	C.	N.	C.	N.	C.	N.	C.
1.1	1.095	2.1	1.742	3.1	2.131	4.2	2.435	6.3	2.841
1.2	1.182	2.2	1.788	3.2	2.163	4.4	2.482	6.5	2.872
1.3	1.262	2.3	1.833	3.3	2.194	4.6	2.526	7.	2.946
1.4	1.336	2.4	1.875	3.4	2.224	4.8	2.569	7.5	3.015
1.5	1.405	2.5	1.916	3.5	2.253	5.	2.609	8.	3.079
1.6	1.470	2.6	1.956	3.6	2.281	5.2	2.649	8.5	3.140
1 7	1.541	2.7	1.993	3.7	2.308	5.4	2.686	9.	3.197
1.8	1.588	2.8	2.030	3.8	2.335	5.6	2.723	10.	3.303
1.9	1.642	2.9	2.065	3.9	2.361	5.8	2.758	11.	3.398
2.	1.693	3.	2.099	4.	2.386	6.	2.792	12.	3.485

S = stroke of piston in inches.
d = distance the piston moves in inches before the steam is cut off.
P = pressure in pounds per square inch of the steam on the piston.
C = tabular quantity in column C, standing against the quantity in column N, that $= S \div d$.

When $S \div d$ = any quantity in column N (above table), then, if $S \div d$ be taken to represent the whole effect of the steam, or effect that would have been produced had the steam not been cut off, the quantity in column C (same table), standing against the quantity $S \div d$, will represent the effect, the steam being cut off as supposed; that is, the effect of the steam, cut off as supposed, will be to what it would have been had no cut-off taken place, as the quantity in column C to the quantity in column N, against it.

$\frac{CPd}{S} = C \times \overline{P \div (S \div d)}$ = mean pressure of steam in pounds per square inch on the cylinder, the steam being cut off *ad libitum.*

EXAMPLE. — The steam enters the cylinder with a force or pressure of 40 pounds to the square inch, is cut off when the piston has moved 32 inches, and the whole stroke of the piston is $7\frac{1}{4}$ feet (87 inches); required the mean pressure per square inch on the cylinder.

$87 \div 32 = 2.7$, the pressure, therefore, is to what it would have been, if the steam had not been cut off at all, as 1.993 to 2.7, as $\frac{738}{1000}$; and the mean force or pressure on the cylinder is

$1.993 \times 40 \times 32 \div 87 = 29.3$ lbs. per square inch. *Ans.*

Steam, under a pressure of 1 atmosphere, flows into a vacuum with a velocity of about 1400 feet a second, and into the air with a velocity of about 650 feet a second; its velocity under a pressure of 20 atmospheres is about 1600 feet a second, either into the air or a vacuum.

Atmospheric air at 60°, b. 30 in., flows into a vacuum with a velocity 1327 feet a second.

The velocity of a cannon ball (maximum) is about 2000 feet a second, when near the muzzle of the gun, but at the distance of 500 yards from the gun it is not above 1300 feet a second.

The greatest velocity of a rifle ball, near the muzzle of the gun, is 2012 feet a second.

The velocity of a musket ball, full charge, 18 balls to the pound, windage .05, near the muzzle of the gun, is 1600 feet a second.

Inflamed gunpowder expands with a velocity of about 5000 feet a second.

OF THE ECCENTRIC IN A STEAM ENGINE.

The *throw* of the eccentric is equal to twice the distance from the centre of formation to centre of revolution; that is, it is equal to the diameter of the circle of revolution minus the diameter of the circle of formation. If r represent the shortest, and R the longest, of all the radii in the eccentric, measuring from the centre of the circle of formation or axis of the revolving shaft; then $(R - r) \times 2 =$ the throw of the eccentric.

The *travel* of the valve is equal to the sum of the widths of the two steam openings, plus a slight excess of length to the valve more than just sufficient to cover the openings.

L = length of lever in inches on weigh or traverse shaft for working the valve.

l = length of lever in inches on weigh-shaft for eccentric rod.

v = travel of valve in inches.

t = throw of eccentric in inches.

$$vl \div \mathrm{L} = t; \quad vl \div t = \mathrm{L}; \quad \mathrm{L}t \div l = v; \quad \mathrm{L}t \div v = l.$$

To find the number of revolutions that each of two wheels that are geared together will make before the same teeth will come together again.

Rule. — Divide the number of teeth in each wheel by the greatest number that will divide both without a remainder; the greater quotient will be the number of revolutions made by the smaller wheel, and the less the number made by the larger. If both wheels cannot be divided by a common divisor, the smaller wheel will make as many revolutions as there are teeth in the larger, and the larger as many as there are teeth in the smaller.

Example. — Required the number of revolutions made by each of two wheels that are geared together, the larger having 66 teeth, and the smaller 21, before the same teeth come together again.

$$\frac{66}{21} \div 3 = \frac{22 \text{ revolutions of smaller wheel.}}{7 \text{ revolutions of larger wheel.}} \Big\} \textit{Ans.}$$

OF CONTINUOUS CIRCULAR MOTION.

When a series of wheels, or wheels, pinions, drums and pulleys, are so arranged that one, being set in motion, imparts motion directly to another, that to a third, and so on, all, at equal distances from their respective centres, will describe equal circles of gyration. As are their radii, diameters, circumferences or number of teeth, therefore, one to another, so are their number of revolutions one to another, or so are their turns in the same space of time.

In every machine there is some first point of impulse, or point at which the motive power is applied, and the circle of gyration described by that point is the periphery of the first moved, or first driven, of that machine. From some point in this circle, or from some circle described upon the shaft which it drives, the power is transmitted to a remoter or next contiguous movement, thereby the point so transmitting becoming the driver thereof. Thus the whole continuous chain consists, alternately, of driven and drivers throughout;

or, if we take the motive power into account of drivers and driven throughout.

Example. — A drum, on the main line of shafting, is 18 inches in diameter, and by means of a belt drives a pulley whose diameter is 12 inches; how many revolutions does the pulley make, to one revolution of the drum?

$$18 \div 12 = 1.5. \quad \textit{Ans.}$$

What portion of a revolution does the drum make, to one revolution of the pulley?

$$12 \div 18 = \tfrac{2}{3} \text{ of one revolution.} \quad \textit{Ans.}$$

Example. — The above drum makes 120 revolutions a minute; how many revolutions does the pulley which it drives make in the same time?

$$18 \times 120 = 2160 \div 12 = 180 \text{ revolutions.} \quad \textit{Ans.}$$

$$\text{Or,} \quad 2 : 3 :: 120 : 180 \text{ revolutions.} \quad \textit{Ans.}$$

Example. — The diameter of the driver is 18 inches, and makes 120 revolutions a minute; what must be the diameter of a pulley which it will drive at the rate of 180 revolutions a minute?

$$120 \times 18 \div 180 = 12 \text{ inches.} \quad \textit{Ans.}$$

Example. — A pinion of 8 teeth drives a wheel of 49; how many revolutions does the pinion make to one revolution of the wheel?

$$49 \div 8 = 6\tfrac{1}{8} \text{ revolutions.} \quad \textit{Ans.}$$

Example. — A pinion has 8 teeth, and makes 80 revolutions a minute; how many revolutions does a wheel make, in the same time, which has 49 teeth, and works in contact?

$$80 \times 8 \div 49 = 13\tfrac{3}{49} \text{ revolutions.} \quad \textit{Ans.}$$

Example. — A wheel has 49 teeth, and makes $13\frac{3}{49}$ revolutions in a given time; how many teeth must a wheel or pinion have to work in contact, and make 80 revolutions in the same time?

$$49 \times 13\tfrac{3}{49} \div 80 = 8 \text{ teeth.} \quad \textit{Ans.}$$

To find the number of revolutions made by the last, to one revolution of the first, in a train of wheels and pinions.

The last wheel, or pinion, in a train, whichever it be, is necessarily a driven; — therefore,

Rule. — Divide the product of all the teeth in the drivers by the product of all the teeth in the driven; the quotient is the number or ratio sought.

Example. — A wheel of 72 teeth drives a pinion of 14, upon whose shaft is a wheel of 56 teeth that drives a pinion of 10, upon whose shaft is a wheel of 35 teeth that drives a pinion of 6; how many revolutions does the last pinion make, to one revolution of the first wheel?

$$\frac{72 \times 56 \times 35}{14 \times 10 \times 6} = 168 \text{ revolutions. } \textit{Ans.}$$

Example. — A wheel of 72 teeth drives another of 29, upon whose shaft is a pulley of 24 inches diameter that gives motion to one of 10; what number of revolutions are made by the last pulley, to one revolution of the first wheel?

$$\frac{72 \times 24}{29 \times 10} = 5.96 \text{ revolutions. } \textit{Ans.}$$

The rule is well established, that, in training for the purpose of accumulating velocity, or for the purpose of diminishing an over accumulated, a certain mean or proportional velocity, between the several movers, should exist; and, further, that without some important reason for a higher ratio, the number of teeth on the wheel should not exceed 6 to 1 on the pinion with which it works.

The mean, or proportional velocity alluded to, is found by the following rule, and is applied as shown in Examples.

Rule. — Multiply the given and required velocities together, and extract the square root of the product, which is the mean sought.

Example. — From a wheel of 72 teeth, making 20 revolutions a minute, motion is to be conveyed, by help of two intermediate wheels, to a pinion having 15 teeth, which is required to make 120 revolutions a minute, or 6 to 1 of the first wheel; what number of teeth should be on each of the intermediate wheels?

$$\sqrt{120 \times 20} = 49 \text{ mean velocity.}$$

$$\left.\begin{array}{l} 72 \times 20 \div 49 = 29.4 \text{ teeth on 1st driven.} \\ 120 \times 15 \div 49 = 36.7 \text{ teeth on 2d driver.} \end{array}\right\} \textit{Ans.}$$

$$\text{Proof. } \frac{72 \times 37}{15 \times 29} 6\tfrac{1}{8}\text{; or, } \frac{72 \times 36}{15 \times 29} = 5.96 \times 20 = 119+ \text{ revolutions.}$$

Example. — A wheel of 72 teeth, making 20 turns a minute, is to drive another, on whose shaft is a wheel of 36 teeth, that is to drive a pinion at the rate of 120 turns a minute; how many teeth must be on the intermediate wheel, and how many on the pinion?

$$\sqrt{120 \times 20} = 49, \text{ and}$$

$$\left.\begin{array}{l} 72 \times 20 \div 49 = 29 \text{ teeth on intermediate wheel.} \\ 36 \times 49 \div 120 = 15 \text{ teeth on pinion.} \end{array}\right\} \textit{Ans.}$$

Example. — A wheel having 72 teeth, and making 20 revolutions a minute, is to drive another wheel, on whose shaft is a pulley of 28 inches diameter, from which pulley motion is to be conveyed to another pulley, required to make 120 revolutions a minute; what number of teeth should the intermediate wheel have, and what must be the diameter of the last pulley?

$\sqrt{120 \times 20} = 49$, and

$72 \times 20 \div 49 = 29$ teeth on intermediate wheel.
$28 \times 49 \div 120 = 11\frac{13}{30}$ inches diameter of pulley. } *Ans.*

The distance, from centre to centre, of two wheels to work in contact, given, and the ratio of velocity between them, to find their requisite diameters.

RULE. — Divide the given distance by the given ratio, plus 1, and the quotient will be the radius of the smaller wheel; subtract the radius of the smaller wheel from the given distance, and the difference will be the radius of the larger, which, multiplied by 2, respectively, gives the required diameters of each.

EXAMPLE. — The distance from centre to centre of two shafts is 28 inches, and one shaft is required to make three revolutions while the other makes one; what must be the diameters of the wheels which turn the shafts, (measured from their pitch lines,) to produce the required effect?

$28 \div 4 = 7$, and $28 - 7 = 21$; hence,
$7 \times 2 = 14$ in., diameter of smaller.
$21 \times 2 = 42$ in., diameter of larger. } *Ans.*

EXAMPLE. — The distance from centre to centre of two shafts is 40 inches; one makes 44 turns a minute, and the other is to make 110; what must be the diameters of the wheels at their pitch lines?

$110 \div 44 = 2.5$ ratio or mean velocity; then,
$40 \div \overline{2.5 + 1} = 11.43 \times 2 = 22.86$ in.
$40 - 11.43 = 28.57 \times 2 = 57.14$ in. } *Ans.*

To find the velocity of a belt.

The velocity of a belt is equal to the surface velocity of any drum or pulley over which it runs, or which it turns, in equal times and terms of measurements.

EXAMPLE. — A drum whose diameter is 6 feet makes 120 revolutions a minute; what is the surface velocity of the drum per minute, or what is the velocity of the belt per minute?

$6 \times 3.1416 \times 120 = 2262$ feet. *Ans.*

Velocity of belt $\div$ revolutions of drum $=$ circumference of drum.
Velocity of belt $\div$ circumference of drum $=$ revolutions of drum

To find the draft on a machine.

GENERAL RULE. — Multiply, continuously, all the driven wheels, by way of their teeth, and the diameter of the front roller, together, and, in like manner, all the drivers, by way of their teeth, and the diameter of the back roller, together, and divide the former product by the product of the latter; the quotient is the draft.

Example. — The driven wheels of a drawing frame head have, one 72 teeth, the other 40, and the diameter of the front roller is $1\frac{1}{10}$ inches; the drivers have, one 25 teeth, the other 30, and the diameter of the back roller is $\frac{9}{10}$ of an inch; what is the draft?

$$72 \times 40 \times 1.1 \div (30 \times 25 \times .9) = 4.69+. \quad \textit{Ans.}$$

Example. — The pinion on the front roller of a spinning frame has 40 teeth, and the diameter of the roller is $1\frac{1}{2}$ inch; the wheel on the back roller has 50 teeth, and the diameter of the roller is $\frac{3}{4}$ of an inch; the stud gears have, driver, 21 teeth, driven, 84; what is the draft?

$$84 \times 50 \times 1.5 \div (40 \times 21 \times .75) = 10. \quad \textit{Ans.}$$

To find the revolutions of the throstle spindle.

Rule. — Multiply the diameter of the cylinder by the number of revolutions it makes in a given time, and divide the product by the diameter of the whir; the quotient will be the number of revolutions of the spindle made in the same time.

Example. — The diameter of the cylinder is 8 inches, and makes 480 revolutions a minute; the diameter of the whir is $\frac{7}{8}$ of an inch; how many revolutions are made by the spindle per minute?

$$480 \times 8 \div .875 = 4388.6 \text{ revolutions.} \quad \textit{Ans.}$$

To find the number of twists per inch given to the yarn by the throstle

Rule. — Divide the number of revolutions of the spindle, in any given time, by the number of revolutions of the delivery (front) roller, multiplied by its circumference, in inches, made in the same time.

Example. — The diameter of the front roller is $\frac{3}{4}$ of an inch, and makes 110 revolutions a minute; the spindle revolves 4388.6 times in a minute; what number of twists per inch has the yarn?

$$.75 \times 3.1416 = 2.3562 \text{ inches circ. of roller; then,}$$
$$4388.6 \div (110 \times 2.3562) = 17 \text{ twists, nearly.} \quad \textit{Ans.}$$

TEETH OF WHEELS, &C.

EXPLANATIONS.

Pitch Line. — A circle defining the base of the working or impinging section of the teeth.

Pitch of a Wheel. — The distance from centre to centre of two adjacent teeth, measured upon their pitch line.

Length of a Tooth. — The distance from its base to its extremity.

Breadth of a Tooth. — The length of the face of the wheel.

Thickness of a Tooth. — The chord of the arc described upon it by the pitch line, the greatest cross section to the breadth.

Circumference of a Wheel. — The pitch line, and its diameter is measured therefrom. See *diagram* — GEOMETRY.

Pitch × 2.5 = breadth.	Pitch × .47 = thickness.
Thickness × 1.5348 = length.	Length × .65154 = thickness.
Thickness × 2.1277 = pitch.	Thickness × 5.31925 = breadth.

Pitch × number of teeth × .318 = diameter.

Diameter ÷ pitch × .318 = number of teeth.

Diameter ÷ number of teeth × .318 = pitch.

As the number of teeth on the wheel, + 2.25, are to the diameter of the wheel, so are the number of teeth or leaves on the pinion, + 1.5, to the diameter of the pinion.

EXAMPLE. — A wheel, 16 inches in diameter, and having 81 teeth, is to pitch with a pinion having 25 leaves; what must be the diameter of the pinion?

81 + 2.25 : 16 :: 25 + 1.5 : 5.093 inches. *Ans.*

As the number of teeth on the wheel, + 2.25, are to the diameter of the wheel, so are half the number of teeth on the wheel, + half the number of leaves on the pinion, to the distance their centres should have.

EXAMPLE. — A wheel is 16 inches in diameter, and has 81 teeth; the pinion with which it is to work has 25 leaves; what should be the distance from the centre of the wheel to the centre of the pinion?

81 + 25 = 106 ÷ 2 = 53; then,
81 + 2.25 : 16 :: 53 : 10.1862 inches. *Ans.*

As half the number of teeth on the wheel and pinion are to the distance from centre to centre of the wheel and pinion, so are the number of teeth on the wheel, + 2.25, to its diameter; or, so are the number of teeth on the pinion, + 1.5, to its diameter.

53 : 10.1862 :: 83.25 : 16 inches.
53 : 10.1862 :: 26.5 : 5.093 inches.

As the number of teeth on the wheel and pinion, — 3.75, are to the distance between the centres of the wheel and pinion, so is the circumference of the wheel to the diameter of the pinion, very nearly, and proving the almost strict accuracy of the foregoing.

106 — 3.75 : 10.1862 :: 16 × 3.1416 : 5.0075 inches.

To find the horse power, at a given velocity, of a cast iron tooth constructed on the foregoing principles.

RULE. — Multiply the breadth of the tooth, in inches, by the square of its thickness, in inches, and divide the product by twice the length, in inches; the quotient, multiplied by the velocity in feet per second, gives the reliable horse power at the velocity specified.

EXAMPLE. — The teeth on a wheel have each a breadth of 10.64 inches, a thickness of 2 inches, and a length of 3.07 inches, required their reliable strength, in horse power, at a velocity of 6 feet per second.

$\overline{10.64} \times \overline{2^2 \times 6} \div \overline{3.07 \times 2} = 41.59$ horse power. *Ans.*

JOURNALS OF SHAFTS.

To find the requisite diameter of a cast iron journal to resist torsion and stress in overcoming a given resistance at a given velocity.

Mr. Buchanan gives deductions, from which are derived the following rule, for ascertaining the proper diameter of the journal of a water-wheel, or first mover, in any machine.

Rule. — Multiply the resistance, in horse power, by 400, and divide the product by the number of revolutions of the wheel per minute; the cube root of the quotient is the requisite diameter of the journal, in inches. For shafts inside of the mill, to drive smaller machinery, use 200, instead of 400, for the multiplier, and, if the shafts are to drive still smaller machinery, use 100 as the multiplier.

Mr. Grier gives the mean of all these, or 240, as the multiplier, to resist torsion alone, and directs to take, for second movers, the diameter thus found, multiplied by 0.8, and for third movers the same diameter multiplied by .793.

If the journal is wrought iron, multiply the diameter, found by the preceding rule, by .963; if of oak, by 2.238.

Example. — The diameter of a water-wheel is 16 feet; the resistance it has to overcome (at its pitch with the *jack*) is 40 horse power, and the surface velocity of the wheel is 6 feet per second; what should be the diameter of its journals?

$\overline{60 \times 6} \div \overline{16 \times 3.1416} = 7$ revolutions of wheel per minute; and $40 \times 400 \div 7 = \sqrt[3]{2286} = 13.2$ inches. *Ans.*

HYDROSTATICS.

All fluids, at rest, press equally in every direction. The pressure exerted by them, therefore, can never be so little as their weight, and may, under circumstances, be to almost any conceivable extent greater. The downward pressure exerted by a fluid is its weight, and its weight is as the quantity; but the lateral pressure exerted is in a measure independent of quantity, being dependent upon depth, or vertical height.

Any given area, in any given section of a containing vessel, is pressed equal to the weight of a column of the fluid whose base is equal to the area pressed, and whose height is equal to the distance of the centre of gravity of that area, below the surface of the fluid;

this is the case whether the sustaining surface be horizontal, or vertical, or oblique.

The bottom of a containing vessel, therefore, whatever be its shape, sustains a pressure equal to the weight of the superincumbent fluid, or equal to the weight of a column of the fluid whose base is equal to the area of the bottom, and height equal to the distance from the bottom to the surface — equal to the area of the bottom, multiplied by the depth of the liquid, multiplied by its weight, in like terms of measurement.

And each side of the containing vessel, whatever number of sides there be, sustains a pressure equal to the area of that side multiplied by half the depth of the liquid, multiplied by its weight, in the same terms of measurement.

Thus, a rectangular vessel, whose sides and bottom are equal, and each two feet square, has a capacity of 8 cubic feet; it will hold, consequently, 8 cubic feet of fresh water, one cubic foot of which weighs $62\frac{1}{2}$ lbs. It will hold, therefore, $62\frac{1}{2} \times 8 = 500$ lbs. of water. Now, if we suppose this vessel filled with water, we have, according to the foregoing, a pressure on the bottom of $2 \times 2 \times 2 \times 62.5 = 500$ lbs.; a pressure exactly equal to the weight of all the fluid. And we have, upon each of the four sides, a pressure of $2 \times 2 \times 1 \times 62.5 = 250$ lbs.; a lateral pressure, therefore, equal to $250 \times 4 = 1000$ lbs., — equal to twice the pressure on the bottom, and showing the entire pressure exerted to be 300 per cent. greater than the weight of the water employed.

Again: if we suppose the above vessel contracted, laterally, to the extent that its sides are but 3 inches, or $\frac{1}{4}$ of a foot apart, throughout, and that its length is so extended that it still holds the 8 cubic feet of water, then we have, upon the bottom, whose area is only 9 square inches, a pressure of $.25 \times .25 \times 128 \times 62.5 = 500$ lbs. as before; and upon each side we have a pressure of $.25 \times 128 \times \frac{128}{2} \times 62.5 = 128000$ lbs.; making in all a pressure of $128000 \times 4 + 500$, — the enormous pressure of 512500 lbs.; and that too exerted by 8 cubic feet or 500 lbs. of water. It is easy to see that the same principles hold good under any extent of lateral area.

Example. — A sluice or flood-gate is 3 feet by $2\frac{1}{2}$, and its centre is 12 feet below the surface of the water; what pressure does the water exert upon it?

$$3 \times 2.5 \times 12 \times 62.5 = 5625 \text{ lbs.} \quad \textit{Ans.}$$

Example. — A dam, that presents a perpendicular resistance to a stream, is 40 feet long and 15 feet high; the water is level with its top; what pressure does the dam sustain, supposing the water at rest, and what is the mean pressure against it per square foot?

$40 \times 15 \times \frac{15}{2} \times 62.5 = 281250$ lbs., pressure against the dam; and

$281250 \div \overline{40 \times 15} = 468\frac{3}{4}$ lbs., mean pressure per sq. foot. *Ans.*

Example.—The same stream, the same length of dam, and the same vertical height as the preceding, and the dam sloping into the stream against the current, 30 feet from its base; required the pressure against the dam, and the average pressure per square foot.

$40 \times 15 \times 7.5 \times 62.5 = 281250$ lbs., press. as before.
$\sqrt{15^2 + 30^2} = 33.541$ feet, slant height of dam; and
$281250 \div \overline{40 \times 33.541} = 209.63$ lbs. av'g pres. per sq. foot. } *Ans.*

HYDRAULICS.

The established law for the velocity of all bodies falling from rest is given under Gravitation, viz., that $\sqrt{\text{height} \times 64.33}$, or $\sqrt{\text{height}} \times 8.02$ = velocity per second, or velocity in one second of time, the velocity and height both being in the same denomination of measure. And from what has been said concerning pressure, under Hydrostatics, it is evident that the same law will cause water, or other fluid, to flow through an opening in the side of a reservoir, or dam, with the same velocity that a body would attain falling perpendicularly through a space equal to that between the surface of the water and the centre of the opening alluded to; and that, consequently, theoretically, the quantity thus discharged, in any given time, will be equal to the product of the velocity and area of the opening, multiplied by that time.

The theoretical law, however, last adduced, under ordinary circum stances, does not apply. And the quantity discharged, owing to the contraction of the fluid vein, caused by the friction of the particles against the sides of the opening, falls short of that theoretically due. The only instance known in which the full force of the law may be obtained, is where the discharge is made to issue through a straight tube whose form is the frustum of a cone, its length being half the diameter of the aperture, and the diameter of the receiving end to that of the discharging end as 5 to 8; when a fluid is allowed to pass through such an opening, no contraction of the vein takes place.

From various carefully conducted experiments by M. Morin, Eytelwein, Bossut, and others, the following practical rules for ascertaining the quantity discharged through different openings, and under different heads, are derived:—

1. When the issue is through a circular opening, its upper vertical point as high as the surface of the fluid, estimate the height or head from the centre of the opening to the surface of the fluid, and use 5.4, instead of 8.02, as the coefficient of quantity.

2. When the opening is circular, and under a head equal to its diameter, estimate the head as in the preceding, and use $7\frac{1}{2}$ as the coefficient.

3. When the issue is through a rectangular orifice, two or more feet beneath the surface, estimate the head from the centre of the orifice to the surface of the water, and use 5.1 as the coefficient.

4. When the discharge is from a rectangular opening, extending as high as the surface of the fluid, estimate the head from the bottom of the opening to the surface of the water, and use 3.4 as the coefficient. This rule applies to water flowing over a dam, or from a notch or slit cut in its side, &c.

It may be proper to add, that if the orifice is small and under considerable head, the quantity discharged, relatively, will be slightly less than would be discharged if the opening were nearer the surface.

From the foregoing we obtain the following

GENERAL RULE.

Multiply the square root of the height, or head, (as estimated in the foregoing,) in feet, by the coefficient of quantity given as pertaining thereto, and the quotient will be the effective velocity in feet per second of the discharge; which, multiplied by the area of the opening in feet, gives the quantity in cubic feet discharged in a single second, or in each second of time.

Example. — A rectangular opening in the side of a dam is 6 feet long and 8 inches deep; and the distance from the centre of the opening to the surface of the water is 4 feet; required the quantity of water discharged in each second of time.

$$\sqrt{4} = 2 \times 5.1 \times 6 \times \tfrac{2}{3} = 40.8 \text{ cubic feet.} \quad \textit{Ans.}$$

Example. — A dam is 60 feet long, and the water flows over its entire length 6 inches, or ½ foot deep; what quantity flows over per second?

$$\sqrt{.50}^{*} = .7071 \times 3.4 \times 60 \times .5 = 72\tfrac{1}{8} \text{ cubic feet.} \quad \textit{Ans.}$$

WATER-WHEELS.

The many uncertainties and doubts which existed until lately concerning the best mode of constructing a water-wheel, with the view to obtain a maximum of effect, the velocity at which the wheel should move, and other requisites pertaining to it generally, have, in a great measure, through investigations and experiments by the Franklin Institute in this country, added to those by M. Morin, in France, and other parties interested, been removed; and the whole seems now to have nearly subsided into the following general and demonstrative conclusions: —

* The decimal .50 is the same value as .5, = ½, but it will be recollected that to obtain the root of a decimal full periods must be used.

1. That, to obtain a maximum of effect by a horizontal water-wheel, the water must be laid upon the wheel on the stream side, and at a point or line on the wheel about 52¾ degrees distant from its summit; or, the effective fall — distance from the centre of the discharging orifice to the bottom of the wheel pit, or water therein, when the wheel is at rest — being 1, the diameter of the wheel should be 1.108.

2. That the periphery of the wheel ought to move at a velocity in feet per second equal to about twice the square root of the number of feet effective fall;* that the number of buckets should equal 2.1 times the wheel's diameter in feet; and that due means be adopted for the escape of the air from the buckets, either by causing the stream to flow some inches narrower than the wheel, or otherwise.

3. That a head of water is requisite sufficient to cause the velocity of its flow to be as 3 to 2 of the velocity of the wheel (ordinarily not less than two nor more than three feet).

4. That a wheel of good workmanship, constructed and geared according to these restrictions, will return, as a maximum, about 80 per cent. of the power employed.

5. That, because of water producing a less effective power by impulse than gravity, turbines, or wheels through which the motion is obtained by reäction, are greatly preferable to undershot or low-breast wheels; that they are, seemingly, as well adapted to great as to small falls, returning, in either instance, under favorable circumstances, an useful effect of from 70 to 78 per cent. of the power expended; that their velocities may vary considerably from that affording the maximum effect; (⅔ of their light velocity,) without materially diminishing their effect; that they are nearly as effective when drowned to the depth of several feet as when working free, thereby making use of a greater fall than can be obtained, at the same locality, for any other wheel; and that they receive variable quantities of water without altering the ratio of the power to the effect.

These considerations, taken in connection with the less important, that they are durable, not more liable than others to require repairs, occupy less room in their position, and cost decidedly less than other motors of equal efficacy, are fast bringing this class of wheels into favor and use.

To find the power of a stream to turn an overshot or pitched-back wheel.

Example. — The entire height — head and fall — of a stream is 19 feet, and the quantity of water flowing is that which may be drawn through a rectangular opening 16 feet long and 3 inches deep; required the greatest exertive force of the stream, in horse-power, applied as above supposed.

$16 \times 12 \times 3 \div 144 = 4$ feet area of discharge.
$19 \times .1221 = 2.32$ feet requisite head.
$\sqrt{2.32} = 1.523 \times 5.1 = 7.76$ feet, effective velocity per second.

* The practice is becoming very general to gear all wheels, great or small, to a velocity of about 6 feet per second.

7.76 × 4 = 31 cubic feet discharged per second.
31 × 60 = 1860 cubic feet discharged per minute.
1860 × 62.5 = 116250 lbs. discharged per minute.

$116250 \times \overline{19 - 2.32} = 1939050$ lbs. momentum.
1939050 ÷ 33000 = 58¾ horses' power. *Ans.*

To find the requisite dimensions of a wheel, based upon the preceding principles, adapted to the foregoing stream.

19 — 2.32 = 16.68 feet effective fall.
16.68 × 1.108 = 18.48 feet diameter of wheel.
360° : 18.48 : : $52^\circ.75$: 2.7 feet of wheel above discharging orifice.
18.48 — 2.7 = 15.78 feet of wheel below discharging orifice.
16.68 — 15.78 = 0.9 foot clearance of wheel.

$\sqrt{16.68} = 4.08+ \times 2 = 8.16$ feet velocity of wheel per second.
18.48 × 2.1 = 39 buckets.
18.48 × 3.1416 = 58.06 feet circumference of wheel.
8.16+ × 60 = 490.17 feet velocity of wheel per minute.
490 ÷ 58 = 8.44 revolutions of wheel per minute.
1860 ÷ 490 = 3.8 feet sectional area of buckets. The buckets, to properly retain the water and avoid waste, should be but half full; therefore —
3.8 × 2 = 7.6 feet practical sectional area; and to allow sufficient room for the escape of the air, the wheel should be, say 9 feet broad.
7.6 ÷ 9 = .85—, say 1 foot depth of shrouding.

$1860 \div \overline{39 \times 8.44} = 5.68$ cubic feet of water received by each bucket.
58 ÷ 39 = 1.49 foot breadth of bucket.
5.68 ÷ 1.49 = 3.8 feet sectional dimensions, as before.
3.8 × 2 × 1.49 = 11.36 feet practical capacity of buckets, more, allowance as above.

From a well constructed wheel the water begins to empty at about 5 feet from the bottom; therefore —

$18.48 - \overline{2.32 + .5 + 5} = 10.66$, ratio of diameter to loaded arch.

18.48 : $\frac{39}{2}$: : 10.66 : 11.2 loaded buckets.

11.2 × 5.68 × 62.5 = 3971 lbs. on loaded arch.

Or, a very good and safe rule for determining the weight, in pounds, constantly on the wheel, is this: — Multiply $\frac{4}{9}$ of the buckets on the wheel by the number of cubic feet of water received by each, and that product by 40. Ex.

39 ÷ 9 = 4⅓ × 4 × 5.68 × 40 = 3938 lbs. load.
3971 × 490 = 1945790 lbs.; or, } momentum, or
39- × 8.44+ × 5.68- × 62.5 × 16.68 = 1939050 lbs. } exertive force.
1939000 ÷ 33000 = 58.75 × .75 = 44 h. p. effective. *Ans.*

From the foregoing a very good practical rule is derived for determining the requisite head for any given velocity of wheel; thus —

$\overline{\text{height} + 8.16} : 2.32 :: \overline{\text{height} + \text{required velocity}}$: required head.

Example. — The entire height is 16 feet, and the velocity of the wheel is to be 6 feet per second; required the necessary head.

$\overline{16 + 8.16} : 2.32 :: \overline{16 + 6} : 2.11$ feet. *Ans.*

It has been demonstrated that in practice nearly two feet head is required to generate a velocity of 5 feet per second, and this rule gives

$\overline{10 + 8.16} : 2.32 :: \overline{10 + 5} : 1.916$; or, $\overline{25 + 8.16} : 2.32 :: \overline{25 + 5} : 2.099$, showing a difference of only .18, or a mean error of about one inch, between the two extremes — 10 feet head and fall, and 25 feet head and fall.

If it were intended to construct a wheel for the foregoing stream, to run at a less velocity, say at 6 feet per second, (the other restrictions to be maintained,) then we should have

6 × 60 = 360 feet velocity of wheel per minute.
360 ÷ 58.83 = 6.12 revolutions of wheel per minute.
1860 ÷ 360 = 5 feet sectional area of buckets. The depth of the buckets should seldom vary much from 12 inches; the required breadth of the wheel, therefore, in this instance, would be about 11 feet. But it is customary, perhaps unadvisedly, when the velocity is intended at 6 feet or less, to place the buckets a little nearer together than com-

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ports with the preceding; in which case, of course, a less breadth of wheel would be required.

If we multiply the number of cubic feet flowing upon the wheel per minute, by the effective fall, and divide the product by 700, we obtain the effective horse-power; thus—

$$1860 \times 16.68 \div 700 = 44.3 \text{ horses' power.}$$

The power should be taken from the wheel on the side to which the water is applied, and at a point horizontal to the centre of the wheel; and belts, instead of gears, for all the prime movers, should be used as far as practicable.

DYNAMICS.

UNIFORM MOTION.

P = mechanical power in pounds, or power in effects.
F = force in pounds, or weight or resistance to be overcome.
V = velocity of the force F in feet per second.
t = time in seconds during which F is in motion.
s = space in feet through which F moves in the time t.
m = momentum of the force F in the time t.
M = mass, or moving matter in effects.
W = work in foot-pounds of power.

$$P = FV = Fs \div t. \quad F = P \div V = Pt \div s. \quad V = P \div F = s \div t.$$
$$t = s \div V = Fs \div P.$$
$$s = Vt = Pt \div F. \quad m = Ft = MV. \quad M = Ft \div V = W \div V^2.$$
$$W = Fs = Ftv = MV^2 = Pt.$$

UNIFORMLY ACCELERATED MOTION.

$$F = V \div gt = 2s \div gt^2 = V^2 \div 2gs. \quad V = 2s \div t = Fgt = \sqrt{(2gFs)}.$$
$$s = \tfrac{1}{2}tv = \tfrac{1}{2}gFt^2 = V^2 \div 2gF. \quad t = 2s \div V = V \div gF = \sqrt{(s \div \tfrac{1}{2}gF)}.$$

MOTION OVER A FIXED PULLEY.

Let A and a represent the opposing forces or weights.

$$F = \frac{A - a}{A + a}. \quad s = \frac{(A - a)gt^2}{2(A + a)} = \tfrac{1}{2}gt^2F. \quad A + a = M.$$

$g = 32.166$ feet per second. (See p. 85.)

HYDROSTATIC PRESS.

A = transverse area of cylinder piston.
D = diameter of cylinder piston.
a = transverse area of forcing-pump piston.
d = diameter of forcing-pump piston.
b = diameter of safety-valve.
f = pressure in pounds on safety-valve that prevents it from rising.

$$a : A :: F : P. \quad d^2 : D^2 :: F : P. \quad D^2 : b^2 :: P : f.$$

GENERAL APPLICATIONS OF PRINCIPLES IN DYNAMICS.

To find the power of a stream issuing from an opening in the side of a reservoir, or flume.

$h' =$ head of water, in feet. See HYDRAULICS, p. 322.
$h =$ effective fall, or height of the column of water, in feet.
$k =$ practical substitute for $\sqrt{(2g)}$ due to the form and position of the opening. See HYDRAULICS, pp. 322, 323.
$V = k\sqrt{h'} =$ velocity of the water in feet per second through the opening, or discharging orifice.
$\Delta =$ area of the opening, in square feet.
$Q = \Delta V =$ quantity of water, in cubic feet, discharged per second.
$H =$ actual or theoretical horse-power of the stream.
$F = 62.5\Delta h =$ weight of the column of water, in pounds.
$W = 60FV =$ work in foot-pounds of power per minute.

$$H = \frac{W}{33{,}000} = \frac{Qh \times 62.5 \times 60}{33{,}000} = 0.1136Qh.$$

To find the power of a bucket-wheel.

$n =$ number of buckets on the wheel.
$q = \frac{1}{2}$ capacity of a bucket, in cubic feet.
$V =$ proposed velocity of the circumference of the wheel, in feet, per second. See WATER-WHEELS, p. 324.

$F = \frac{62.5qn}{\pi} \times 0.8936 = \frac{160qn}{9} =$ weight of water, in pounds, that the wheel can constantly carry.

$$W = 60FV. \quad H = \frac{W}{33{,}000} = 0.3232qnV.$$

To find the power of a turbine wheel.

$h =$ height, in feet, of the column of water resting on the wheel.
$\Delta =$ area of the ventage of the wheel, in square feet.
$V = \sqrt{(2g)}\sqrt{h} = 8.02\sqrt{h} =$ velocity, in feet, per second, at which the water enters the wheel.
$Q = \Delta V = \Delta\sqrt{(2gh)} =$ quantity of water, in cubic feet, discharged per second.

$$F = 62.5\Delta h. \quad W = 60FV. \quad H = \frac{W}{33{,}000} = 0.1136Qh.$$

To calculate the power of a hydraulic ram.

$F = 62.5\Delta h. \quad W = 60FV. \quad H = W \div 33{,}000 = 0.1136Qh.$

In the preceding calculations I have thought proper to express the *actual* power, force, and velocity, in all cases, and to avoid any allusion to what is called the *effective*. The effective power of a machine depends upon so many conditions, — the unavoidable friction of its parts, varying more or less in different machines, supposed to be in all respects equal; the mechanical adjustments and adaptations of the parts; the material, or materials, of which it is composed, and the workmanship generally, — that it admits of no fixed mathematical formula, correctly expressive of its worth, unless the working or sensible force is known; of which I am about to incidentally speak, and which I shall define.

QUERY IN DYNAMICS. — Is there a *quantity*, which, if taken as a constant, will equalize FORCE with TIME and SPACE?

Now, to the mere mathematician this proposes nothing; but to science it propounds an important question, which, I believe, has not been answered, if ever started before. But science answers nothing *per se*. She does, however, suggest, and seems to whisper that there is such a quantity. Then, if so, what is it?

SCIENCE: We have seen by the law of uniformly accelerative velocity, or gravitation, that $V = \sqrt{(2g)}\sqrt{s} = gt$ for any length of time, in which V represents the velocity at the close of the time, g the constant acceleratrix, s the space described in the time, and t the time in units of time.

MATHEMATICS: Wherefore $V = g$, and $s = \frac{1}{2}g = \frac{1}{2}V$ in the first unit of time.

SCIENCE: But $\frac{1}{2}g$ now represents a space described by a uniformly accelerative velocity.

MATHEMATICS: The mean of which is equal to $\sqrt{(\frac{1}{2}g)} = \sqrt{(\frac{1}{2}V)}$.

LOGIC: And, therefore, equal to the *uniform* velocity, which is equal to the space, in the first unit of time.

SCIENCE: But, in uniform velocity, spaces and velocities are equal to each other in all equal times.

LOGIC: Therefore $\sqrt{(\frac{1}{2}g)}$ represents the space in uniform velocity, under the influence of gravity, that is equal to each unit of time; and, consequently, since the velocity divided by the product of the space and time is equal to the force in all cases, that is equal to a unit of force in each unit of time.

Thus, there being no fallacy in the preceding, we have answered the first query by showing that there positively is such a quantity, and the second, by showing its algebraic value. The law of uniform velocity, therefore, and the law of mechanical force and velocity, are involved in, and derivable from, the law of gravitation.

The position taken, then, is this, viz., that one unit of mechanical force, constantly applied, will move a body equal in uniform velocity to $\sqrt{(\frac{1}{2}g)} = 4.01$ feet a second, the influence of gravity upon the body being with the same kind of force counterpoised; or, in other words, that the addition of one unit of constantly applied force to that of the same kind required to overcome the *vis inertia*, or resistance to motion in a machine, will move it, equal in uniform velocity to $\sqrt{(\frac{1}{2}g)} = 4.01$ feet a second, and so, proportionally, for superadded force or forces.

I will now introduce rules predicated upon this law, in place of the common rules, for calculating the power, &c., of steam-engines; where, if the law be thought to be empirical, it can easiest be tested by experiments.

$\Delta =$ transverse area of steam-cylinder piston, in inches.

$d =$ diameter of steam-cylinder piston, in inches.

$P =$ pressure per steam-gauge (the vacuum pressure included for a condensing engine).

$p =$ pressure per steam-gauge (the vacuum pressure included for a condensing engine) that is required to barely set the unloaded machine in motion.

$V =$ velocity of piston in feet per second due to a unit of sensible or working force, $= \sqrt{(\frac{1}{2}g)} = 4.01$.

$P - p =$ sensible, or working force.

$$F = \Delta(P - p). \quad W = 60FV. \quad H = \frac{W}{33{,}000} = \frac{d^2(P-p)}{174.6335} \text{ effective.}$$

Thus we have the *effective* horse-power of a steam-engine at given pressures, P and p; and the *normal* horse-power of a steam-engine, p being practically known or assigned, and P being the maximum pressure that may be constantly carried with safety; which last pressure may be assigned by theory, based on the thickness of the plate, the quality of the material, &c.

$$P - p = f = s \div Vt. \quad P = s \div Vt + p.$$

EXAMPLE. — What working-force, f, must be constantly applied to a body, in order to move it s, 25 feet, in t, $2\frac{1}{2}$ seconds of time?

$25 \div (4.01 \times 2.5) = 2.494$ units of working-force. *Ans.*

EXAMPLE. — What total force, P, must be constantly applied to a body whose *vis inertia*, p, is 4 pounds, in order to move it s, 18 feet, in t, 2 seconds of time?

$18 \div (4.01 \times 2) = 2.2444 + 4 = 6.2444$ lbs. *Ans.*

HEAT — CALORIC.

SENSIBLE, LATENT, SPECIFIC, ETC.

SENSIBLE HEAT is that description of heat in a body which is sensible to the touch, and which is indicated by a common thermometer.

Latent heat is that description of heat in a body which is insensible to the touch, and which the thermometer does not indicate; but which, by chemical, or other forces, can be, more or less, set free and made sensible.

The specific heat of a body' is the *quantity* of the substance (?) heat, that it contains at a given density and temperature. It is usually, however, expressed in comparison with that of water as the unit, or with that of atmosphere as the unit, and for equal weights at like temperatures, about 60° Fahrenheit.

Let $l + t$ represent the sum of the latent and sensible heat of saturated steam, at $t°$ of sensible heat, under a pressure in units of force, p, of 1 atmosphere; then

$\frac{l+t}{pg} = \frac{1178.6}{472.752} = 2.493062$, the specific heat of saturated steam at $t = 212°$ F., under a pressure of 1 atmosphere, $p, = 14.7$ pounds per square inch.

$\sqrt{\frac{pt}{\frac{1}{2}g}} + 1 = 14.9214213$, the atomic weight of saturated steam at $t = 212°$ F., under a pressure of 1 atmosphere.

$\frac{l+t}{pg} \times \left(\sqrt{\frac{pt}{\frac{1}{2}g}} + 1\right) = 37.2$, a constant, equal to the product of the specific heat and atomic weight of all substances at $t = 212°$F.: whence $37.2 - 1 = 36.2$, the atomic weight of the atmosphere; and $2.493062 + 1 \div (9 + 1) = 0.3493$, the specific heat of steam under a pressure of 1 atmosphere, water as unity.

And, since the specific heat of the atmosphere is known to be 0.2669 when that of an equal weight of water is 1, both at 60°, F., we have

$(9 + 1) \div 0.2669 = 37.4672$, a constant, equal to the product of the specific heat and atomic weight of all substances, at $t =$ 60°, F.

$\frac{pt}{\sqrt{(\frac{1}{2}g)}} - [\sqrt{(\frac{1}{2}g)} + 1] = 772.1471$, the mechanical equivalent of a unit of heat in foot-pounds; i.e., one unit of heat will raise 1 pound weight 772.147 feet high.

Now, in regard to the numerical values assigned to the elements introduced in the foregoing, no warranty, of course, can be offered that they are strictly correct; since, at best, they are mostly but settled conclusions, based on the results of carefully conducted experiments. But the theory introduced, it is believed, will be found tenable; nevertheless, it is offered, open to further and more critical investigation.

The specific heats of the gases increase as their densities decrease, but somewhat less rapidly than the diminutions in density; the exact ratio, I believe, is not known.

The sensible heat of water is augmented by violent agitation, according to Mr. Dalton, as follows; viz., from 60° F., in ½ hour, 10°; in 1 hour, 14.5°; in 2 hours, 19.5°; in 3 hours, 29.5°; in 5 hours, 39.5°; and in 6 hours, 42.5°.

The sensible heat of atmospheric air is increased by sudden compression to ⅕ its normal bulk about 500°.

The sensible heat of soft iron is increased by receiving a few rapid and strong blows with a common hammer to a bright red heat in day-light, equal to about 900°, and always, as in the case of water, more by the first blow than by the next succeeding, and so on.

Sensible and latent heat of steam. — Regnault.

Sensible°.	Latent °.	Sensible°.	Latent °.	Sensible°.	Latent °.	Sensible°.	Latent °.
32	1092.6	176	991.8	248	936.6	338	874.8
104	1042.2	212	966.6	266	927.0	374	849.6
140	1017.0	230	952.2	302	901.8	410	822.6

Latent heat of different substances — equal weights.

Alcohol,	364°	Ice,	142.6°	Steam,	966.6°
Ammonia,	860°	Lead,	162°	Sulphur,	17°
Beeswax,	175°	Mercury,	157°	Tin,	500°
Bismuth,	22°	Phosphorus,	9°	Water,	966.6°
Ether,	163°	Spermaceti,	148°	Zinc,	493°

Specific heat of equal weights of different substances (as commonly reported) compared with that of water as 1.

Water,	1.	Hydrogen,	3.405 / 3.294	Platinum,	0.034
Air,	0.2669			Silver,	0.057
Alcohol,	0.7	Ice,	0.504	Steam,	0.475 / 0.347 / 0.365
Antimony,	0.054	Iron, wrought,	0.113		
Bismuth,	0.023	Lead,	0.031 / 0.029		
Brick, clay,	0.202			Steel,	0.116
Carb. acid,	0.219	Lime,	0.217	Spts. turpentine,	0.467
Carb. oxid gas,	0.288	Linseed oil,	0.53	Sulphur,	0.209 / 0.188
Cast iron,	0.13	Mercury,	0.032		
Charcoal,	0.241	Nickel,	0.104	Sulphuric acid,	0.334
Cobalt,	0.15	Nitric acid,	0.661	Tin,	0.048 / 0.056
Coke,	0.202	Nitrogen,	0.275		
Copper,	0.095	Nitro-oxid gas,	0.231	Tellurium,	0.091
Ether,	0.517	Olive oil,	0.31	Woods, avg.,	0.51
Glass, crystal,	0.195	Oxygen,	0.218 / 0.361	Zinc,	0.095
Gas of oils,	0.421				
Gold,	0.029 / 0.032	Petroleum,	0.468		
		Phosphorus,	0.189		

Capacity for heat expresses the relative ability of a body to retain its respective *specific heat*, compared, usually, with that of water as the unit, and at equal weights, or equal volumes; and the capacities for heat of different bodies are inversely as their densities.

Relative capacities for heat of different substances (as commonly reported).

Names of bodies.	Equal weights.	Equal volumes.	Names of bodies.	Equal weights.	Equal volumes.
Water,	1.000	1.000	Iron,	0.126	0.993
Brass,	0.116	0.971	Lead,	0.043	0.487
Copper,	0.114	1.027	Mercury,	0.036	....
Glass,	0.187	0.443	Silver,	0.082	0.883
Gold,	0.050	0.966	Tin,	0.060	....
Ice,	0.900	....	Zinc,	0.102	....

SECTION VI.

COVERINGS OF SOLIDS OR PROBLEMS IN PATTERN CUTTING.

Under this head, I propose chiefly to contemplate patterns that are applicable to the wants and purposes of *Tin-Plate and Sheet-Iron Workers;* and, in treating of the construction of these, my main purpose will be to offer clear and unmistakable step-by-step directions for constructing them *practically*, and, as far as admissible by *mechanical* means; to the end that the student unacquainted with the principles involved in his tasks, and reluctant to enter into mathematical calculations, can nevertheless accomplish his purposes, and with accuracy and despatch: and I shall accompany the proceedings with diagrams for illustration and reference. But since I shall be obliged to view the patterns *theoretically* and *analytically* in all their parts, in order to devise the best rules for constructing them practically, I shall deem it not unadvisable, with reference to many of them at least, to state the laws and the mathematical data upon which the directions for their construction will be predicated. Moreover, many of the patterns will be found of a high geometrical type, governed by inflexible laws, and capable of mathematical investigation and measurement in all respects; and to present these only in their naked aspects of mere mechanical contrivances, to be fixed in the memory, or copied at will, would seem out of place in a work of this kind.

Problem 1 of the following series will embrace in its solution all the principles involved in the construction of the whole class of patterns to which it will relate; and, more or less interwoven with its solution mechanically, I shall, once for all, with reference to the class, enunciate the laws and define their bearings, to the full end of constructing them *theoretically* and mathematically; and I shall do this in as brief a manner and as free from technicalities as the circumstances will admit of.

But, before proceeding to the solution of problems, it may be proper, perhaps, to explain the meaning of some terms that I shall be liable to make use of; and it may as well be done here, perhaps, as elsewhere.

A vessel in the form of a frustum of a cone or truncated cone, is a *flaring* vessel having circles for its bases.

The lateral portion of a conical vessel or cylindrical vessel is the *side* or *body*.

The bases of a vessel are the *ends*.

The fixed base of a vessel is called the *bottom;* and the movable base, the *cover*.

The slant depth of a vessel (and none but flaring vessels have slant depths) is the depth of the side.

The diameters of a vessel are the diameters of the bases or ends.

The perpendicular depth of a vessel is its depth.

The circumference of a circle is equal to the diameter multiplied by 3.1416; or it is equal to the diameter multiplied by 355, and divided by 113; or it is nearly equal to the diameter multiplied by 22, and divided by 7.

The diameter of a circle, therefore, is equal to the circumference divided by 3.1416; or it is equal to the circumference multiplied by 113, and divided by 355; or it is nearly equal to the circumference multiplied by 7, and divided by 22. The Greek letter π, if met with in connection with the problems, will invariably mean 3.1416, or the ratio of the circumference to the diameter, the latter being 1.

$\Theta =$ Solidity; $\Delta =$ Area; $\Omega =$ Curved or convex surface of a Solid.

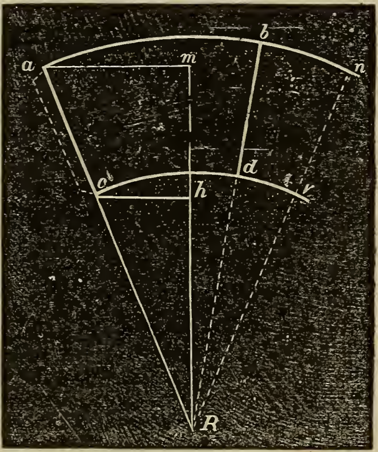
In geometry, written lines are limited by the letters or characters that are placed at their extremities; and, in the text, they are announced by the same letters or characters written with a space between them. Thus *a b*, *k m*, *c z*, *&c.*, in the text refer to the lines limited by *a b*, *k m*, *c z*, *&c.*, in the diagrams; but the values of these lines, that is their lengths, when they are introduced into equations by the letters that limit them, are otherwise expressed: thus $\overline{a\,b}$ or $(a\,b)$ in the text or an equation means the *length* of the line *a b*, or that is limited by *a* and *b*; $\overline{a\,b}^2$ or $(a\,b)^2$ means the square of the length or line *a b*; $2\,\overline{a\,b}$ or $2(a\,b)$, twice the length of the line *a b*, *&c.*

In algebraic notations, factors and numeral co-efficients and factors are usually written without the sign of multiplication or a space between them; thus abc, $2a$, $12ab$, $(ab - d)(cd + m)$, are to be read, $a \times b \times c$, $2 \times a$, $12 \times a \times b$, $(a \times b - d)(c \times d + m)$, that is, the *difference* of *d* and the product of *a* into *b* is to be multiplied by the *sum* of the product of *c* into *d* and the quantity *m*.

PROB. 1. — *To construct a Pattern for the Lateral Portion of a vessel in the form of a Frustum of a Cone of given Diameters and Depth.*

The chief principle involved in the construction of this description of patterns is easily explained: it is that of a right cone placed upon its side, and rotating on a plane. If a cone so placed and starting from rest make one revolution, or, in other words, roll once over, its whole lateral surface, correctly delineated, may be supposed to be described upon the plane; if it make a half-revolution or roll half-over, half its lateral surface in like manner delineated may be supposed to be described; and so on for any partial or fractional rotation whatever: thus the slant height of the rotating cone will be the radius of the arc that will be described by the rotating base, and the arc so described will be that of the lateral surface, lateral portion, side, body, or covering. The rotating cone, then, that will describe the greater arc of the lateral surface of a frustum, must be a cone including the frustum; and that that will describe the lesser must be the same cone, less the frustum.

RULE. — Place the square suitably on the plate from which the pattern is to be taken, as *a m R*, diagram, and draw to any sufficient length a line *m R*; and from *m*, on the other arm of the square, set off a known measure (the whole or any desired aliquot part, as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$) of the greater given base, *m a*, and draw that measure; then drop the square perpendicularly down on the line *m R*, from *m*, equal to the given depth of the vessel, *m h*, and set off in like manner the same known measure of the lesser given base, *h c*, and draw that measure; then draw a line, *a c R*, through the points *a* and *c*, to the line *m R*, and cutting that line in *R*.



Next, with the distance *R a* in the dividers or on the beam compasses, and *R* the centre, describe an arc, as *a b*, to any sufficient length; and with the distance *R c*, and

R the centre, describe a parallel arc, as *c d*, to any sufficient length.

We have now defined the curves for the level surfaces (top and bottom) of the given vessel, and the depth of its sides; or, in other words, we have defined the *ratio* of the given diameters and the slant depth of the vessel; and have thus far a pattern, in some sort, for a vessel of this general form, having the same slant depth, the same ratio of diameters, and the diameters varying from almost nothing to almost twice the radii to which the respective arcs have been drawn; or, in another point of view, having the same slant depth, the same ratio of diameters, and a depth varying from almost nothing to almost the depth of the side. A right section taken out indiscriminately, *a c v n*, for example, would be a pattern for this kind of vessel at a fixed slant depth and a fixed ratio of diameters, and such patterns are sometimes used; but clearly it would be no pattern for a vessel of a given perpendicular depth and given diameters, since it would be no known measure of the lateral surface required. The slant depth and the ratio of the diameters remaining constant, the perpendicular depth varies, as we have seen, with the diameters; and the given diameters, it appears, have as yet in no degree been fixed or defined, only their ratio has been defined. But since the perpendicular depth varies with the diameters, and the ratio of the given diameters has been defined, it follows, that, if we were to fix the given depth, we should thereby fix the given diameters; so, if we were to fix one of the given diameters, we should thereby fix the given depth and the other given diameter. But we cannot fix the depth of a flaring vessel by describing it upon the side; nor can we in any way fix the given diameters, *except by their circumferences upon the arcs which we have drawn.*

It thus appears, that, before we can proceed to complete our pattern, we must know the circumference of one of the given bases, and must decide what the measuring unit of the pattern shall be; whether one covering the whole lateral surface of the vessel, or only a right section of that surface, covering a known aliquot part, as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, or less, of that surface.

Suppose, now, for example, that one of the given diameters of the pattern thus far drawn, we will say the greater, is 18 inches, and that we would take out a pattern covering say one-fourth part of the lateral surface or side; then $3.1416 \times 18 = 56.5488$ inches, the circumference of the greater given base; and $56.5488 \div 4 = 14.137$, or $14\frac{1}{8}$ inches nearly, the length of the greater arc that will contain one-fourth part of the circumference. Then, with a strip of flexible tin cut to that length, and bent to the curve, or by any other mechanical means, measure off on the greater arc from

a, in the direction n, $14\frac{1}{8}$ inches large, as from a to b; and from the new-found point, b, draw a right line to the central point, R, as $b\ d\ R$: then will the section $a\ b\ d\ c$ be the pattern required. This may be taken out with or without the requisite margins for locks, burrs, &c., as desired, properly marked, and kept for future use.

NOTE.—In order to avoid uncertainty and confusion, I shall in all cases in this work confine the directions to the construction of the DIMENSIONS-PATTERN, leaving the workman to allow the requisite margins for *locks*, *seams*, *burrs*, and rolled or wired *rims*, as his taste and the circumstances may require. I will suggest, however, that, for a straight lock, the allowance should be equal on each side, and that two-thirds of the allowance on a side, less the thickness of the plate, should be turned when neatness and accuracy of dimensions are intended; also that, ordinarily, three-eighths of an inch is sufficient for the lock on tin-plate, and one inch for that on stove-pipe sheet-iron.

By Mathematics.

R = radius, or slant height, of generating cone = $R\ a$, diagram.
s = slant height of given frustum = $c\ a$, diagram.
H = perpendicular height of generating cone = $R\ m$, diagram.
h = perpendicular depth of given frustum = $h\ m$, diagram.
D = diameter of base of generating cone, or of greater base of frustum.
d = diameter of lesser base of frustum.
r = radius, or slant height, of cone whose base is the lesser base of the given frustum = $R\ c$, diagram.

$$R = s + r = \frac{Dr}{d} = \frac{Ds}{D-d} = s + \frac{sd}{D-d} = \sqrt{H^2 + \left(\frac{D}{2}\right)^2},$$

$$r = R - s = \frac{Rd}{D} = \frac{ds}{D-d} = \sqrt{(H-h)^2 + \left(\frac{d}{2}\right)^2},$$

$$s = R - r = \frac{Rh}{H} = \frac{(D-d)r}{d} = \sqrt{h^2 + \left(\frac{D-d}{2}\right)^2},$$

$$H = \frac{Dh}{D-d} = \frac{Rh}{s} = \sqrt{R^2 - \left(\frac{D}{2}\right)^2} = h + \sqrt{r^2 - \left(\frac{d}{2}\right)^2},$$

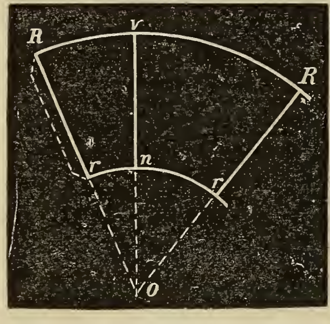
$$h = \frac{H(D-d)}{D} = \frac{Hs}{R} = \sqrt{s^2 - \left(\frac{D-d}{2}\right)^2} = H - \sqrt{r^2 - \left(\frac{d}{2}\right)^2},$$

$$D = \frac{Hd}{H-h} = \frac{Rd}{r} = \frac{ds}{r} + d = 2\sqrt{R^2 - H^2} = 2\sqrt{s^2 - h^2} + d,$$

$$d = \frac{(H-h)D}{H} = \frac{Dr}{R} = D - \frac{Ds}{R} = 2\sqrt{r^2 - (H-h)^2}$$
$$= D - 2\sqrt{s^2 - h^2}.$$

PROB. 2.—*To construct a Pattern for the Body of a Vessel in the form of a Frustum of a Cone of given Dimensions, without plotting the dimensions, and to take it out a known measure of the lateral surface required.*

RULE. — Take the radius of the arc of the greater given base in the dividers, or beam compasses, and from any desired centre on the plate, from which the pattern is to be taken, as *o*, diagram, describe an arc, as *R v*, to any sufficient length; then with the radius of the arc of the lesser given base in the dividers, or beam compasses, describe, from the same centre, a parallel arc, as *r n*, to any sufficient length; then draw a right line from the central point to the outer arc, as *o r R*, diagram. Next apply the requisite measure to the arc of the base whose circumference is known, as from *R* to *v*, diagram; and from the point *v* draw a right line to the central point, as *v n o:* the section *R v n r* will be the pattern proposed.



EXAMPLE. — The depth of a vessel is to be 10 inches; the diameter of one of its bases, 8 inches; that of the other, 6 inches; and a pattern containing one-half the lateral surface is required.

Now, by the foregoing formulæ, $R = \sqrt{H^2 + (\frac{1}{2}D)^2}$, and

$$H = \frac{Dh}{D - d}; \text{ therefore}$$

$$R = \sqrt{\left(\frac{Dh}{D-d}\right)^2 + (\tfrac{1}{2}D)^2}. \quad r, \text{ by the formulæ} = \frac{Rd}{D}; \text{ then}$$

$\sqrt{\left(\frac{8 \times 10}{8 - 6}\right)^2 + (\frac{8}{2})^2} = 40.2$ inches, the radius of the arc of the greater given base, and $\frac{40.2 \times 6}{8} = 30.15$ inches, the radius of the arc of the lesser given base.

$8 \times 3.1416 \div 2 = 12.5664 = 12\frac{6}{10}$ inches, short; the required length of the arc of the greater base; or $3.1416 \times 6 \div 2 = 9\frac{4}{10}$ inches, large, the required length of the arc of the lesser given base.

NOTE.—As before stated, it is wholly immaterial which of the arcs is known, for by defining one we define the other; but generally the greater arc can be much more readily measured by mechanical means than the lesser, and it may be trimmed to facilitate the act, in which case the measuring-tape may be used. When the lesser arc is to be measured mechanically, the measure should be a strip of flexible plate, cut to the required length, and bent to the curve.

If it is desired to convert the decimal part of an inch into eighths of an inch, multiply it by 8; if into sixteenths, multiply it by 16. Thus, the decimal .5664 × 8 = 4.5312 eighths = 9.0624 sixteenths. The decimal .5644, therefore, is practically equal to 9-16.

PROB. 3.—*To construct a Pattern for the Lateral Portion of a vessel in the form of a Frustum of a Cone, of given relative proportions or symmetry of outline, and given Capacity; any two of its dimensions, and one of them a base, being given.*

The following table has been calculated for relative proportions as set down at the top of the columns, and for portions, P'n, or parts, of lateral surface, as set down in the left-hand column.

R represents the radius of the arc of the given base, and *D*, the diameter of that base. Thus, if *D* be taken for the greater base, then *R* will represent the radius of the arc of that base, and the slant height of the cone from which the frustum is to be taken; but if *D* be taken for the lesser base, then *R* will represent the radius of the arc of the lesser base, and the slant height of the cone that will be left after the frustum has been taken from it.

H represents the perpendicular height of the cone having *D* for its base.

c represents the chord of the required arc, or chord that will subtend the arc that must be on the pattern or portion set down in the left-hand column.

S represents the cubic contents of the cone having *D* for its base.

P'n	$R = D$.	$R = 1\frac{1}{3}D$.	$R = 2D$.	$R = 2\frac{1}{2}D$.	$R = 3D$.	$R = 4D$.	$R = 6D$.
	$R \times = c$	$R \times = c$	$R \times = c$	$R \times = c$	$R \times = c$	$R \times = c$	$R \times = c$
1	2	1.8478	1.4142	1.1755	1	.7654	.5176
$\frac{1}{2}$	1.4142	1.1113	.7654	.618	.5176	.39	.2611
$\frac{1}{3}$	1	.7654	.5176	4158	.3473	.2611	.1743
$\frac{1}{4}$	.7654	.5806	.39	.3129	.2611	.1961	.1308
$\frac{1}{5}$	.618	.4669	.3129	.2506	.2091	.1569	.1047
$\frac{1}{6}$	.5176	.39	.2611	.2091	.1743	.1308	.0872
	$D \times = H$	$D \times = H$	$D \times = H$	$D \times = H$	$D \times = H$	$D \times = H$	$D \times = H$
q	.866	1.236	1.9365	2.4495	2.958	3.9686	5.9791
	$D^3 \times = S$	$D^3 \times = S$	$D^3 \times = S$	$D^3 \times = S$	$D^3 \times = S$	$D^3 \times = S$	$D^3 \times = S$
k	.2267	.3236	.507	.6413	.7744	1.039	1.5653

When $R = 5D$, and pattern covering the whole lateral portion of the vessel is required, $c = .618R$; covering $\frac{1}{2}$, $c = .3129R$; $\frac{1}{3}$, $c = .2091R$; $\frac{1}{4}$, $c = .1569R$; $\frac{1}{5}$, $c = .1257R$; $\frac{1}{6}$, $c = .1047R$; $H = 4.975D$; $S = 1.3024D^3$.

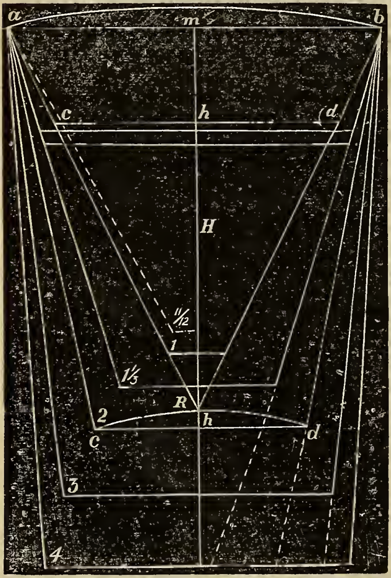
General Explanation of the Table. — $R = D$; then $R \times 2$ is equal the chord of the arc that is equal to the whole circumference of the base, and $R \times 1.4142$ is equal the chord of the arc that is equal to half the circumference of the base, &c.

$R = 2D$; then $R \times .7654$ is equal the chord of the arc that is equal to half the circumference of the base, and $R \times .5176$ is equal the chord of the arc that is equal to one-third part of the circumference of the base, &c.,

The annexed diagram is drawn to radii 1, $1\frac{1}{3}$, 2, 3, and 4 times the diameters of the ends, and exhibits symmetrical proportions for vessels of this general class; and, by help of the preceding table, a pattern containing the whole lateral surface of either special figure, or such portion thereof as set down in the left-hand column, may be readily and correctly obtained.

When the greater Base of the Vessel is given.

Rule. — Place the square suitably on the plate from which the pattern is to be taken, and scribe to its blades, as *a m*, *m R*. Make *m R* of sufficient length, and make *m a* equal to any desired aliquot part of the greater given base; then, with the square or straight-edge in the position *a R*, and that measure 1, $1\frac{1}{3}$, 2, or more times the diameter of the greater base, according to the table and the special figure intended, draw a line, or that measure, as *a R*. Next, if the perpendicular depth be given, space it off on the perpendicular, as *m h*; and from *h*, with the square as at first dropped down to that point, draw a line, *h c*, parallel to *m a*, which will be the corresponding measure of the lesser base of the vessel. If the slant



depth of the vessel, instead of the perpendicular, be given, space it off on the slant depth, as *a c*; and from *c*, with the square as before, draw a line, *c h*, the corresponding measure of the lesser base, as before. If the lesser base of the vessel, instead of the perpendicular depth, or slant depth, be given, take the same relative measure of that on the square that was taken of the greater base; and with the square as at first dropped down on the perpendicular, until that measure is exactly included between the lines *a R* and *m R*, as *h c*, draw a line, *h c*, which will be the required measure of the lesser base. Next, with the radius *a R*, and *R* the centre, describe an arc, as *a b*, to any sufficient length; and with the radius *c R*, and *R* the centre, describe a parallel arc, as *c d*, to any sufficient length.

Suppose now, for example, that we have drawn the arcs to radii once the diameters of the vessel; that the diameter of the greater base is 18 inches; and that we would take out a pattern containing say $\frac{1}{4}$ part of the lateral surface required: then, on turning to the table, we find in the column headed $R = D$, and against $\frac{1}{4}$ in the column of portions, the constant, or co-efficient, 0.7654; and we are told at the top of the column, that the radius multiplied by that co-efficient is equal the chord of the required arc. Then $18 \times .7654 = 13.779$ inches, the chord of the arc that is equal to $\frac{1}{4}$ part of the circumference of the greater base. Take, therefore, 13.779 inches in the dividers, and from the point *a*, with that measure, cut the arc, as at *b*, and from the new-found point, *b*, draw a right line to the point *R*, as *b R*; then will the section *a c d b* be the pattern sought. This may be taken out, and marked with the values of *R*, *d*, and $\frac{1}{4}$, with other distinguishing marks, as the depth, or capacity, or both, and kept for future use.

Suppose again, for further illustration of the practical use of the TABLE, that we have drawn the arcs to radii $1\frac{1}{3}$ the diameters of the bases; that the diameter of the greater base is 10 inches; and that we would take out a pattern containing $\frac{1}{2}$ the side of the vessel: then, on referring to the column headed $R = 1\frac{1}{3} D$, in the table, we find in that column, against the portion $\frac{1}{2}$ in the column of portions, the co-efficient 1.1113; and $10 \times 1\frac{1}{3} \times 1.1113 = 14.82$ inches, the chord of the arc required. Take, therefore, 14.82 inches in the dividers, or on the square, and from the point *a*, with that measure, cut the arc as at *b*, and draw a line *b R*. The section *a c d b* will be the pattern demanded.

When the Lesser Base of the Vessel is given, and the Greater is unknown.

RULE. — Place the square suitably on the plate from which the pattern is to be taken, as *c h R*, diagram, and draw to any sufficient length a line, *h R*; and from *h*, on the other arm of the square, set off any aliquot part of the base, as $h\ c$, and draw that measure. Next, with the square, or straight edge, in position, as $c\ R$, and that measure 1, $1\frac{1}{3}$, 2, or more times the diameter of the base, according to the table and the special figure intended, draw a line, as $c\ R$, and produce it sufficiently in the direction a. Next produce $R\ h$ sufficiently in the direction m. Next, if the perpendicular depth is known, space it off on the perpendicular produced, as $h\ m$, and, with the square raised up to the point m, draw a line, $m\ a$, parallel to $h\ c$, which will be the corresponding aliquot part of the greater base. If the slant depth is given instead of the depth, space it off on the slant depth produced, as $c\ a$, and, with the square raised up on the perpendicular till the shorter arm cuts the point a, draw a line, $a\ m$, which will be the required aliquot part of the greater base. Next, with $R\ a$ the radius, and R the centre, describe an arc, as $a\ b$, to any sufficient length; and with $R\ c$ the radius, and R the centre, describe an underlying arc, as $c\ d$, to any sufficient length.

Suppose now, for example, that we have drawn the arcs to radii six times the diameters of the bases, that the diameter of the lesser base is 11 inches, and that we would take out a pattern containing one-third part of the side. On referring to the table we find, in the column headed $R = 6D$, and against $\frac{1}{3}$ in the column of portions, the constant .1743, and $11 \times 6 \times 0.1743 = 11\frac{1}{2}$ inches, the chord of the arc of the lesser base that is equal to one-third part of the circumference of that base. Take, therefore, $11\frac{1}{2}$ inches on the square; and with that measure, as from c to d, cut the arc as at d; and from R, through the point d, draw a right line to the greater arc, as $R\ d\ b$: then will the section $a\ b\ d\ c$ be the pattern demanded.

It may be proper to state that the table is also applicable in finding the sides or ends of a vessel in the form of a prismoid, or of a frustum of a pyramid, of any number of sides from three up, corresponding to the denominators of the fractional portions tabulated.

PROB. 4. — *The special tabular figure, the Diameter of one end, and the Cubic Capacity of the vessel, being given, to find the Diameter of the other end.*

D, d = diameters of bases.
a = cubic capacity of vessel.
k = tabular constant for S.

$$D = \sqrt[3]{\frac{d^3k + a}{k}}\ ;\ d = \sqrt[3]{\frac{D^3k - a}{k}}.$$

EXAMPLE. — A vessel in the form of a frustum of a cone is to be constructed to radii 4 times the diameters of the bases: the diameter of the greater base is to be 7 inches, and the capacity exactly 1 gallon (231 cubic inches). What must be the diameter of the other base?

Seeking k in the column headed $R = 4D$, in the table, and under $D^3 \times = S$, we find it to be 1.039; then

$$\sqrt[3]{\frac{7^3 \times 1.039 - 231}{1.039}} = 4.9416 \text{ inches. } Ans.$$

Having now both diameters of the vessel, proceed for a pattern containing the desired portion of the side, as directed in the foregoing.

EXAMPLE. — A measure of the exact capacity of 2 gallons is to be constructed to radii 3 times the diameters of the bases: the diameter of the lesser base is to be $5\frac{1}{2}$ inches. What must be the diameter of the greater base?

k, by the table, $= .7744$; then

$$\sqrt[3]{\frac{5.5^3 \times .7744 + 462}{.7744}} = 9.1377 \text{ inches. } Ans.$$

EXAMPLE. — What will be the depth of the last-mentioned vessel?

Seeking q in the column headed $R = 3D$, in the table, under $D \times = H$, we find it to be 2.958; $\therefore H = 9.1377 \times 2.958 = 27.0293$ inches, and $h = H - \frac{Hd}{D} = 2.958\ (D - d) = 10.76$ inches. *Ans.*

EXAMPLE. — A pan in the form of a truncated cone is to be constructed to radii once the diameters of the bases: the diameter of the greater base is to be $15\frac{1}{2}$ inches, and the capacity 8 wine quarts (462 cubic inches). What must be the diameter of the other base?

The constant for the solidity, in the column headed $R = D$, is .2267, and

$$\sqrt[3]{\frac{15.5^3 \times .2267 - 462}{.2267}} = 11\tfrac{9}{10} \text{ inches. } Ans.$$

EXAMPLE. — What will be the slant depth, and what the perpendicular, of the aforementioned pan?

$s = R - r$, and $h = Hs \div R$. But R in this case $= D$, and $r = Rd \div D = d$; therefore $s = 3.5976$ inches. *Ans.*

H, by the Table, $= D \times .866 = 13.423$, and $h = 13.423 \times 3.5976 \div 15.5 = .866\ (D - d) = 3.116$ inches. *Ans.*

By Mathematics.

$D, d =$ diameters of bases.
$R, r =$ radii of arcs of bases.
$H =$ perpendicular height of generating cone.
$h =$ perpendicular depth of vessel.
$S =$ cubic contents of generating cone.
$a =$ cubic capacity of vessel.

$$D = \sqrt[3]{\frac{12S}{\pi H}} = \sqrt[3]{\frac{Sd^3}{S-a}} = \sqrt[3]{\left(d^3 + \frac{12da}{\pi(H-h)}\right)}.$$

$$d = \sqrt[3]{\frac{D^3(S-a)}{S}} = \sqrt[3]{\left(D^3 - \frac{12Da}{\pi H}\right)}.$$

$$H = \frac{12S}{\pi D^2} = \frac{12Da}{\pi(D^3-d^3)},\quad h = \frac{12a}{\pi(Dd+D^2+d^2)}$$

$$= \frac{H}{D}\left(D - \sqrt[3]{\frac{D^3(S-a)}{S}}\right).$$

$$S = \frac{\pi D^2 H}{12} = \frac{D^3 a}{D^3-d^3} = \frac{\pi D^3 h}{12(D-d)}.$$

$$a = \frac{S(D^3-d^3)}{D^3} = \frac{\pi H(D^3-d^3)}{12D} = \frac{\pi h(Dd+D^2+d^2)}{12}.$$

For other forms of expression, and other applicable equations, see p. 331.

Prob. 5. — *To construct a Pattern for the Lateral Portion of a vessel in the form of a Frustum of a Cone, of given Tabular Outline, and given Dimensions, without plotting the dimensions.*

$s =$ slant depth of vessel.
$g =$ tabular ratio of R to $D = 1, 1\frac{1}{3}, 2, 3,$ &c., top line.
$q =$ tabular ratio of H to D, third line from bottom.
$k =$ tabular constant for S, bottom line.
$y =$ tabular constant for chord of arc.
Other symbols as in the foregoing.

When both Bases of the Vessel are given,

$$R = Dg, \text{ and } r = dg;\ h = q(D-d),\ s = R - r.$$

When the Greater Base and the Depth are given,

$$R = Dg;\ r = \frac{g(Dq-h)}{q};\ d = \frac{Dr}{R} = \frac{r}{g} = \frac{Dq-h}{q}.$$

When the Greater Base and the Slant Depth are given,

$$R = Dg, \text{ and } r = R - s.$$

When the Lesser Base and the Depth are given,

$$R = \frac{r(dq+h)}{dq};\ r = dg;\ D = \frac{Rd}{r} = \frac{R}{g} = \frac{dq+h}{q}.$$

When the Lesser Base and the Slant Depth are given,

$$R = r + s;\ r = dg.$$

EXAMPLE. — Given the tabular outline $R = 2\,D$, the diameter of the lesser base $d = 5$, and the depth $h = 8$, to find the diameter of the greater base D, the radii of the arcs R, r, and the chord of the greater arc c, that will subtend $\frac{1}{2}$ the circumference of the greater base.

By the Table, when $g = 2$, and $p = \frac{1}{2}$; that is, when $R = 2\,D$, and a pattern containing one-half the lateral surface is required, $q = 1.9365$, and $y = .7654$;

then $r = 5 \times 2 = 10$; $R = \dfrac{10(5 \times 1.9365 + 8)}{5 \times 1.9365} = 18.26$;

$D = \dfrac{18.26}{2} = 9.13$; and $c = 18.26 \times .7654 = 13.976$. *Ans.*

EXAMPLE. — Given $D = 10$, $s = 4$, $g = 3$, $p = \frac{1}{3}$, to find R, r, and c.

By the Table, $q = 2.958$, and $y = .3473$; then $R = 10 \times 3 = 30$; $r = 30 - 4 = 26$; and $c = 30 \times .3473 = 10.419$. *Ans.*

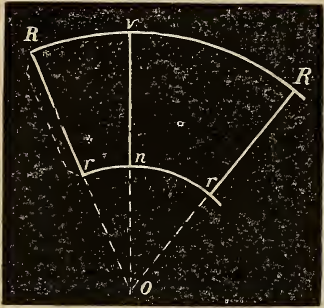
When the tabular outline $R = Dg$, the assumed diameter, and the cubic capacity only, are given, the other diameter must be found, and by one of the following formulas, viz.: —

$$D = \sqrt[3]{\frac{d^3k + a}{k}} = \sqrt[3]{\left(d^3 + \frac{12a}{\pi q}\right)}.$$

$$d = \sqrt[3]{\frac{D^3k - a}{k}} = \sqrt[3]{\left(D^3 - \frac{12a}{\pi q}\right)}.$$

RULE. — Take R in the dividers, or beam compasses, and from any suitable centre, o, describe an arc, $R\ v$, to any sufficient length. Next, with r the radius, and the same centre, describe an arc parallel to the former, $r\ n$, to any sufficient length; then draw a right line from the central point to the outer arc, and cutting the arcs, as at r and R.

Suppose, now, for example, that the arcs are drawn to radii once and one-third the diameters of the bases; that the radius of the greater arc is 16 inches; and that you wish to take out a pattern containing one-fifth part of the side of the vessel. Referring to the table, column $R = 1\frac{1}{3} D$, and against $\frac{1}{5}$ in the column of portions, you find .4669; and $16 \times .4669 = 7.47$ inches, the chord required. Take therefore 7.47 inches in the dividers or on the square, and from the point R, with that measure, cut the greater arc as at v, and from the new-found point, v, draw a right line to the central point o, and cutting the lesser arc, as at n; then will $R\ v\ n\ r$ be the pattern sought.



On the contrary, suppose you have drawn the arcs to radii four times the diameters of the bases, that the diameter of the lesser base is 5 inches, and that a pattern containing one-third part of the lateral portion of the vessel is required: then, by the table, column $R = 4D$; and against $\frac{1}{3}$ in the column of portions, the constant, .2611 appears; and $5 \times 4 \times .2611 = 5.222$ inches, the chord of the arc of the lesser base that is equal to one-third part of the circumference of that base. Take, therefore, 5.222 inches on the square, and from the point r, with that measure, cut the lesser arc, as at n; and from the central point o, through the point n, draw a right line to the greater arc, and cutting that arc, as at v: then will the section $R\,r\,n\,v$ be a pattern for the vessel, and contain one-third part of the side.

PROB. 6. — *The Capacity in gallons of a Vessel in the form of a Frustum of a Cone being given, and any two of its dimensions, to find the other Dimension.*

It has been shown that $\frac{\pi h(Dd + D^2 + d^2)}{12} =$ solidity, or capacity, of a frustum of a cone, the solidity and the dimensions being in the same terms of measurement;

$$\therefore\ d = \sqrt{\left(\frac{12a}{\pi h} - \frac{3D^2}{4}\right)} - \frac{D}{2}.$$

But since a gallon contains 231 cubic inches, and

$$\frac{\pi}{4 \times 231} = .0034,$$

it follows that $\frac{.0034h(Dd + D^2 + d^2)}{3} = .0011\frac{1}{3}h(Dd + D^2 + d^2)$ = solidity, or capacity in gallons; the dimensions being in inches.

Therefore, putting C to represent the capacity of the vessel in gallons,

$$h = \frac{3C}{.0034(Dd + D^2 + d^2)};\quad D = \sqrt{\left(\frac{3C}{.0034h} - \frac{3d^2}{4}\right)} - \frac{d}{2};$$

$$d = \sqrt{\left(\frac{3C}{.0034h} - \frac{3D^2}{4}\right)} - \frac{D}{2}.$$

EXAMPLE. — A measure in the form of a frustum of a cone is to be constructed to the exact capacity of 3 gallons; the diameter of one of its bases is to be 11 inches, and the perpendicular depth 12 inches. What must be the diameter of the other base?

$$\sqrt{\left(\frac{3 \times 3}{12 \times .0034} - \frac{11^2 \times 3}{4}\right)} - \frac{11}{2} = 5.895 \text{ inches. } \textit{Ans.}$$

EXAMPLE. — A vessel in the form of a frustum of a cone is to be constructed to the capacity of 3 gallons; the diameters are to be 11 inches, and $8\frac{3}{4}$ inches. What must be the depth?

$$\frac{3 \times 3}{.0034(11 \times 8.75 + 11^2 + 8.75^2)} = 9.01 \text{ inches. } \textit{Ans.}$$

EXAMPLE. — A measure in the form of a truncated cone is to be constructed to the capacity of $\frac{1}{2}$ gallon; the diameter of one end is to be $5\frac{1}{2}$ inches, and that of the other 4 inches. What must be the depth?

$$\frac{3 \times \frac{1}{2}}{.0034(22 + 30.25 + 16)} = 6.464 \text{ inches. } \textit{Ans.}$$

EXAMPLE. — A measure is to be constructed to the capacity of one wine quart; the diameter of one of the bases is to be 4 inches, and the depth $5\frac{1}{2}$ inches. What must be the diameter of the other base?

$$\sqrt{\left(\frac{3 \times \frac{1}{4} = .75}{.0034 \times 5.5} - \frac{3 \times 4^2}{4}\right)} - \frac{4}{2} = 3\tfrac{3}{10} \text{ inches. } \textit{Ans.}$$

EXAMPLE. — A measure of the capacity of one wine pint is to be constructed; the depth is to be $4\frac{1}{2}$ inches, and one of the diameters is to be $3\frac{1}{2}$ inches. What must be the other diameter?

$$\sqrt{\left(\frac{3 \times \frac{1}{8} = .375}{.0034 \times 4.5} - \frac{3 \times 12\frac{1}{4}}{4}\right)} - 1.75 = 2.1644 \text{ inches. } \textit{Ans.}$$

EXAMPLE. — A measure is to hold $\frac{1}{2}$ wine pint, and each base is to be $2\frac{3}{8}$ inches in diameter. What must be the depth?

$$\frac{3 \times \frac{1}{16}}{.0034 \times 3d^2} = \frac{.0625}{.0034d^2} = \frac{.0625}{.0034 \times 2.375^2} = 3.259 \text{ inches. } Ans.$$

PROB. 7. — *To construct Patterns for flaring oval Vessels of different Eccentricities and given Dimensions.*

The solids here contemplated, and for which coverings are to be constructed, closely resemble the frustums of elliptic cones; but they are not identical with that figure, since an oval is made up of circle arcs, while no part of an ellipse whatever is strictly the arc of a circle. The leading principle, however, for this class of patterns, so far as regards their sides, is expressed by the act of an elliptic cone rotating on its side, on a plane, from the line where the plane of the transverse axis of its base is at right angles to the plane on which it rotates, to the extent of one revolution of the cone.

D represents the transverse diameter, and d the conjugate, of the greater base; D' the transverse, and d' the conjugate, of the lesser base; h the perpendicular depth of the vessel: $H = \frac{Dh}{D - D'}$, the perpendicular height of generating cone; $M = \sqrt{H^2 + (\frac{1}{2}D)^2}$, the maximum slant height of generating cone; $S = \sqrt{H^2 + (\frac{1}{2}d)^2}$, the minimum slant height of generating cone.

No. 1. — *The Lateral Portion.* D : d : : D$'$: d'.

$tf = F$; $ta = P$; $fa = R$; $fc = N$; $fv = S$; $fA = M$; diagram.

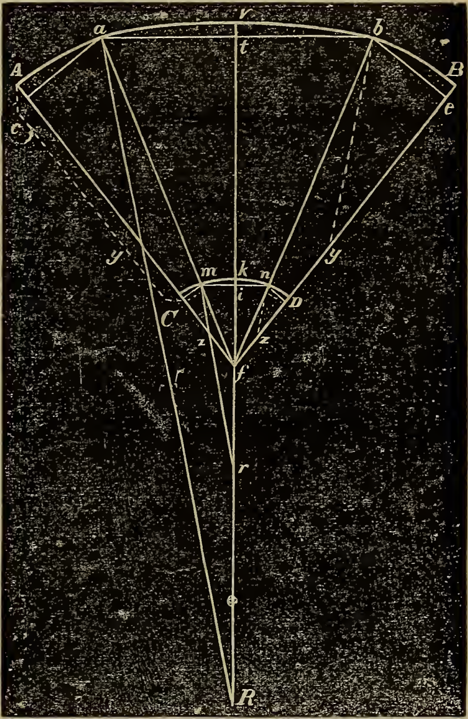
RULE. — Place the square suitably on the plate from which the pattern is to be taken, and scribe to its edges $a\,t$, $t\,R$, making both lines of sufficient length; and produce $t\,R$ sufficiently in the direction v. Make $tf = \sqrt{H^2 + \frac{D^2}{12}}$, and $ta = \sqrt{\left(\frac{D}{3}\right)^2 + \left(\frac{(2 - \sqrt{3})D}{3}\right)^2} = 0.345092D$, and draw fa. Make $ti = \sqrt{h^2 + \frac{(D - D')^2}{12}}$, and draw im parallel to ta. Next, with $cf = \sqrt{R^2 - \frac{D^2}{9}}$ on one blade of the square, and the limit of that measure in the point f, the other blade cutting the point a, as $f\,c\,a$; draw fc, ca, and produce

$f\,c$ in the direction A. Make $t\,R = F + \frac{MN}{R}$, and draw $a\,R$.

Make

$$i\,r = \frac{D'(t\,R)}{D},$$

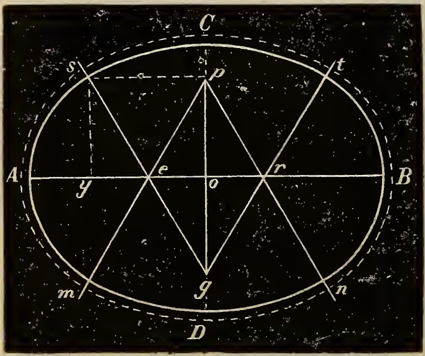
and draw $m\,r$, which will be parallel to $a\,R$. Then with $a\,R$ in the dividers, and R the centre, describe $a\,v$; and with $m\,r$ in the dividers, and r the centre, describe $m\,k$; also with $a\,y$ the radius, and y the centre, describe $a\,A$; and with $m\,z$ the radius, and z the centre, describe $m\,C$. Thus $A\,a\,v\,k\,m\,C$ will be the unit measure of the pattern, and will contain one-fourth part of the side.



NOTE.—It will be perceived that the section on the right of the perpendicular $v\,R$, diagram, is an exact duplicate of that on the left, only reversed; thus $A\,v\,B\,D\,k\,C$ is a pattern for the vessel, and contains one-half its lateral surface. The last-mentioned pattern may be obtained by draft, by repeating the lines and arcs on the left; or it may be obtained by scribing to the unit measure in the two positions. In making up, the side should consist of a single piece when practicable, and the lock or seam should be under the handle.

To construct the Bases for No. 1.

RULE. — Lay off $A\,B$, diagram, equal in length to the transverse diameter of the base, and divide it into three equal parts, e and r; also bisect it, and through the point of bisection, o, draw an indefinite perpendicular, as $C\ D$. Make $e\ p$, $e\ g$, each equal to $A\ e$ or $B\ r$; and from the points p and g, through the points e and r, draw right lines, as $g\ e\ s$, $g\ r\ t$, $p\ e\ m$, $p\ r\ n$. Then with $A\ e$ or $B\ r$ the radius, and e and r the centres, describe $m\ A\ s$, $t\ B\ n$; and with $g\ s$ or $p\ n$ the radius, and g and p the centres, describe $s\ t$ and $m\ n$.



By Mathematics.

$$A\ B = D = \frac{3d}{4-\sqrt{3}} = 1.322781d; \quad C\ D = d = \frac{(4-\sqrt{3})D}{3} = 0.7559831D;$$

$$o\ p = o\ g = s\ y = \frac{D}{\sqrt{12}} = 0.288675D;\ p\ C = g\ D = \frac{(2-\sqrt{3})D}{3} = 0.0893164D;$$

$$p\ s = e\ p = A\ e = o\ y = \frac{D}{3} \qquad g\ s = A\ r = g\ C = \frac{2D}{3};$$

$$A\ y = e\ y = o\ r = \frac{D}{6};\ s\ g\ t = 60^\circ; \quad m\ e\ s = 120^\circ.$$

No. 2.—The Lateral Portion. $D : d :: D' : d'$.

D represents the transverse, and d the conjugate, diameters of the greater base; D′ the transverse, and d' the conjugate diameter of the lesser base; h the perpendicular depth of the vessel. $H = \frac{Dh,}{D-D'}$, the perpendicular height of the generating cone; $S = \sqrt{H^2 + (\frac{1}{2}d)^2}$, the minimum slant height of generating cone; $M = \sqrt{H^2 + (\frac{1}{2}D)^2}$, the maximum slant height of generating cone.

$$tf = F;\ t\,a = P;\ f\,a = R;\ f\,c = N;\ f\,v = S;$$
$$f\,A = M; \text{ diagram.}$$

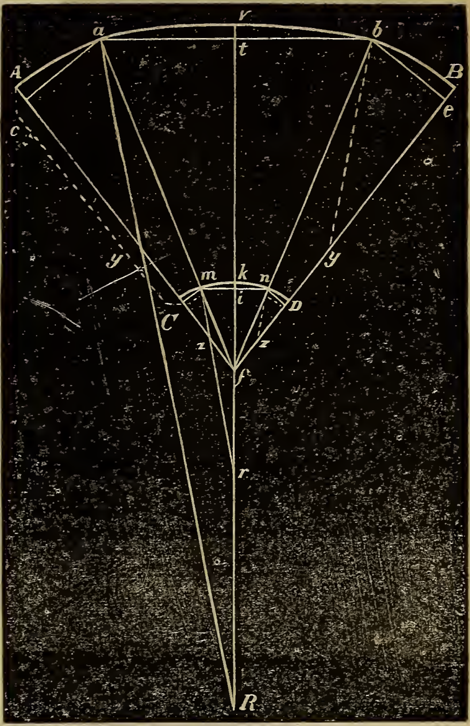
Rule.—Place the square suitably on the plate from which the pattern is to be cut ($a\ t\ R$, diagram), and draw lines $a\ t$, $t\ R$, each of sufficient length; and produce $R\,t$ sufficiently in the direction v. Make $t\,f = \sqrt{H^2 + \frac{D^2}{25}}$, and $t\,a = \sqrt{\frac{4D^2 + d^2}{25}} = 0.421637D$, and draw $f\,a$. Make $t\,i = \sqrt{h^2 + \frac{(D - D')^2}{25}}$, and draw $i\,m$ parallel to $t\,a$. Next, with $c\,f = \sqrt{R^2 - \frac{D^2}{20}}$ on one edge of the square, the limit of that measure in the point f, the edge of the other blade cutting the point a, as $f\,c\,a$, draw $f\,c$, $c\,a$, and produce $f\,c$ in the direction A. Make

$$t\,R = F + \frac{M\,N}{R},$$

and draw $a\,R$. Make

$$i\,r = \frac{D'\,(t\,R)}{D},$$

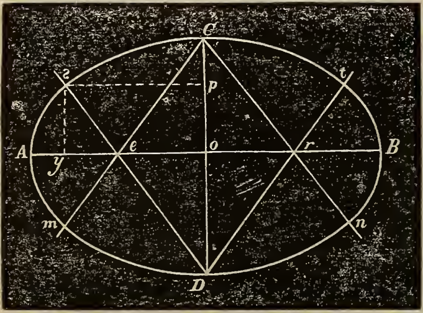
and draw $m\,r$, which will be parallel to $a\,R$: then with $a\,R$ the radius, and R the centre, describe $a\,v$; and with $r\,m$ the radius and r the centre, describe $m\,k$: also with $a\,y$ the radius, and y the centre, describe $a\,A$; and with $m\,z$ the radius, and z the centre, describe $m\,C$. Then will



$A\,a\,v\,k\,m\,C$ be the unit measure of the lateral portion of the vessel, and contain one-fourth part of that portion. See Note at the close of the directions for constructing the lateral portion of No. 1.

To construct the Bases for No. 2.

RULE.—Make $A\ B$ equal to the transverse diameter of the base, and divide it into four equal parts, in e, o, and r; and through o draw a perpendicular, $C\ D$, of sufficient length. Make $o\ C$, $o\ D$, each equal to one-third of $A\ B$, and from C and D, through e and r, draw $C\ e\ m$, $C\ r\ n$, $D\ e\ s$, $D\ r\ t$. Then with $D\ C$ the radius, and D and C the centres, describe $s\ C\ t$, and $m\ D\ n$; also with $A\ e$ or $B\ r$ the radius, and e and r the centres, describe $m\ A\ s$ and $t\ B\ n$.



BY MATHEMATICS.

$$A\ B = D = \frac{3d}{2};\ d = \frac{2D}{3};\ o\ C = o\ D = \frac{D}{3};\ o\ p = s\ y = \frac{D}{5};$$

$$p\ C = \frac{2D}{15};\ \ p\ s = o\ y = \frac{2D}{5};\ A\ y = \frac{D}{10};\ p\ D = \frac{8D}{15};$$

$$s\ D\ t = 73^\circ.74;\quad t\ r\ n = 106^\circ.4.$$

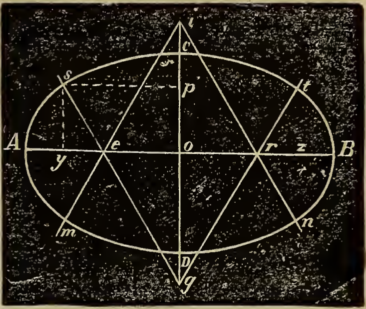
No. 3.—*The Lateral Portion.* $D : D' :: d : d'$.

Symbols, same as for No. 1 and No. 2.

RULE.—Draw $a\ t$, $t\ R$, at right angles to each other, and of sufficient length; and produce $R\ t$ sufficiently in the direction v. Make $t\ f = \sqrt{H^2 + \frac{3D^2}{64}}$, and $t\ a = \sqrt{.1507214D^2} = 0.3882285D$, and draw $f\ a$ Make $t\ i = \sqrt{h^2 + \frac{3(D-D')^2}{64}}$, and draw $i\ m$ parallel to $a\ t$. Next, with $c\ f = \sqrt{R^2 - \frac{D^2}{16}}$ on one blade of the square, and the limit of that measure in the point f, the other blade cutting the point a, as $f\,c\,a$, draw $f\,c$, $c\,a$; and produce $f\,c$ sufficiently in the direction A. Make $t\,R = F + \frac{MN}{R}$, and draw $a\,R$. Make $i\,r = \frac{D'(t\,R)}{D}$, and draw $m\,r$, which will be parallel to $a\,R$. Then with $a\,R$ the radius, and R the centre, describe the arc $a\,v$; and with $r\,m$ the radius, and r the centre, describe the arc $m\,k$; also with $a\,y$ the radius, and y the centre, describe the arc $a\,A$; and with $m\,z$ the radius, and z the centre, describe the arc $m\,C$: then will the section $A\,a\,v\,k\,m\,C$ be the unit measure of the pattern, and contain one-fourth part of the side.

To construct the Bases for No. 3.

RULE. — Make $A\,B$ equal to the transverse diameter of the base, and divide it into four equal parts, in e, o, r; then bisect it with a perpendicular of sufficient length. Make $e\,i$, $e\,g$, each equal to half $A\,B$, and from the points i and g, through the points e and r, draw $i\,e\,m$, $i\,r\,n$, $g\,e\,s$, $g\,r\,t$, all of sufficient length. Next, with $A\,e$ or $B\,r$ the radius, and e and r the centres, describe the arcs $m\,A\,s$, $t\,B\,m$; and with $g\,s$ or $g\,t$ the radius, and g and i the centres, describe the arcs $s\,C\,t$, $m\,D\,n$.



BY MATHEMATICS.

$$A\,B = D = \frac{2d}{3 - \sqrt{3}} = 1.57814d;\quad d = \frac{(3 - \sqrt{3})D}{2} = .6339746D;$$

$$p\,s = o\,y = \frac{3D}{8};$$

$$o\,p = \frac{D\sqrt{3}}{8} = .21650635D;\quad p\,C = \frac{(6\sqrt{3} - 9)D}{8\sqrt{3}} = .100481D;$$

$$A\,y = \frac{D}{8};$$

$$o\,i = o\,g = \frac{D\sqrt{3}}{4};\ g\,s = A\,r = \frac{3D}{4};\ s\,g\,t = 60°;\ t\,r\,n = 120°.$$

The Lateral Portion by Mathematics, be the Eccentricity what it may.

$p\,s = s$, sine of the arc $s\,C$, base;

$o\,p = g$, sine of the arc $s\,A$, base (see diagram of base).

D, D', d, d', h, H, as in the foregoing, $D : D' :: d : d'$.

$t f = F = \sqrt{(H^2 + g^2)}$; $t a = P = \sqrt{[s^2 + (\frac{1}{2}d - g)^2]}$
$= \sqrt{(R^2 - F^2)}$;

$f a = R = \sqrt{(F^2 + P^2)} = \sqrt{(N^2 + p^2)}$; $f c = N = \sqrt{(R^2 - p^2)}$;

$f v = S = \sqrt{[H^2 + (\frac{1}{2}d)^2]}$; $f A = M = \sqrt{[H^2 + (\frac{1}{2}D)^2]}$;

$c a = p = \sqrt{(R^2 - N^2)}$; $c y = f = FN \div (t R) = \sqrt{(r^2 - p^2)}$;

$a y = r = \sqrt{(f^2 + p^2)}$; $t i = \sqrt{\left(h^2 + \frac{g^2(D - D')^2}{D^2}\right)}$;

$m r = \sqrt{[(i r)^2 + (i m)^2]} = \frac{D'(a R)}{D}$; $i m = \sqrt{[(m r)^2 - (i r)^2]}$

$= \sqrt{[(m f)^2 - (i f^2)]} = \frac{D'P}{D}$; $i r = \sqrt{[(m r)^2 - (i m)^2]} = \frac{D'(t R)}{D}$;

$t R = F + \frac{MN}{R} = \sqrt{[(a R)^2 - P^2]}$; $a R = \sqrt{[(t R)^2 + P^2]}$;

$m z = r' = D' r \div D$; $x m = p' = D' p \div D$; $x z = f' = D' f \div D$;

$i k = D'(S - F) \div D$; $f i = F - (t i)$; $C x = D'(M - N) \div D$;

$c A = M - N$; $t v = S - F$; also, l or $A\,a\,v$, diagram,

$$= 0.8825 \sqrt{\frac{\pi D d}{4} + \frac{(D - d)^2}{\pi}} *$$

nearly; and $4l \div \pi =$ diameter of generating circle of ellipse.

Of Cylindrical Elbows.

The solid to be taken into view in reference to the construction of the arms of a hollow cylindrical elbow is a cylinder of the diameter proposed for the arms, having one of its bases oblique to the sides to the extent of half the angle proposed for the elbow; and the best mode of laying off the arms, generally speaking, is expressed by the act of this cylinder rotating on its side on a plane, from the line where the plane of the transverse axis of its oblique base is at right angles to the plane on which it rotates, to the extent of one revolution of the cylinder.

* This formula for one-fourth of the perimeter of an ellipse affords almost strict accuracy when the conjugate diameter is not greater than two-thirds, nor less than one-third, of the transverse. It furnishes it too short by 1-1400 when the conjugate is equal to three-fourths of the transverse, and too long by 1-1100 when the conjugate is equal to one-fourth of the transverse. See Conic Sections, *Ellipse*.

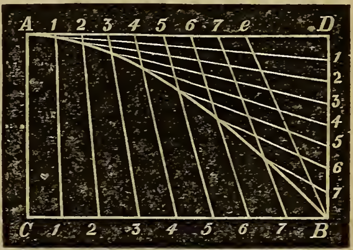
It is apparent, then, that the true face, or outline, of the joint of a cylindrical elbow, when the cylinder in continuance is rolled into place, is an ellipse, whatever be the angle of the elbow. And it may be shown that if a hollow cylinder having an oblique base be bisected by the plane of either axis of that base, and the semi-cylinders opened to plane surfaces, the portion of the curve that will be on each will be a hyperbola, or two equal and similar semi-hyperbolas united, and forming a cima, or facing in opposite directions, according as the plane of the conjugate axis, or that of the transverse, be made the cutting plane alluded to; and it is clear that the curve that will be on one piece will be equal and similar to that on the other in all particulars. Thus the curve of the joint of a cylindrical elbow, when laid off on a plane, is made up of four equal and similar semi-hyperbolas for either arm of the elbow; the method of locking the joint not now being taken into account.

From the foregoing analysis, the following rules and directions for practice have been derived; and it is believed they will be found not only as correct and simple, but as ready of execution, as any that have been or can well be devised for the purposes proposed.

PROB. 8.—*For a Right-angled Elbow.*

$d =$ diameter of pipe.
$d \times .7854 = \frac{1}{4}$ circumference of pipe.

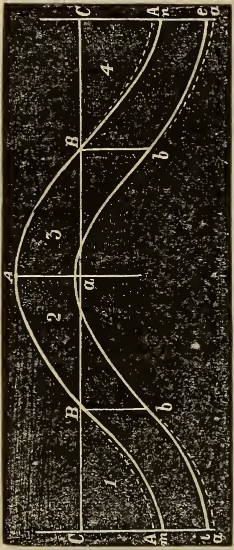
RULE.—Construct on any plate suitable for the purpose, and for taking out, a rectangle, *A C B D*, in length, *A D*, equal to one-fourth part of the circumference of the pipe, and in breadth, *A C*, equal to one-half the diameter of the pipe. Make *A e* equal to $d \times 0.5708$, and divide the space into any number of equal parts. Divide *C B* and *B D* each into the same number of equal parts that you divide *A e* into. Connect the points of division in



A e and *C B* directly, by right lines, as 1 1, 2 2, 3 3, &c. Then, from the points of division in *B D*, draw right lines tending directly to the point *A*, as 1 *A*, 2 *A*, 3 *A*, &c. The intersections of the lines bearing like numbers will be so many points in the *locus* (place or line) of the required curve; and the more numerous these, the more completely, of course, will the curve be defined. The practice may be called locating the curve by intersecting lines, and is equivalent to locating it by ordinates. Trace the curve, and the unit measure, *A B C*, of the true arm of the contemplated elbow will be constructed, and may be taken out for use.

Prob. 9. — *To apply the Unit Measure to the Construction of a Full Pattern for the Arms of the Elbow.*

Rule. — Draw a guide-line, *C a C*, suitably on the plate from which the pattern is to be taken, and of sufficient length. Then, with the measure in position 1, scribe to its sides *C A*, *A B;* with it in position 2, scribe to its sides *B A*, *A a;* with it in position 3, scribe to its curve *A B;* and with it in position 4, scribe to its sides *B A*, *A C*. This practice, care being taken during the proceedings to keep the base of the measure directly in the guide-line *C a C*, and that the extremities of the curve are made to unite in the same points, will construct the curve *A A A*, the curve proposed.

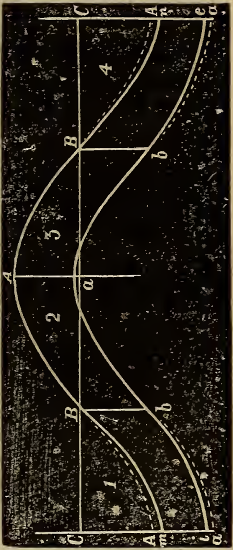


Now, as may be readily inferred, were the workman to cut out both arms of the elbow by this curve, and then to roll them properly into cylinders, they would unite by their oblique bases uniformly throughout; and would form a right angle, or stand to each other at an angle of 90 degrees. Moreover, were he to turn a burr on the oblique base of one of them, and a ledge and lip to match on that of the other, both parallel in their practical bearings to the plane that is common to both the said bases, the same state of things would still be maintained; and the arms would lock with a close uniform flange bisecting the angle.

We have thus far spoken of the true or geometrical cylindrical elbow; but we come now to speak of the arms as they ordinarily come from the *edging machine*, which, short of the strictest handling with reference thereto, can scarcely be made to turn the wards as above proposed, and, moreover, the flexibility of the plate is usually insufficient to admit of it.

The work, however, as it ordinarily comes from the edging-machine, lacks uniformity and definiteness in nearly all particulars. The wards, beside being out of parallelism to the plane that is common to the oblique bases, are more or less irregular, and out of harmony with each other. Scarce any two workmen turn them alike; and scarce any workman, it may almost or quite be said, ever turns them twice alike. The condition of the machine at one time compared with another; the thickness of the plate employed; the customary manner of handling the work in the machine; and the fact that one of the arms has usually to be made less in circumference at its extremity than at the joint, that it may enter the flush end of a straight pipe of the same nominal diameter, which tends to displace the curve on that arm, — these circumstances, I say, render the irregularity and indefiniteness spoken of unavoidable, in a greater or less degree.

There is one point, however, which we can fix upon, viz., — the wards come from the edging-machine invariably less oblique in their practical bearings to the sides of the cylinders than comports with the angle proposed for the elbow, unless that angle is considerably greater than 90 degrees.



It is the practice with many workmen to retain the true curve for the outside arm of the elbow, and after closing the straight lock to turn its ward unhesitatingly upon it; and then, after closing the straight lock of the inside arm, and before turning the burr, to cut away or *trim* its upper limb, or crown, until it will properly enter and lock. But this practice does not tend to correct the angle; and unless the wards are turned well down in the throat, and rather narrow at the crown, the angle will be greater than demanded.

The best general rule that I have been able to put to a practical test for constructing a general pattern for both arms of the elbow, of from 4 to 8 inches in diameter, is the following: —

Rule. — Construct the curve *A A A*, with its lateral and central guides, as already directed; then with the measure successively in positions 1, 2, 3, 4, but dropped down equally in each position, according to the desired width of the pattern, construct the curve *a a a;* taking care the while to keep the perpendicular of the measure directly in the guide-lines, *C A*, produced; and that the extremities of the measure unite in the same points. This will duplicate the true curve, and place the curves parallel in position. At this stage, it will be well to make the central line

A a distinct and permanent, and to drop the permanent perpendiculars *B b*, *B b*. Next, make *A m*, *A n*, each equal to one-fourth of an inch, and make *a i*, *a e*, each equal to one-eighth of an inch. Then, with the measure nearly in position 1, its angle *B* in the point *B*, and its curve tending to the point *m*, scribe to its curve, as *B m;* and with its angle *B* in the point *b*, and its curve cutting the point *i*, scribe to its curve *i b;* also, with the measure nearly in position 4, its angle *B* in the point *B*, and its curve tending to the point *n*, scribe to its curve, as *B n;* and, with its angle *B* in the point *b*, and its curve cutting the point *e*, scribe to its curve *e b:* then *m A n e a i* will be the pattern contemplated.

Prob. 10. — *To apply the Pattern to the Construction of the Arms of the Elbow*, Prob. 9.

Rule. — Place the pattern suitably on the plate from which the arms are to be taken, and scribe to its curve *m A n;* then raise it up on the plate till the points *b b* are in the points *B B*, and scribe to the curves *i b*, *e b:* then the curve *m A n*, with the continuation of the plate, will be the outside arm of the elbow, and the curve *i A e*, with the continuation of the plate, will be the inside arm.

Prob. 11. — *To find A C of the Unit Measure for any Angle of Elbow whatever.*

Rule. — Draw three sides of a parallelogram, as *A C*, *A D*, *D B*, diagram. Make *A D* equal to half the diameter of the arms, and make *A C*, *D B*, each of sufficient length; then, with the bevel square set to half the angle proposed for the elbow, as *C A B*, and one of its arms directly in *A C*, scribe to its arm *A B:* then will *D B* be equal to *A C* required.

By Mathematics.

For the Unit Measure of an Elbow of any given Angle.

$d =$ diameter of arms $=$ conjugate axis of hyperbola.

$A\,D = a = \pi d \div 4 = .7854d = \frac{1}{4}$ circumference of arms.

$V =$ angle of elbow in degrees.

$A\,e = 2a - d = \frac{1}{2}\pi d - d = .5708d.$

$$A\,C = b = \frac{\frac{1}{2}d}{\tan\frac{1}{2}V} = \frac{\frac{1}{2}d \times \cos\frac{1}{2}V}{\sin\frac{1}{2}V}.$$

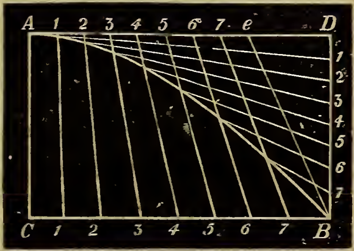
$$t = \text{transverse axis of hyperbola} = \frac{db}{a^2}\left(\tfrac{1}{2}d + \sqrt{a^2 + \tfrac{1}{4}d^2}\right)$$

$$x = \text{abscissa from vertex} = \frac{t}{d}\sqrt{(\tfrac{1}{4}d^2 + y^2)} - \tfrac{1}{2}t.$$

$$y = \text{ordinate} = \frac{d}{t}\sqrt{(tx + x^2)}.$$

PROB. 12. — *To construct a Right-angled Elliptical Elbow.*

RULE. — Construct a rectangle, *A D B C*, in length, *A D*, equal to one-fourth part of the circumference of the ellipse, or elliptic collar, and in breadth, *A C*, equal to half the conjugate diameter of the ellipse. Make *A e* = 0.18169 multiplied by the circumference of the ellipse, and in all other respects construct the unit measure by rule, Prob. 8.



Next construct the full curve, *A A A*, by rule, Prob. 9, and take out both arms by that curve; then lock the arms by their straight locks, turn the proper ward on the outside one, and trim the inside one, if necessary, to match.

PROB. 13. — *To find A C of the Unit Measure of an Elliptic Elbow of any given Angle.*

RULE. — Make *A D* equal to half the conjugate diameter of the ellipse, and in all other respects proceed by rule, Prob. 11.

BY MATHEMATICS.

For the Unit Measure of an Elliptic Elbow of any given Angle.

C = circumference of ellipse.
V = angle of elbow in degrees.
c = conjugate diameter of ellipse.
$d = C \div \pi$ = conjugate axis of hyperbola.
$A\ e = \frac{1}{2}C(\pi - 2) \div \pi = \frac{1}{2}\pi d - d.$
$A\ D = a = \frac{1}{4}$ circumference of ellipse.

$$A\ C = b = \frac{\frac{1}{2}c}{\tan \frac{1}{2}V} = \frac{\frac{1}{2}c \times \cos \frac{1}{2}V}{\sin \frac{1}{2}V}.$$

$$t = \frac{db}{a^2}\left[\tfrac{1}{2}d + \sqrt{(a^2 + \tfrac{1}{4}d^2)}\right] = \text{transverse axis of hyperbola.}$$

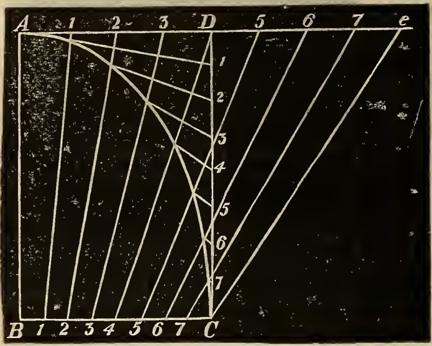
$$x = \frac{t}{d}\sqrt{(\tfrac{1}{4}d^2 + y^2)} - \tfrac{1}{2}t = \text{abscissa from vertex.}$$

$$y = \frac{d}{t}\sqrt{(tx + x^2)} = \text{ordinate.}$$

Prob. 14.—*To construct the Quadrant of a given Circle by intersecting lines.*

Rule.—Construct a square, *A B C D*, making each side equal to the radius of the proposed circle, and make *A e* equal to the diameter of the circle; thence by rule, Prob. 8.

Prob. 15.—*To construct the Quadrant of a given Ellipse by intersecting lines.*



Rule.—Construct a rectangle, *A B C D*, making *A B* equal to half the transverse diameter, and *A D* equal to half the conjugate. Make *A e* equal to the conjugate; thence by rule, Prob. 8.

Prob. 16.—*To apply the Quadrant of a Circle, or Quadrant of an Ellipse, to the Construction of the Circle or Ellipse.*

Rule.—Draw a guide-line of sufficient length; then, with *A B* of the quadrant in that line, in the four requisite positions, scribe to the curve and to *B C*, as required.

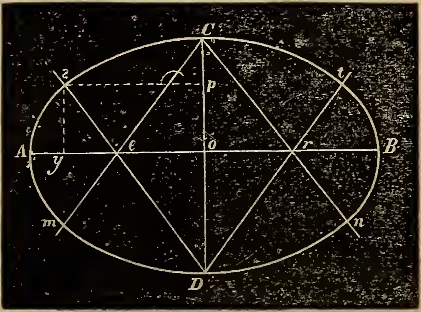
Prob. 17.—*To construct the Quadrant of a Cycloidal Ellipse by intersecting lines.*

Rule.—Construct a rectangle, *A B C D*, making the transverse diameter to the conjugate as π to 2; in all other respects proceed by rule, Prob. 15.

Prob. 18.—*To describe an Ellipse of given diameters, by means of two Posts, a Pencil, and a String.*

Let *A B* be the transverse diameter, and *C D* the conjugate.

Rule.—Lay down the given diameters at right angles to each other, and bisecting each other, as in *o*. Next, with half *A B* in the dividers, and one foot in *C* or *D*, cut *A B* in *y* and *z*. Next, strike in a pin at *y* and another at *z*. Next, pass a string around the pins, and tie it at such a length that the loop may be extended to *C* or *D*. Next, introduce a pencil, and, bearing upon the string, carry it around the centre, to the construction of the full circumference.



NOTE.—If we let l represent the circumference of an ellipse, D the transverse diameter, and d the conjugate, then $l = 3.53\sqrt{\left(\frac{\pi D d}{4} + \frac{(D-d)^2}{\pi}\right)}$, very nearly. And I may add, that the expression affords almost strict accuracy when d is not greater than two-thirds nor less than one-third of D. It gives l too long by 1·274 when $d = \frac{1}{4}D$, and too short by 1·351 when $d = \frac{3}{4}D$. (See CONIC SECTIONS, *Ellipse.*)

PROB. 19.—*To construct a Semi-parabola by intersecting lines.*

RULE.—Construct a rectangle, *A B C D*, making *A B* equal to the altitude of the parabola, and *B C* equal to half the base; make *A e* = *B C* = *A D*; thence by rule, Prob. 8.

PROB. 20.—*To construct a Right-angled Circular-Elbow of* 3, 4, 5, 6, 7, *or* 8 *pieces, of any given Diameter, and any given Radius of Curve.*

d = diameter of pipe.
r = radius of throat (usually $= \frac{1}{2}d$, $\frac{2}{3}d$, $\frac{3}{4}d$, or d).
$\pi d \div 4 = .7854d = \frac{1}{4}$ circumference of pipe.
V = angle at centre in degrees.

The Unit Measure.

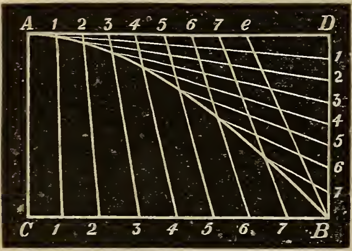
$$A\,C = \frac{d \times \tan\ V}{2}.$$

$$A\,D = C\,B = .7854d.$$

$$A\,e = .5708d.$$

No. of pieces.	Angle at Centre.	$Aa = (d+r) \times$	$Ab = d \times \tan V$	$ba = r \times$	$AC = d \times \frac{1}{2} \tan V$
3	22° 30′	.41421	.41421	.41421	.207105
4	15°	.26795	.26795	.26795	.133975
5	11° 15′	.19891	.19891	.19891	.099455
6	9°	.15838	.15838	.15838	.079190
7	7° 30′	.13165	.13165	.13165	.065825
8	6° 25′ 43″	.11266	.11266	.11266	.056331

RULE. — Construct a rectangle, $A\ C\ B\ D$, in length, $A\ D$, equal to one-fourth part of the circumference of the pipe, and in width, $A\ C$, equal to $\frac{1}{2}d \times \tan V$. Make $A\ e = d \times 0.5708$, and in all other respects proceed by rule Prob. 8.



Suppose the unit measure of a right-angled circular elbow of 3 pieces is required, and that the diameter of the pipe is to be 7 inches; then, on turning to the table, column headed

$$A\ C = d \times \tfrac{1}{2} \tan V,$$

and opposite 3 in the column containing the number of pieces, we find the co-efficient, or half the tangent of 22° 30′ to be 0.207105; therefore $A\ C$ of the unit measure $= 7 \times 0.207105 = 1.449735$ in.; $A\ D = 0.7854 \times 7 = 5.4978$ in.; and $A\ e = 0.5708 \times 7 = 3.9936$ inches.

Suppose the unit measure of an elbow of 5 pieces is required, and that the diameter of the pipe is to be 8 inches, then

$A\ D = .7854 \times 8 = 6.2832$ inches;
$A\ C = 8 \times .099455 = .79564$ inch;
$A\ e = 8 \times .5708 = 4.5664$ inches.

On the contrary, suppose the unit measure of an elbow of 4 pieces is required, and that the diameter of the pipe is to be 6 inches, then

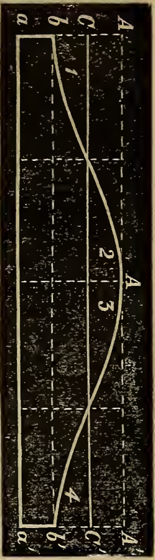
$A\ D = .7854 \times 6 = 4.7124$ inches;
$A\ C = .133975 \times 6 = .80385$ inch;
$A\ e = .5708 \times 6 = 3.4248$ inches.

PROB. 21. — *To apply the Unit Measure to the Construction of the several Segments of a Circular Elbow.*

RULE. — Draw a guide-line $C\ C'$, equal in length to the circumference of the pipe, and construct the curve, $b\ A\ b'$, by scribing to the measure in its several positions, 1, 2, 3, 4, as by rule, Prob. 9. This will make $A\ C = C\ b = \frac{1}{2}d \times \tan V$, and will make $A\ b = 2(C\ b) = d \times \tan V$. Make $b\ a$, $b'\ a'$, each equal to $r \times \tan V$, and draw the line $a\ a'$, which will make $A\ a$, $A'\ a'$, each equal to $(d + r) \times \tan V$. Thus $b\ A\ b'\ a'\ a$ will be the unit segment of the elbow; and, with the requisite continuation of the plate, will form one of the two equal and similar outside pieces of the elbow. The inside segments are equal one with another, and each is equal to two of the outside segments, in all cases.

Suppose a right-angled circular elbow of 3 pieces is required; that the diameter of the pipe is to be 7 inches, and that the throw or radius of the throat, r, is to be 6 inches; then

$C\ C'$ or $a\ a'$ of the segment $= 3.1416 \times 7 = 21.9912$ inches.
$A\ C = C\ b = d \times \frac{1}{2} \tan 22° 30' = 7 \times .207105 = 1.449735$ in.
$b\ a = r \times \tan 22° 30' = 6 \times .41421 = 2.48526$ in.
$A\ a = (d + r) \tan 22° 30' = (7 + 6) \times .41421 = 5.38473$ in.



The middle segment, therefore, of a 3-piece circular elbow will take the form No. 2, or No. 3, following diagram (No. 2, with reference to economy of stock, when the outside segments are made to lock at the throat). Thus the maximum width of the middle piece of a circular elbow consisting of 3 pieces $= 2\ (A\ a) = 2(d + r) \tan 22° 30'$; and the minimum width of the same piece $= 2\ (b\ a) = 2r \times \tan 22° 30'$.

Suppose a 5-piece elbow is required; that the diameter of the pipe is to be 6 inches, and that the throw, or radius of the throat, is to be equal to $\frac{2}{3}$ the diameter, or 4 inches; then

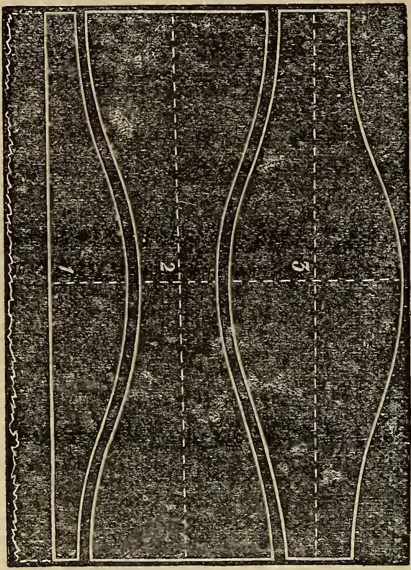
$C\ C' = 3.1416 \times 6 = 18.8496$ inches;
$A\ C = C\ b = .099455 \times 6 = .59673$ inch;
$b\ a = .19891 \times 4 = .79564$ inch;
$A\ a = (6 + 4) \times .19891 = 1.9891$ inches.

On the contrary, suppose that a 6-piece elbow is required; that the diameter of the pipe is to be 8 inches, and that the radius of the throat is to be equal to $\frac{1}{2}d$, or 4 inches; then

$C\ C' = 3.1416 \times 8 = 25.1328$ inches; and, by the table,

$$A\ C = C\ b = .07919 \times 8 = .63352 \text{ inch};$$
$$b\ a = .15838 \times 4 = .63352 \text{ inch};$$
$$A\ a = .15838 \times (8 + 4) = 1.90056 \text{ inches}.$$

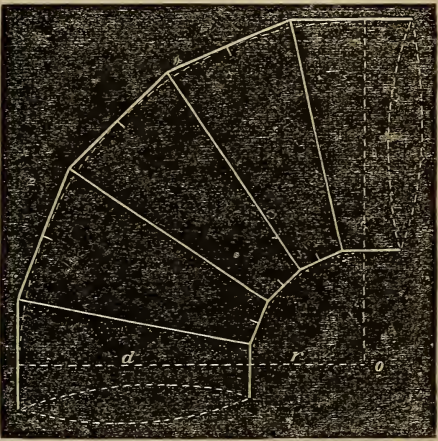
From the foregoing it may be perceived that to cut stock with reference to economy, we must make use of the three annexed general forms for the segments of a circular elbow whenever the elbow is to consist of more than three pieces; whereby one or more of the segments will have its straight lock at the crown, and the other, or others, at the throat; and, thus, an elbow consisting of an odd number of pieces will make up without waste.



The two outside segments of a circular elbow are equal and similar, one to the other, in all cases; and, if they are to have their straight locks at the throat, are but exact copies of the unit segment above described; thus, the segment *b A b′ a′ a*, preceding diagram, is identically the same as segment No. 1, of the diagram here presented. It may be perceived, also, that the half of segment No. 1, by its dotted transverse, is identically the same as one-fourth of segment No. 2, by its dotted transverse and longitudinal sections; and that a right quarter section of No. 2 is identically the same as a right quarter section of No. 3, only reversed in position.

NOTE. — The half of segment No. 2 or 3 by the dotted cross-section is a good working measure for the segments, and it may be taken out bearing the requisite margins for locks or laps, both along the curves and at the ward end. The allowance along the curve on one side should be with reference to the *burr*, and, on the other, to the *ledge* and *lip*. One of the pieces carrying the continuation for the pipe (one of the outside pieces) should be fitted last, and trimmed to correct the angle, if necessary. But, with proper allowance for locks, and correct handling in the edging-machine, trimming will seldom be required.

The annexed diagram presents a side view of a 5-segment elbow, or, in other words, it shows the outline of the several segments when they are rolled into place. *d* represents the diameter of the pipe, and *r* the radius of the throat. The dotted arcs have their centres at *o*. In this diagram, *r* is taken equal to $\frac{1}{2}d$, and it should seldom or never be taken at less; in practice it may be taken to any extent greater, as circumstances may require. A



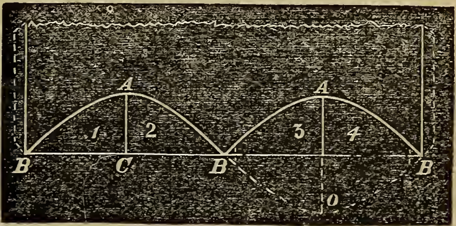
3-piece elbow is by no means a handsome structure, but it is easily made. A 4-piece elbow looks well, though it has one of the angles of its perimeter vertical to the centre.

PROB. 22.— *To construct a Collar for a Cylindrical Pipe of the same Diameter as the Receiving-pipe.*

The unit measure of this description of collar has already been treated of, and the manner of constructing it explained. It is identically the same as that of a right-angled cylindrical elbow of the same diameter. Directions for its construction are given under Prob. 8.

PROB. 23.— *To apply the Unit Measure to the Construction of the Collar*, PROB. 22.

RULE.— Measure down from the top of the plate from which the collar is to be taken, equal to the intended length of the collar and half its diameter, and draw a horizontal

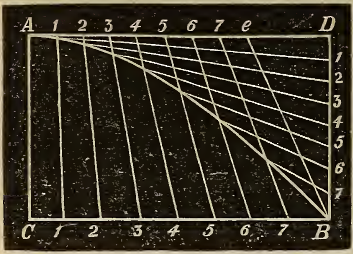


guide-line, *B B B*; next with the base of the measure, *B C*, directly in the guide-line, scribe to its curve and its perpendicular in the several positions, 1, 2, 3, 4; then, *B A B A B*, with the continuation for the pipe above, will be the collar proposed.

NOTE.—This collar will lock at one of its angles; and, ordinarily, this is the best practice. If it be desired to lock the collar from the crown of one of the arches, construct it by positions 2, 3, 4, and place position 1 in continuation of position 4.

PROB. 24.— *To construct a Cylindrical Collar of a given Diameter to fit a Receiving-pipe or Cylinder of a greater given Diameter.*

RULE.— Construct a rectangle, *A C B D*, in length, *A D*, equal to $\frac{1}{4}$ part of the circumference of the collar; and in breadth, *A C*, equal to $\frac{1}{2}$ the diameter of the collar multiplied by the diameter of the collar and divided by the diameter of the receiving-pipe: make *A e* equal to the diameter of the collar multiplied by 0.5708. Proceed in all other respects for the unit measure by rule, Prob. 8.



PROB. 25.— *To apply the Unit Measure to the Construction of the Collar*, PROB. 24.

RULE.— Proceed in all respects by rule, Prob. 23.

BY MATHEMATICS.

For the Unit Measure of a Right Cylindrical Collar of any given Diameter to fit a Cylinder of a given Diameter.

$d =$ diameter of collar $=$ conjugate axis of hyperbola.
$D =$ diameter of receiving-pipe or cylinder,
$C\,B = y = .7854d = \frac{1}{4}$ circ. of collar $=$ base of measure.
$A\,C = x = d^2 \div 2D =$ perpendicular of measure.
$A\,e = \frac{1}{2}\pi d - d = .5708d.$

$$t = \frac{dx}{y^2}\left(\tfrac{1}{2}d + \sqrt{(\tfrac{1}{2}d)^2 + y^2}\right) = \text{transverse axis of hyperbola.}$$

$y' = \frac{d}{t}\sqrt{(tx' + x'^2)}$; $x' = \frac{t}{d}\sqrt{((\frac{1}{2}d)^2 + y'^2)} - \frac{1}{2}t$; x' being any abscissa or part of x, reckoned from the origin A, and y' the ordinate to abscissa x'.

Prob. 26. — *To construct a Cylindrical Collar to fit an Elliptic-cylinder at either right section of the Ellipse.*

Rule. — Find, by any mechanical means, the radius of a circle arc that coincides best with the arc of the ellipse to be covered by the collar, and take twice that radius for the practical diameter, D, of the receiving pipe. Then proceed strictly by rule, Prob. 23, for the unit measure. Apply the measure to the construction of the collar by rule, Prob. 23.

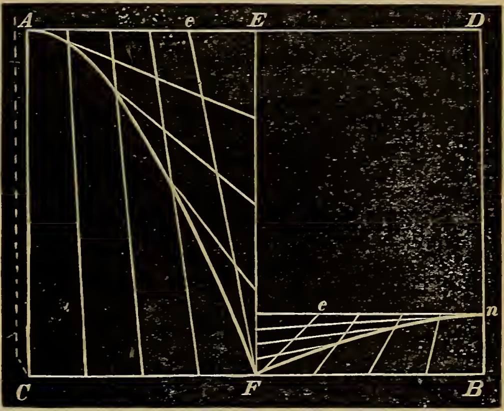
Note. — It is not in keeping with the laws of geometry to suppose that an ellipse and a circle can have a portion of their perimeters common to both; but it is customary with sheet-iron workers to fix a cylindrical collar to an elliptic stove; and by proceeding by the foregoing rule, a collar will be obtained which will practically fit without trimming, unless the diameter of the collar is unusually large for the size of the stove.

Remark. — Section $B\ A\ B\ o\ B$ of diagram, Prob. 22, shows the opening for the reception of the collar (or to be circumscribed by the collar) as it appears upon a plane surface. It may be formed upon a plane surface by scribing to the respective unit measure in the four positions indicated, in all cases.

Prob. 27. — *To construct a Cylindrical Collar of a given Diameter, to fit a Cylinder of the same Diameter, at any given Angle to the side of the Cylinder.*

$d =$ diameter of collar, $V =$ angle of collar to side of cylinder.

Rule. — Construct a rectangle, $A\ C\ B\ D$, making $A\ D$ equal to half the circumference of the collar, and $A\ C$ equal to $\frac{1}{2}d \div \tan\frac{1}{2}V$. Bisect $A\ D$ in E and drop the perpendicular $E\ F$, which will divide the rectangle $A\ C\ B\ D$ into two equal and similar rectangles. Make $B\ n$ equal to $\frac{1}{2}d \times \tan\frac{1}{2}V$, and draw the line $n\ o$ parallel to $B\ F$. Next, make $A\ e$ and $n\ e$



each equal to $0.5708d$, and proceed for the curves in all respects by rule, Prob. 8: thus, $A\ F\ n\ D$ will be the unit measure of the collar, and with the requisite continuation of the plate above $A\ D$, for the pipe, together with the necessary margins for the rivets and straight lock, will be equal to one-half of it.

EXAMPLE. — A cylindrical collar of $4\frac{1}{2}$ inches in diameter is to be constructed to fit a cylinder of the same diameter at an angle of 35° to the side of the cylinder; then

$A\ D$, diagram, $= \pi d \div 2 = 3.1416 \times 4.5 \div 2 = 7.0686$ inches; and, by the table of natural sines, cosines, and tangents of different angles, it is found that the tangent of half the angle of 35° (tan 17° 30′) is .31530; therefore,

$A\ C$, diagram, $= \frac{1}{2}d \div \tan \frac{1}{2}V = 2.25 \div .3153 = 7.13606$ in., and
$B\ n$, diagram, $= \frac{1}{2}d \times \tan \frac{1}{2}V = 2.25 \times .3153 = .709425$ in.

PROB. 28. — *To construct a Cylindrical Collar, or Spout, of a given Diameter, to fit a Cylinder of a greater given Diameter at a given Angle to the side of the Cylinder.*

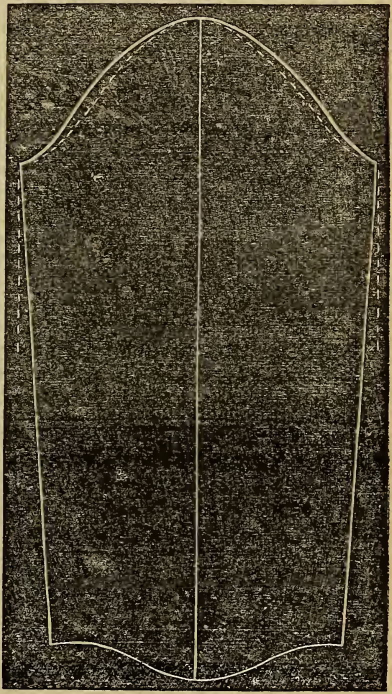
$d =$ diameter of collar.
$D =$ diameter of cylinder.
$V =$ proposed angle.

RULE. — Make $A\ D$ equal to $\pi d^2 \div 2D$, and in all other respects proceed strictly by rule, Prob. 27.

NOTE. — Diagram, Prob. 27, represents a right semi-section of a collar, to fit a cylinder of the same diameter as the collar, at an angle of 45° to the side of the cylinder. It is a general guide for oblique collars.

OF SPOUTS.

SPOUTS for vessels are usually made conical, more or less, according to the stake on which they are turned; and, by common consent, are divided into two classes; viz., "*tea-kettle spouts*," to fit cylindrical vessels; and "*coffeepot spouts*" or "*teapot spouts*," to fit flaring vessels.



In practice, no definite geometrical relations between the spout and the vessel it is intended for are sought to be maintained. Thus, the diameter of the spout relative to that of the body, its length, flare, angle of inclination to the body, and place of attachment, are matters of taste or convenience, or both, with the workman who constructs them. Nevertheless, the ideas of symetry and practical utility are not to be outraged, but, on the contrary, should be kept in view.

Rules, strictly geometrical, might be given, covering probable cases; but the workman, with a little practice, can much sooner design a becoming spout, and fit it satisfactorily by "trial and trimming," than in any other way.

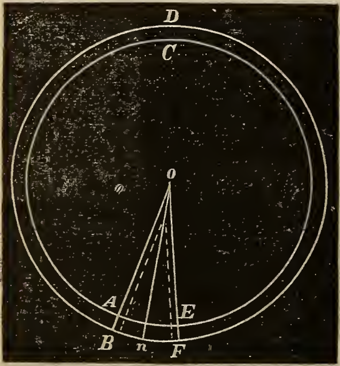
The annexed diagram represents the general outline of spouts. It is given in comparison with the true arm of a right cylindrical elbow, which it in a considerable degree resembles, that it may the more readily be understood. The full-lined figure is that of the spout; that by the dotted lines, the arm of the elbow.

Of Pitched or Bevelled Covers.

Prob. 29. — *To construct a Bevelled Circle, or Circular Cover, of a given Rise and given Diameter.*

h = rise, or perpendicular height.
d = initial diameter, or diameter of hoop.
s = slant height, or radius of chief arc, = $o\ A$, diagram.
m = sum of the widths of the burr and edge (about $\frac{1}{4}$ inch), = $A\ B$, diagram.
$s = \sqrt{((\frac{1}{2}d)^2 + h^2)}$; $R = s + m$, = $o\ B$, diagram.

Rule. — From a common centre, o, properly chosen on the plate, describe two circles, taking R for the radius of one of them, and s for that of the other. Next, draw a radius from the centre to the outer circle, as $o\ B$, diagram, and cut out that circle. Next, cut a narrow flexible measure, in length equal to $\pi(2R - d)$, and from the point B, with that measure bent to the circumference, measure off its length on the circumference, as from B to F, and from the new-found point, F, draw a radius to the centre, as $o\ F$. Next, bisect the arc $B\ F$, and draw the bisecting radius, $o\ n$. Next, allow the requisite margins for laps, seam, or lock, as by the dotted lines, and cut out by those lines. Lastly, cut open the remaining portion of the bisecting radius, to the centre, o: then will the sector, $B\ D\ F\ o\ B$, be the cover; and the arc, $A\ C\ E$, will be equal to the circumference of the rim, or hoop.



Example. — A bevelled cover is to be constructed having a rise of $1\frac{1}{4}$ inches, and to fit a cylinder of 12 inches in diameter: then

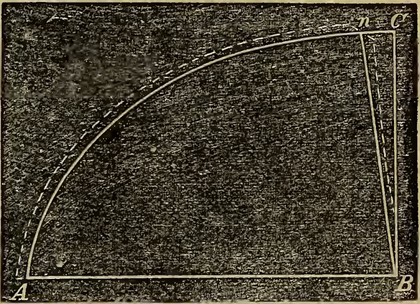
$$s = \sqrt{[(\tfrac{12}{2})^2 + 1.25^2]} = 6.1289 \text{ inches};$$
$$R = 6.1289 + \tfrac{1}{4} = 6.3789 \text{ inches; and}$$
$B\ n\ F$, the length of the flexible measure,
$$= 3.1416(6.3789 \times 2 - 12) = 2.381 \text{ inches.}$$

Note. — The value of s in all cases coming under this problem, since it is the length of the hypotenuse of a right-angled triangle whose base and perpendicular are given, may be found by mechanical means, thus: construct a right-angle by draft on a plane, making one of the legs equal to half the initial or given diameter (in the last example 6 inches), and the other equal to the given rise or perpendicular of the triangle (in the last example $1\frac{1}{4}$ inches), then will the rectilinear distance between the extremities of the legs, opposite the angle, be the hypotenuse, or value of s, required; thus, either side of a right-angled triangle may be found, by mechanics, the other two sides being known. But I must not be understood by this as encouraging a desire to avoid the extraction of the square root of numbers by arithmetic; without the ability to do that, the student will find himself often perplexed, and occasionally defeated.

PROB. 30.— *To construct a Pattern for a Bevelled Elliptic Cover of a given Rise, to fit an Elliptic Boiler of given Diameters.*

$D =$ transverse diameter of boiler.
$d =$ conjugate diameter of boiler.
$h =$ rise or perpendicular height of cover.

RULE.— Construct a right quarter-section of an ellipse, $A\ C\ B$, by rule, Prob. 15 or 18, making $A\ B$ equal to $\sqrt{[(\frac{1}{2}D)^2 + h^2]}$, and $B\ C$ equal to $\sqrt{[(\frac{1}{2}d)^2 + h^2]}$. Make $C\,n = \frac{1}{4}(P - p)$, P being the circumference of an ellipse whose semi-axes are $\sqrt{[(\frac{1}{2}D)^2 + h^2]}$ and $\sqrt{[(\frac{1}{2}d)^2 + h^2]}$, and p the circumference of one whose diameters are D and d; and from the point n draw a line $n\ B$: then will $A\ n\ B$ be the unit measure of the cover, and will contain one-fourth part of it. Allow, as by the dotted lines, the necessary margins for the edge and half-lock.

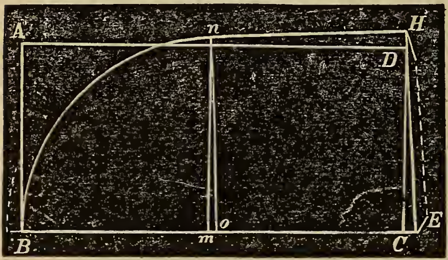


$A\ B$ and $B\ C$ may be found by mechanics (see NOTE appended to Prob. 29).

PROB. 31.— *To construct a Bevelled Cover of a given Rise, to fit a False-oval Boiler of given length and width.*

$D =$ length or transverse diameter of boiler.
$d =$ width or conjugate diameter of boiler.
$h =$ rise or perpendicular height of cover.
$S = \sqrt{[(\frac{1}{2}D)^2 + h^2]} = B\ E$, diagram.
$N = (S \times d) \div D = B\ m$, diagram.
$h' = (d \times h) \div D.$
$F = \sqrt{[(\frac{1}{2}d)^2 + h'^2]} = m\ n$, diagram.

RULE. — Construct a rectangle, $A\ B\ C\ D$, making $A\ D$ equal to $\frac{1}{2}D$, and $A\ B$ and $B\ m$ each equal to $\frac{1}{2}d$. Make $B\ E$ equal to $\sqrt{(\frac{1}{2}D)^2 + h^2}$, and $m\ n$ equal to $\sqrt{(\frac{1}{2}d)^2 + h'^2}$. Next, with the square in position $E\ H\ n$, one of the blades cutting the points E and D, and the other cutting the point n, draw the lines $E\ H$, $H\ n$. Next, with the dividers, find a radius, $o\ B$, that will cut the points B and n, and with o the centre, describe the arc $B\ n$: then will $B\ n\ H\ E$ be the unit measure of the cover, and contain one-fourth part of it, less the allowance, as by the dotted lines, for the edge and half the lock.



NOTE. — When $d = \frac{1}{2}D$, $h' = \frac{1}{2}h$, and F, N, and $B\ o = \frac{1}{2}S$. S and F may be found by *Mechanics*, as by rule given in Note appended to Prob. 29. In practice, if h be taken equal to $\frac{1}{10}D$, the rise will generally be sufficient.

OF CAN-TOPS.

Can-Tops are simply truncated cones, and the cones themselves are pitched or bevelled circles. They may be defined in part by their *pitch*, which I shall here define to be the angle of the side of the cone to the base; or they may be defined by their bases and perpendicular height. The body of a common tunnel is a two-thirds pitched can-top, or a can-top having a pitch of 60°; or, in other words, it is the frustum of a cone, or pitched circle, whose slant height was equal to the diameter of the base: it is therefore made up of a semi-circle whose radius is equal to the greater base; but can-tops are rarely pitched as steep as 60°. They may be constructed in a single piece, and should be when practicable; or they may be composed of two or more right-sections, as the body of a common flaring vessel; so they may be pieced transversely, when desirable, after the manner of piecing a large tunnel.

PROB. 32. — *To construct a Can-top of a given Depth and given Diameters.*

RULE. — Proceed in all respects by rule, Prob. 1 or Prob. 2.

Prob. 33. — *To construct a Can-top of a given Pitch and given Diameters.*

D = diameter of greater base.
d = diameter of lesser base.
V = pitch, or angle of the side to the base.
H = perpendicular height of generating cone, having D for its base.
R = slant height of generating cone, having D for its base.
r = slant height of cone, having d for its base.

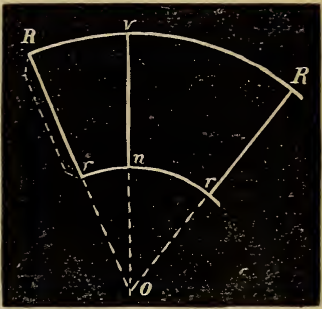
$$H = \frac{\frac{1}{2}D \times \sin V}{\cos V} = \frac{1}{2}D \times \tan V = \sqrt{R^2 - (\frac{1}{2}D)^2} = \frac{Dh}{D-d}.$$

$$R = \frac{\frac{1}{2}D}{\cos V} = \frac{H}{\sin V} = \sqrt{H^2 + (\frac{1}{2}D)^2}.$$

$$r = \frac{\frac{1}{2}d}{\cos V} = \frac{Rd}{D} = \sqrt{(H-h)^2 + (\frac{1}{2}d)^2}.$$

$$h = \frac{D \times \tan V(D-d)}{2D} = \frac{H(D-d)}{D} = H - \sqrt{r^2 - (\frac{1}{2}d)^2},$$ the height of the frustum.

Rule. — From a common centre describe two concentric arcs of circles of sufficient length, taking R for the radius of one of them, and r for that of the other. Next, draw a radius from the centre to the outer arc, as *o r R*, diagram. Next, with a flexible measure, cut to the required length, (the whole circumference of one of the bases, when practicable), and bent to the proper curve, measure off that length on the curve, as from R to v, and from the new-found point, v, draw a radius to the centre, o: then will $R\ v\ n\ r$ be the top, or a known aliquot part of the top, required.



Special Cases.

For a two-thirds pitch, or pitch of 60° (sometimes erroneously called a half-pitch, because the angle of the side and axis is half as great as that of the side and base).—From a common centre describe two concentric semi-circles, taking the diameters of the given bases, respectively, for the radii: take the greater semi-circle for the top required.

For a half-pitch, or pitch of 45°.—Describe two concentric circles, taking the diameters of the bases multiplied by 1.4142, respectively, for the radii; take three-fourths of the greater circle for the body, less one-fourth of its radius as a chord.

For a third-pitch, or pitch of 30° (often erroneously called a half-pitch, because the angle of the base and side is half as great as that of the side and axis).—From a common centre describe two circles, taking $D \times 1.1549$ for the radius of one, and $d \times 1.1549$ for that of the other. Next, draw a radius from the centre to the outer circumference. Next, take $3D \div \pi = D \div 1.0472$ in the dividers, and from the summit of the radius just drawn, space it off on the outer circumference, as a chord; and from the last found point draw a radius to the centre: the greater sector, or sector of the re-entrant angle, will be the body or top required.

For a quarter-pitch, or pitch of $22\frac{1}{2}$°.—From a common centre describe two circles, taking $1.0824D$ for the radius of one, and $1.0824d$ for that of the other. Next, draw a common radius to the circles. Next, take $\frac{1}{2}D$ in the dividers, and from the point where the radius cuts the circumference of the greater circle, step it off on that circumference as a chord; and from the last found point draw a radius to the centre: take out the lesser sector, and the remainder of the circle will be the top required.

NOTE 1.—The foregoing special cases are all covered, of course, by the preceding general rule; but it is more expeditious to measure an arc by its chord, when the latter is known or can be readily found, than to measure it by the flexible measure, more generally employed.

2.—Having laid off the dimensions pattern, as above directed, allow the requisite margins for burr and lock, or seam, as in other like cases.

3.—A *fifth-pitch*, or pitch of 18°, is sometimes, but incorrectly, called a quarter-pitch, because the angle of the base and side is one-fourth as great as that of the axis and side.

4.—The prevailing tendency of a can-top, or frustum of a cone, to "lop at the lock," as it is called, is clearly due to the careless manner of turning the wards and closing them. If the workman will turn the wards parallel to the lines of the dimensions pattern, and will use neither more nor less for the lock than the margins he allows, the tendency complained of will not obtain (see Note, Prob. 1).

OF LIPS FOR MEASURES.

Lips for measures, when laid off on a plane, are simply lunes, or crescents. In practice, no arbitrary relations between the lip and the vessel it is intended for are sought to be maintained. Thus, the length of the lip, its width, and its angle with the side of the vessel, are matters of taste or choice with the workman, limited, of course, by the purpose the lip is intended to subserve.

The following rule, Prob. 34, which sets the lip at an angle of 41° 50′ to the side, very nearly, is the one most commonly in practice; and it appears to be as good a general rule as can be offered.

Prob. 34. — *To construct a Lip for a Measure, the Diameter of the Top of the Measure being given.*

Rule. — Take three-fourths of the diameter of the top of the measure in the dividers, and with that as radius describe a circle. Next, with the same radius, from a new centre, taken about midway between the centre of the circle and the circumference (more or less, according to the desired width of the lip, wired-edge included), describe an arc cutting both sides of the circumference: then will the crescent thus formed be the lip intended.

Note. — To diminish the pitch, which serves to make the lip longer, take the radius at a greater ratio to the diameter of the lesser base, or top, than 3 to 4. But, even when the measure has little or no flare, the radius should not much, if any, exceed seven-eighths of the diameter of the top.

Of Sheet Pans.

Sheet pans are vessels intended to hold fluids at a temperature above the melting point of solder. They are constructed of a single rectangular sheet, by turning up the sides, and folding the surplus surfaces at the corners upon the sides.

Dripping-pans and *baking-pans* are commonly constructed with sides slightly flaring, while *evaporating-pans*, most generally, have perpendicular sides.

In geometry, those having oblique sides are called prismoids, or prismoidal vessels, and those having perpendicular sides are called parallelopiped vessels, or prisms.

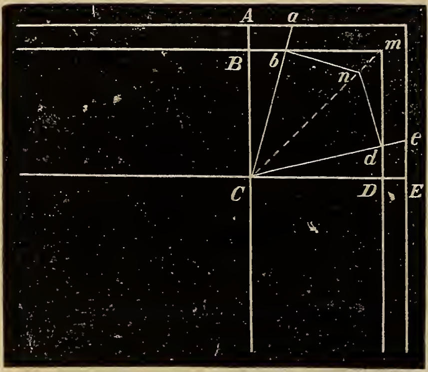
These vessels are commonly constructed with wired tops, or rims, partly for the purpose of stiffening them, and partly to hold the sides in place.

Sheet pans may be constructed to given dimensions, — length, breadth, and depth, — and thus to given capacities; so they may be constructed to given ratios of parts; but, generally, economy of stock and utility of purpose, without further specifications, are allowed to govern.

With these remarks (and the additional one, perhaps, that the sides of a sheet pan are to be of equal width), we might dismiss the subject, were it not that rules for cutting the corners so as to bring the edges up to, or under, the wire, seem to be called for.

PROB. 35.—*To cut the Corners for a Perpendicular-sided Sheet Pan.*

RULE.—Let *A B* and *D E*, diagram, represent the width required to cover the wire; *B C* and *D C*, the width of the sides. Remove the section *A B m D E*: then will the square *B m D C*, folded diagonally, and reflexed upon the side or end, form the edge proposed.



PROB. 36.—*To cut the Corners for an Oblique-sided Sheet Pan.*

RULE.—Let *A B* and *D E*, diagram, represent the width or margins required to cover the wire; *B C* and *D C*, the width of the sides; *B b* and *D d*, the flare of the sides. Draw *b n* perpendicular to *a C*, and *d n* perpendicular to *e C*. Next, remove the section *a b n d e*: then will the trapezium *b C d n*, folded diagonally from *n* to *C*, and reflexed upon one of the sides, form the edge contemplated.

PROB. 37.—*To construct a Heart, or Heart-shaped Cake-cutter.*

RULE.—Construct the Gothic arch, No. 1; this work, and upon its chord, describe two equal semi-circles, in diameter equal to half the length of the chord.

PROB. 38.—*To construct a Mouthpiece for a Speaking-tube.*

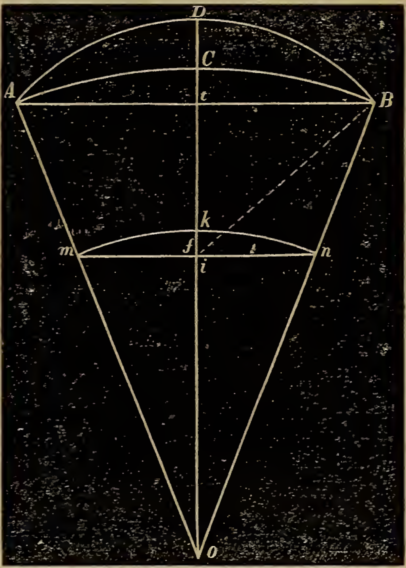
The curve for this structure consists of four cymas of unequal branches, in pairs. The structure itself is little else than two short, decidedly flaring teakettle spouts, in a single piece, having a side common to both.

Prob. 39. — *To construct a Pattern for the Body of a Circular-bottomed Flaring Coal-hod, all the curves to be arcs of circles.*

$D =$ nominal diameter of greater base.
$d =$ nominal diameter of lesser base.
$h =$ perpendicular depth of vessel.

$$H = \frac{Dh}{D-d}; \; R = \sqrt{(0.61685D^2 + H^2)}; \; r = \frac{Rd}{D}.$$

Rule. Place the square on the plate and scribe to its edges, as $A\ t\ o$, diagram; produce $A\ t$ in the direction B, and $o\ t$ in the direction D, making all the lines from t of sufficient length. Next, make $t\ B$ equal to one-fourth part of the circumference of the greater base $= 0.7854D$, and make $t\ A$ equal to $t\ B$. Next, make $t\ o = H$, $= Dh \div (D-d)$, and draw the radii $o\ A$, $o\ B = R$. Next, drop the square perpendicularly down on the line $t\ o$, equal to the given perpendicular depth, h, as from t to i, and through the point i draw the line $m\ i\ n$, parallel to the line $A\ t\ B$. Next, take the nominal diameter of the greater given base (D) in the dividers, and with one foot in the point B, and the other in the line



$t\ o$, as at f, and f the centre, describe the arc $B\ D\ A$. Next, with $o\ A$ the radius and o the centre, describe the arc $A\ C\ B$. Next, with $o\ m$, $= r$, or $o\ n$ the radius and o the centre, describe the arc $m\ k\ n$; then will $A\ D\ B\ n\ k\ m$ be the front half of the body, or a pattern covering that portion; and $A\ C\ B\ n\ k\ m$, in a separate and distinct piece, will be the back half of the body, or a pattern covering that portion. Both portions are introduced here in the same diagram, that their relations may be perceived and readily comprehended.

NOTE.—This style of hod is introduced, partly to meet the popular demand with regard to the spout, partly with reference to economy in stock, and partly because of the readiness with which it may be plotted upon a plane surface, compared with the labor of plotting for an oval or elliptical bottom. The workman will find no difficulty in constructing it, except, perhaps, in turning the rim for the wire in the immediate vicinity of the side-locks: this he will probably be obliged to do on the stake. When locked at the sides and wired, it is to be compressed along the upper front half, and also in front, so as to form nearly a perpendicular-sided spout, of about four inches in width at the lip.

The real bases of the vessel will be somewhat greater than the nominal, because the chords of the arcs are made equal to the given half-circumferences, instead of the arcs themselves; the diameter of the bottom, therefore, will be equal to twice the arc $m\ k\ n$, divided by 3.1416, instead of being equal to d.

In practice, if the nominal diameter of the greater base be taken equal to once and one-half that of the lesser, or the nominal diameter of the latter be taken equal to two-thirds that of the former, and the perpendicular depth be taken equal to the nominal diameter of the lesser base, the proportions will be found satisfactory; moreover, if the nominal diameter of the greater base be taken at 12 inches, and the foregoing proportions maintained, a very fair medium-sized hod will be obtained, particularly if the requisite margins be allowed.

The hoop for the bottom, which should be about one and a half inch in width after it is wired, may have the same flare as the body, and the radius of its lesser arc will be the same as that for the lesser base of the body.

SOLDERS, ALLOYS, AND COMPOSITIONS.

Hard solder. — Copper 2 parts, zinc 1 part; — used with powdered borax.
Pewterer's solder. — Tin 2 parts, antimony 1 part.
Tinman's solder. — 1 part each, lead and tin.
Plumber's solder. — Tin 2 part, lead 5 parts; or, pewter 4 parts, tin 1, and bismuth 1. Resin is used with the last three.
Solder for iron. — Tough brass, used with borax.
Silver solder. — 1 part brass, and from 2 to 5 parts fine silver.
Spelter " for brass, copper, and German silver. — 2 parts brass, 1 part zinc.
Solder for copper. — Brass 6 parts, tin 1, zinc 1.
Dentist's solder. — 4 parts 22 carat gold, 1 part silver, 1 part copper.
Dentist's gold. — 10 parts 22 carat gold, 1 part silver, 1 part copper.
Dentist's compound for clasps. — 5 parts 22 carat gold, 1 part platinum.
Yellow brass. — Copper 3 parts, zinc 1 part.
Spelter. — Copper 2 parts, zinc 1 part.
For lathe bushes. — Copper 16 parts, tin 4 parts, zinc 1 part.
" " " harder. — Copper 16 parts, tin 4 parts, zinc 2 parts.
Improved Babbit metal. — This composition, for the lining of boxes, shaft bearings, &c., — which, from the satisfaction it has thus far given, bids fair to come into general use, — is composed of *tin* 12 parts, *antimony* 3 parts, and *copper* 2 parts. The original recipe for this alloy was, tin 6 parts, antimony regulus 2 parts, and copper 1 part, as its prime equivalents, to which, when about to be remelted for use, 2 parts of copper to 1 of the composition were added.
For pulley blocks. — Copper 7 parts, tin 1 part.
" *wheels, boxes, and cocks.* — Copper 8 parts, tin 1 part.
Bronze — government gun-metal. — 9 parts copper, 1 part tin. The specific gravity of this composition is greater than the mean of its constituents.
For valves. — 10 parts copper, 1 part tin.
Bell metal. — 39 parts copper, 11 parts tin.
Gong metal — 40 parts copper, 5 tin, 2.8 zinc, 2.15 lead.
Bath metal. — 32 parts brass, 9 parts zinc.
Blanched copper. — 16 parts copper, 1 part arsenic.
Britannia metal. — 1 part each, — brass, tin, bismuth, antimony.
Petong, or Chinese white copper. — 20.2 parts copper, 15.8 nickel, 12.7 zinc, 1.3 iron.
German silver. — 2 parts copper, 1 nickel, 1 zinc; when intended to be rolled into plates, it is composed of 60 parts copper, 25 parts nickel, 20 of zinc, and 3 of lead.
Manheim gold. — 3 parts copper, 1 of zinc, and a small quantity of tin.
Mock gold. — 16 parts copper, 7 parts platinum, and 1 part zinc.
Mock platinum. — 8 parts brass, 5 parts zinc.
Speculum metal. — 7 parts copper, 3 zinc, 4 tin; or, 6 parts copper, 2 of tin, and 1 of arsenic.
Tombac, or gilding metal. — 9 parts copper, and 1 part zinc.
Mock iron — expanding alloy. — Lead 9 parts, antimony 2 parts, bismuth 1 part. This composition expands in cooling, and is used in filling small defects in iron castings.
Ring, or jeweller's gold. — 150 parts pure gold, 39 parts copper, 22 parts pure silver.
Queen's metal. — Tin 9 parts, antimony 1, lead 1, bismuth 1.
Pewter, common. — Tin 4 parts, lead 1.
" best. — Tin 100 parts, antimony 17.

Steel alloyed with $\frac{1}{500}$ part of platinum, or to the same extent with silver, is rendered harder, more malleable, and better adapted for every kind of cutting instrument.

Solder for gold. — 3 parts gold, 1 part silver, 1 part copper.

Solder for Britannia. — Tin, 7 parts; lead, 4 parts.

Yellow solder. — Copper, 1 part; zinc, 1 part.

Black solder. — Copper and zinc, each 8 parts; tin, 1 part.

Pewterer's soft solder. — Bismuth, 1 part; tin, 2 parts; lead, 1 part.

Common Britannia metal. — Tin, 100 parts; copper, 2 parts; antimony, 1 part.

Common bronze metal. — Copper, 4 parts; zinc, 2 parts; tin, 1 part.

White metal. — 5 parts copper, 3 zinc, 1 lead, 1 tin.

Silver-colored metal. — Tin, 50 parts; copper, 3 parts; antimony, 3 parts; bismuth, 1 part.

Imitation silver. — 16 parts copper, 1 part zinc.

Pinchbeck. — Copper, 4 parts; zinc, 1 part.

Metal for taking Impressions. — Bismuth, 6 parts; lead, 2 parts; tin, 1 part.

Rivet metal. — Copper, 10 parts; tin, 5 parts; zinc, 2 parts.

Fusible alloy (melts at 200°). — Bismuth, 2 parts; lead, 1 part; tin, 1 part.

Muriate of zinc. — Muriatic acid, holding in solution all the zinc it will dissolve.

Acid for soldering tin. — Muriate of zinc, 1 part by volume; soft water, 2 parts by volume; add a trifle of Sal. Ammoniac.

Acid for soldering zinc. — Muriate of zinc, 10 ounces; Sal. Ammoniac, 1 ounce; water, 1 pint.

Acid for soldering brass or copper. — Muriate of zinc, 5 parts; Sal. Ammoniac, 1 part.

Acid for soldering gold or silver. — Muriatic Acid, 2 parts; sperm tallow, 1 part; Sal. Ammoniac, 1 part; all by weight.

Acid for soldering iron. — Muriatic Acid, 16 parts; sperm tallow, 6 parts; Sal. Ammoniac, 4 parts; all by weight.

Tinning acid, for brass or copper. — Muriate of zinc, 4 parts; soft water, 4 parts; Sal. Ammoniac, 1 part; all by weight.