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Physical and Chemical Tests of Cast Iron

Mechanical Engineering

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PHYSICAL AND CHEMICAL TESTS OF CAST IRON

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Walter John Kanne

THESIS FOR THE DEGREE OF BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

IN THE COLLEGE OF ENGINEERING OF THE UNIVERSITY OF ILLINOIS PRESENTED JUNE, 1906

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June 1, 1906

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

WALTER JOHN KANNE

ENTITLED PHYSICAL AND CHEMICAL TESTS OF CAST IRON

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE

OF Bachelor of Science in Mechanical Engineering

L. P. Brickenridge

HEAD OF DEPARTMENT OF Mechanical Engineering

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CONTENTS

Chemical and Physical Tests of Cast Iron.

- I. Preliminary Remarks
- II. Purpose of Tests

III. Physical Tests

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- 1. Selection of Test Bars
- 2. Standard Test Bars
- 3. Moulding Test Bars
- 4. Determining Strength of Cast Iron
 - a. Transverse Test
 - b. Pure Tension
- 5. Results of Tests (Table)
- IV. Chemical Tests
 - 1. Sampling Iron Filings for Analysis
 - 2. Determination of Elements
 - a. Carbon
 - b. Silicon
 - c. Manganese
 - d. Sulphur
 - e. Phosphorus
 - 3. Results of Chemical Analysis (Table)
- V. Effects of Various Elements
 - 1. Carbon
 - 2. Silicon

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- 3. Sulphur
- 4. Phosphorus
- 5. Manganese
- VI. Relation between the Thickness of Metal and Chemical Composition - (Table)
- VII. Conclusions
 - 1. Disadvantages of Chemical Analysis
 - 2. Advantages of Physical Tests



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Chemical and Physical Tests of Cast Iron.

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1. Preliminary Remarks.

Cast Iron is an alloy composed of iron, silicon, manganese, sulphur and phosphorus. Since its strength and many of its physical properties depend upon its chemical composition, we must know in what ratio to combine these elements in order to produce castings at a low cost which will meet commercial requirements. It becomes necessary, therefore, to know the effects of the predominant elements and the results of different chemical combinations. A few of the leading foundries of this country have installed separate testing departments to determine the elements of the cast iron by means of chemical analysis. All of their cupola charges are made according to the direction of the chemist. Mechanical tests are made on the final product and if the castings do not contain the properties which are required the chemical composition is changed until such a combination is made that it will give the required physical properties. Although it is tedious and rather expensive to make both chemical and physical tests, yet on account of the information obtained from the mechanical tests, these foundries have decreased the expense of production, increased the quality of the castings and turned out a better grade of iron at a smaller cost. They are able to make use of a cheaper iron and by combining it properly, by the aid of physical tests, they have not only kept up the standard but

and the second s have increased the quality of the castings.

II. Purpose of Tests

A series of tests were made at the University Experiment Station to show that a good grade of castings could be made by determining the physical properties of cast iron and thus controlling the chemical composition. This can be accomplished by means of mechanical tests which can be made by anyone of ordinary intellegence. Such tests show the founder exactly what physical properties the castings have, and what change to make in the chemical composition to bring the physical properties of the castings up to the standard. Such tests show whether more or less silicon is needed to overcome all influences whether due to chemical changes or to adverse conditions which cannot be foreseen.

III. Physical Tests

1. Selection of Test Bars

In order to obtain accurate and reliable data such a test bar must be selected which will give the most uniform results for all cast-iron mixtures. The test bar must give the greatest variation in strength and shrinkage for the smallest variation in chemical composition. The one-half inch square bar is recommended by Keep. When it is made of gray iron, it cools so quickly that its grain is influenced very little by the time occupied in cooling; consequently the variation in

* From " Cast Iron " by Keep.

- 4 -

shrinkage of such a bar more nearly agrees with the variation in silicon. The one lnch square bar gives results that agree very well with those obtained from the one-half inch bar. These two bars are being used to a considerable extent in many commericial foundries because they are easy to make and easy to test and because they are subjected to the same conditions as the ordinary castings. They seem to be the only standard sizes which are suited for general foundry testing and for that reason they were used for these tests.

2. Standard Test Bars

Plate I consists of a drawing of the most important test pieces which have been adopted by the American Foundrymen's Association.

The one-half inch square test bar is as small a bar as it is practicable to make of gray iron. It cools so quickly that the variation of shrinkage nearly agrees with the variation of silicon. The measurement of the shrinkage of such a bar is a mechanical analysis for * silicon.

The one inch square bar has been more generally used than any other size.

The 2" x 1" bar tested with the flat side down has also been used because it is thought that such bars represent castings one inch thick better than the one inch square bar. Such a bar is more inconvenient to handle, cools more slowly and requires a larger testing machine.

* From " Cast Iron " by Keep

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The 2"x2", 3"x3" and the 4"x4" bars are seldom used on account of the difficulty of breaking them. The results of these bars are not satisfactory because they cool so slowly that the effect of the heat exerts a greater influence than the chemical composition.

On account of the difficulty of moulding the square bars horizontally the American Foundrymen's Association has recently adopted a round bar one and one-quarter inches in diameter and thirteen inches long which is cast on end. The bar is tested transversely on supports twelve inches apart. This bar gives better results than the square bar when subjected to the transverse test. If the shrinkage is to be determined separate bars must be cast for that purpose.

3. Moulding Test Bars

The one inch square bars were moulded horizontally in the ordinary moulding sand a day or two before the iron was poured. The patterns were not rapped any more than was necessary for drawing. Two gates large enough to prevent shrink holes, were cut into each mould at a distance of two inches from the ends. This left the ends free so that the shrinkage could be measured directly. The iron entered on the upper side of the test bars. Iron chill blocks were placed at one end of some of the patterns to determine the chill of the iron. The sand between the block and the pattern was carefully

* From " A.F.A. " Feb. 1906

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1" Square Bar and Keep's 1/2" Square Bar

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Keep's 1/2" Square Bar, Yokes and Follow Board

removed. Plumbago was used as a facing.

The one-half inch bars were moulded horizontally acrecording to Keep's method. The iron entered from below and all the slag remained in the skim gate. Chill blocks were placed at the ends of the patterns.

- 10 -

4. Determining Strength of Cast Iron

a. Transverse Test

The strength of the cast iron was determined by both the transverse and the tension methods. Tn the transverse test, the bars were placed horizontally in the machine and supported at two places a distance of ten inches apart. The load was applied half way between the supports. The shrinkage of each bar was determined, also the maximum deflection and the breaking load. The ultimate breaking strength was calculated by the formula SI M = -- where M = the maximum bending moment, I the moment of inertia, and c the distance from the center to the most remote fiber. The cross sectional area was taken as the average, because at the best it can be measured only approximately; for in any case the surface is uneven, being a series of elevations and depressions, corresponding to the shape of the grain of the sand which composed the mould. For this reason the patterns were not rapped more than was necessary for drawing and all precautions were taken in moulding to get the same sized cross section, as far as possible, in each test bar.

* From " Cast Iron" by Keep

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b. Pure Tension

In order to bring out the differences in the results of the different methods of determining the strength of cast iron, some of the bars were pulled apart in a vertical testing machine. The test bars broke at nearly one-half the stress which was indicated by the transverse method. More or less eccentric loading took place each time, consequently the results are low.

5. Results of Tests

The following tables show the variation in strength and shrinkage of cast iron: also the discrimination between the tension test and the transverse test. The ultimate unit stress S of cast iron, determined by the formula $S = \frac{MC}{I}$, is far too great. For materials such as stone and cast iron, which have a variable coefficient of expansion, this formula does not hold: beyond the elastic limit it gives results at the breaking point in excess of the real tensions set up in the material. For this reason the formula must be modified; thus $kS_{b} = \frac{MC}{I}$.

The constant k depends not only upon the material but also on the form of the cross section of the test bar. It is determined by the following equation: $k = u \sqrt{\frac{c}{z}}$, where



u is a constant which depends upon the material and the shape of the edge in tension, c is the distance from the neutral axes to the most remote fibre, and z the distance

- 11 -

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from the neutral axis to the center of gravity of the section lying below the neutral axis. The value of u must be determined for different materials. For cast iron bars, rectangular pieces, I beams or channels, tested with the flat side down u = 1.33: for cast iron bars square, rectangular or triangular, tested on edge u = 1.2. Thus the value of k can be determined for any shaped bar. The correct unit stress of cast iron may now be calculated by the formula $kS_t = S_b$, where S_t is the unit breaking strength, k the constant just found and S_b the ultimate unit stress given by the formula $S_b = \frac{M}{I} \frac{c}{c}$. Substituting for S its value kS in the last equation, we have $S_t = \frac{M}{k} \frac{c}{l}$, which gives the correct unit stress.

The values given in the tables for tension are low. The tension machine in the laboratory was not provided with a ball-bearing socket, consequently, the load was not distributed uniformly over the entire cross section, but the bar was loaded eccentric and hence the machine did not show the maximum unit stress.

* From " Elesticitat und Festigkeit" by Bach.
*** From " Metallurgy of Cast Iron" by West.

- 12 -

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Table I

Transverse Test Heat No.1

		Chuinh	7 2	Lbs. per sq. in.			
No.	Defl	age in inches	in lbs.	$S = \frac{MC}{I}$	S= <u>Mc</u> Ik	Remarks	
6	.081	.10	3730	55700	32000		
3	.060	.12	2915	43500	25000	Blow hole	
13	.062	.12	3000	45000	26000		
15	.070	.10	3400	51000	29500		
7	.095	.12	3420	51100	29600		
2	.040	.10	2925	43800	25400	Blow hole	
1	.068	.10	3570	53200	30700		
15	.065	.12	2965	44300	25600		
5	.061	.12	3300	49400	28400		
17	.055	.12	2900	43500	25000		
4	.053	.10	2860	43000	24800	Defective	
14	.066	.09	2615	39700	23000	Defective	
19	.060	.14	3565	53500	30700		
14	.065	.09	3540	53000	30600		
12	.058	.10	2970	44500	25600		
Tension							
11		.10	16700	16700		Defective	
18		.10	23500	23500			
8		.12	18140	18140		Defective	
NOTE Distance between support - 2011 422 - 21							

<u>'E</u> Distance between supports = 10". All pieces 1" square.

k = 1.73; Value given by Bach.

Silicon = 2.76

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	T T C	110 4 61 00	ICSU HCC	0 110	
 Size	Shrink- age in inches	Load in lbs.	Lbs. per S= <u>Mc</u> I	sq. in. S= <u>Mc</u> Ik	Remarks
•5"	.09	390	47000	27200	
1.0"	.08	4080	60000	34600	
1.0"	.10	3335	49850	28600	
1.0"	.10	3 9 10	58800	34000	
1.0"	.08	3665	55000	31600	
1.0"	.09	3360	50500	29300	
	0.0				

Transverse Test Heat No.2

120	1.0"	.09	3360	50500	29300		
121	1.0"	.08	3850	57600	33100		
122	1.0"	.09	2620	54100	31500		
123	1.0"	.10	3490	52100	30100		
124	1.0"	.08	2745	41000	23700		
125	1.0"	.11	3460	51890	30000		
126	•5"	.09	460	49700	29600	9" between supports	
Tension							
112	1.0"	.10	19720	19720			
113	1.0"	•08	16000	16000		Defective	
114	1.0"	.08	16850	16850		Defective	
115	1.0"	.08	18850	18850			

NOTE

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Distance between supports = 10"

k = 1.73; Value given by Bach.

Silicon = 2.87

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	Transv	erse	Test	Heat	No.	3
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		Chrink	Lood	Lbs. per	sq. in.			
No.	Size	age in inches	in lbs.	S= <u>Mc</u> I	S= <u>Mc</u> Ik	Remarks		
134	1.0"	.11	2920	43700	27300			
: 135	1.0"	.13	2670	40000	23100			
136	1.0"	.12	2525	38000	22000			
137	1.0"	.10	2895	43300	25000			
138	1.0"	.12	2980	44600	25400			
139	.5"	.12	410	49100	28400			
140	.5"	.12	420	50200	29000	6		
141	.5"	.12	395	47400	27900			
142	.5"	.12	385	46100	26700			
144	.5#		415	49800	28800			
Tension								
130	1.0"	.12	13000	13000				
131	1.0"	.10	12000	12000				
132	100"	.06	15000	15000				
133	1.0"	.06	18520	18520				

Note-

Distance between supports = 10"

Silicon = 2.80

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Table IV

Transverse Test Heat No. 4

		Shrink	Dood	Lbs. per	sq. in.	
No.	Size	age in inches	in lbs.	S=MC 1	S=MC Ik	Remarks
154	1.0"	.10	3060	46000	26600	
155	1.0"	•11	3555	53300	30800	
156	1.0"	.10	3380	50600	29300	
157	1.0"	•09	3170	47600	27500	
158	1.0"	.10	2870	43100	25000	
159	1.0"	.13	2750	41300	23800	
160	1.0"	.12	3030	45300	26200	
161	1.0"	.09	3080	45300	26200	
162	•5"	.12	440	52700	30300	
163	.5"	.12	430	51600	29800	
164	•5"	.13	430	51600	29800	
165	•5"	•11	430	51600	29800	
166	.5"	.12	370	44400	25700	
167	•5"	.12	410	49100	28400	
168	•511	.12	415	49800	28800	
169	•5"	.12	390	46800	27000	
Tension						
150	1.0"	.10	13230	13230		
151	1.0"	.13	13930	13930		Defective
152	1.0"	.12	12740	12740	_	
153	1.0"	.12	20120	20120		

Note-

Distance between supports = 10"

k = 1.73 ; Value given by Bach Silicon = 2.77

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Table V

Transverse Test Heat No. 5

		Shnink-	Load	Lbs. per	sq. in.	
No.	Size	age in inches	in lbs.	s= <u>Mc</u> I	S=MC Ik	Remarks
180	1.0"	.11	2350	35400	20200	
181	1.0"	.13	2560	38400	22100	
182	1.0"	.12	2510	37700	21800	
183	1.0"	.11	2560	38400	22100	
184	1.0"	•11	3040	45000	26000	
186	1.0"	.11	2670	40000	23100	
187	1.0"	.12	2390	35800	20600	
188	.5"	.13	360	31200	18000	
189	.5"	.13	350	30000	16900	
190	•5"	.14	350	30000	16900	
191	.5"	.13	350	30000	16900	
192	•5"	.13	350	30000	16900	
193	•5"	.13	360	31200	18000	
194	.5"	.13	380	33700	19500	

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Note-

k= 1.73 ; Value given by Bach

Silicon = 2.60

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Table VI

		Charimle	Incol	Lbs. per	sq. in.	
No.	Size	age in inches	in lbs.	$S = \frac{MC}{I}$	S=Mc Ik	Remarks
200	1.0"	.13	2900	43600	25100	
201	1.0"	.13	2810	42300	24500	
202	1.0"	.14	2900	436 00	25100	
203	1.0"	.12	2960	44400	25600	
205	1.0"	.12	2660	40000	23200	Defective
206	1.0"	.10	2970	44500	25700	
207	.5"	.14	410	49100	28400	
208	.5"	.13	415	4960 0	28700	
209	.5"	.13	385	46200	26700	
210	.5"	.13	360	43300	25000	Defective

Transverse Test Heat No. 6

Note-

No Tension Test Made

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Distance between supports = 10"

Silicon = 2.64



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Table	V	Ι	Ι
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		Shnink	Logd	Lbs. per	sq. in.	
No.	Size	age in inches	in lbs.	$S = \frac{MC}{I}$	S= <u>Mc</u> Ik	Remarks
211	.5"	.14	420	50400	29000	
212	.5"	.14	440	52600	30400	
213	•5"	.14	300	36000	20800	Defective
214	•5"	.14	420	50400	28100	
215	1.0"	.14	3260	49000	28300	
216	1.0"	.14	2830	42500	24600	Defective
217	1.0**	.14	3270	49100	28400	
218	1.0#	.14	3260	49000	28300	

Transverse Test Heat No. 7

Note-

No tension test made

Distance between supports = 10"

Silicon = 2.59%











Table VIII

Transverse Test Heat No.	8&9
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		Chrink	Lood	Lbs. per	sq. in.	
No.	Size	age in inches	in lbs.	S= <u>Mc</u> I	S= <u>Mc</u> Ik	Remarks
220	1.0"	.11	2650	39800	27600	
221	1.0"	.11	2700	40500	23400	
222	1.0"	.14	2690	40400	23380	
223	1.0"	.13	2620	39200	22600	
224	1.0"	.11	2370	35600	20500	
225	1.0"	.14	2300	34500	19900	
226	.5"	.15	390	46700	27000	
227	.5"	.15	450	54000	31000	0 ve r Size
228	1.0"	.14	2335	35000	20200	
229	1.0"	.14	2510	37700	21700	
230	1.0"	.12	3090	45100	26100	
231	1.0"	.14	3230	48500	28000	
234	1.0"	.14	2640	39700	23000	
235	1.0"	.08	2880	43300	25000	
236	.5"	.14	460	55200	31900	

Note-

No tension test made

Distance between supports = 10"

Silicon = 2.50%



IV. Chemical Tests

1.Sampling Iron Filings for Analysis

The samples for the chemical analysis were collected as follows, Ten of the test bars which were . supposed to represent the average quality of the iron of that heat, were drilled at different places. The hard crust of the iron and the sand was removed and the sample was taken out of the interior of the bars. One sample of ten ounces, free from oil, was taken from each heat and given to the chemical department for analysis.

- 2. Determination of Elements
 - a. Carbon
 - 1. Total Carbon

Weigh out two grams of pig iron. Put them into a beaker, and add 50cc of the copper solution for each gram of iron taken. Stir the solution continuously until the iron is dissolved. The completion of the reaction is easily recognized by the residue becoming light. At first more or less copper will separate, but stirring and time will bring it into solution. Now add 5 to locc of concentrated HCl, and when all the free copper is dissolved, filter on an asbestos filter. The asbestos must be thoroughly ignited in air before using to remove any carbonaceous matter.

When the liquid has run through, wash out the beaker and transfer all adhering carbon to the filter, using

- 21 -

HCl diluted with its own volume of water. Wash the carbon on the filter twice with the acid, letting it run through slowly to give it time to act. Now wash with water until all the HCl acid is removed and the filtrate does not react with AgNO_g. The filtrate will be dark colored at first, but when diluted with HCl and water it will become light, and then must be carefully examined to see that no particles of carbon have run through the filter.

Transfer the residue to the flask, using a little water to wash out the filter tube. The total amount of liquid in the flask must not exceed 30cc; dissolve four grams of chromic acid in four cc of water, and pour it into the flask through the funnel tube. Wash out the tube with 2 or 3cc of water. Estimate the amount of liquid in the flask by comparison with a similar one, and put into the bulb of the funnel a quantity of pure concentrated H_2SO_4 equal to about twice the volume of the liquid in the flask.

Allow the acid to run into the flask gradually and carefully to avoid too violent action. Shake the flask carefully to mix the solution. The evolution of CO₂ will begin at once. Finally heat with a lamp until the liquid begins to boil. Continue heating until no more bubbles come over through the bottles in the train. A sharp rattling sound usually marks this point. The time of boiling should not exceed one or two minutes. Now withdraw the lamp and immediately open the stop

- 22 -

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cock of the funnel tube to admit air and prevent back suction. Connect the funnel tube with the soda lime guard tube, and let the apparatus cool a few minutes. Aspirate carefully four or five litres of air. Detach the absorption apparatus and weigh as explained under "Setting up the Apparatus". The total gain in weight will be the amount of CO_2 and this multiplied by 0.2727will give the amount of carbon.

Preparation of the Chloride of Copper and Potassium Solution

Prepare a solution of the double chloride of copper and potassium salts. Use the purest crystallized salt obtainable. Dissolve one part in three parts of pure water, then add NH₄HO drop by drop until a slight permanent precipitate forms. Let settle, decant off the clear solution, and filter the turbid portions through ignited asbestos.

Arrangement of the Apparatus

Take an Erlenmeyer flask of about 250cc fitted with a two hole rubber cork. Into this insert a bulb funnel tube having having a glass stop cock, and a delivery tube, for the gas. Fit a small guard tube filled with soda lime into the top of the funnel tube. Connect the delivery tube with the following purifying and absorbing apparatus, arranged in the order given.

1. A small bottle containing "pyro solution" made by mixing 0.2 grams of pyrogallic acid with 5 grams of

- 23 -

neutral potassium oxalate, adding water enough to make 20cc and then 2 drops of H_2SO_4 which must make the solution distinctly acid.

2. A similar bottle containing about 20cc of an acid solution of silver sulphate.

3. A bottle containing 20 to 30cc of concentrated pure H_0SO_4 .

4. A "U" tube containing granular $CaCl_2$. Fill about an inch of the tube, on the side next to the H_2SO_4 with cotton and moisten the tip of this with a drop of water.

5. Trebrg's potash bulbs containing a clear solution of KHO of about 1.27 specific gravity.

6. A small "U" tube the limb next to the potash bulbs filled with granular soda lime which should not be too dry. The other limb is filled with CaCl₂.

7. A "U" tube similar to the last, but larger, having the limb next to 6 filled with CaCl₂ and the other with granular soda lime.

8. An aspirator for sucking air slowly through the apparatus.

Setting up the Apparatus

The connections are made by glass tubes united by short rubber tubes. These must be carefully tied with thread or wire, as it is essential that the whole apparatus be air tight. Rubber corks are, of course the best, but good ordinary corks can be used if rolled soft and carefully bored and fitted. Sealing wax is

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often recommended to make joints tight, but it is a bad thing to use as it will crack and leak unexpectedly. The potash bulbs and the "U" tubes must be 'capped' with short rubber tubes closed with bits of glass rod. These must always be removed for a moment and then replaced just before weighing, that the air pressure inside and outside may equalize itself.

It is necessary to pass some CO_2 through the apparatus in order to saturate any alkaline material present in the CaCl₂,etc. When this is done the weighed part of the train is omitted.

Connect up the train omitting the parts numbered 5-6-7. Put a little marble in the flask, add a little dilute H_2SO_4 so as to generate a slow stream of CO_2 . Let this run through this portion of the train for thirty minutes. Disconnect and wash out the flask, replace it and now aspirate air alone until six or eight litres have been slowly drawn through.

Now connect up the whole apparatus and attach the aspirator. Close the stop cock in the funnel tube of the flask and see if all connections are tight. This is proved by the water ceasing to flow from the aspirator Cautiously let in air by opening the stop cock. Attach the soda lime guard tube to the funnel and aspirate slowly one or two litres. Disconnect the bulbs and the "U" tubes, cap them and wipe them carefully. Set them in or near the balance case until they attain the temperature of the air. Uncap them a moment, replace the caps

and then carefully weigh them. Replace the apparatus and aspirate three or four litres of air the second time and reweigh, as before. The KHO bulbs will lose weight and the "U" tubes will gain. The loss in one must equal the gain in the other. The total weight of the absorption apparatus must not change more than one-half milligram.

2. Graphite

Treat two grams of cast iron filings ina a beaker with 50cc of HCl, specific gravity 1.12. Cover and boil briskly for thirty minutes. Dilute, filter on an asbestos filter and wash first with hot water and then with a solution of caustic soda, then with water , then with alcohol, then with ether and finally with water, first hot then cold,till every trace of ether is removed. Transfer to the carbon apparatus and treat with chromic acid and sulphuric acid as in the determination of the total carbon.

The combined carbon is determined by subtracting the amount of free carbon or graphite from the amount of total carbon.

- 26 -

b. Silicon

Weigh out .9404 grams of cast iron. Put into a casserole or dish and cover with a large watch glass. Add carefully 30c.c. of a cold mixture of eight parts by volume of concentrated H N 0_3 , five parts of concertrated H₂S 0_4 and seventeen parts of H₂O.

Warm until action ceases, then boil down rapidly until the $\operatorname{Fe}_2(S \circ_4)_3$, separates as a white mass. Continue the heating until dense fumes of $\operatorname{H}_2S \circ_4$ are evolved. These have a peculiar suffocating odor, easily recognized. Their formation indicates the total expulsion of the H N \circ_3 , and this is absolutely necessary in order to make the silicon insoluble,

Let cool; add 10 cc of concentrated H Cl and wash off the cover into the dish. Dilute to 150 or 200 cc, cover set over a lamp and boil until all $\operatorname{Fe}_2(\operatorname{SO}_4)_3$ is dissolved. This can be recognized by the disappearance of the silky precipitate in the liquid. Continue the boiling for five minutes, as this will cause the solution to filter more easily; wash off the cover, and let the liquid stand until all of the SiO_2 settles. Decant the clear liquid through a 7 cm ashless filter, previously washed out with boiling water. Finally transfere and wash the residue with hot water. When partially washed, drop a little HCl on the filter and residue, then wash again with hot water till the filter no longer tests acid. Without drying, transfer the filter to a crucible and ignite, gently at first, finally with high heat, until the carbon

- 27 -

is burned and the SiO₂ is white. If this is done in a platinum crucible and over a blast lamp, the "burning off" of the carbon need not take more than a few minutes.

It is important that the temperature be low at first, not exceeding a dull red until the paper and the amorphous carbon are burned out, as overheating at first will cause the carbon to lump together and it will then burn very slowly. During the final heating over the blast lamp keep the crucible partially covered. The burning may be hastened by directing a gentle current of oxygen gas into the crucible, but if due care is taken this is not necessary. If oxygen is used care must be taken not to blow any particles of SiO₂ out of the crucible by forcing in the gas toorapidly.

The weight of SiO in milligrams divided by two gives 2 * the silicon in tenths of one percent.

c. Manganese

Violard's Process for Manganese

This is a volumetric process depending upon the reaction between potassium permanganate and manganous salts by which all manganese is precipitated as MnO_3 ; $3MnSO_4 + 2KMnO_4 + 2H_2O = 5MnO_2 + 2KHSO_4 + H_2SO_4$.

The solution must be neutral or nearly so. The titration must be made with the solution nearly at the boiling point and very dilute or the precipitate will settle quickly, and it will be difficult to see the end

* From "Note on Metallurgical Analysis" by Lord

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reaction. For the same reason the solution must be vigorously shaken or stirred and the permanganate added rather slowly, especially at first. Too rapid addition of the permanganate will cause the MnO₂ to precipitate on the glass, forming a firmly adherent yellowish brown stain which makes it difficult to see the final pink when the titration is finished.

Process for Cast Iron

Weigh out 1 gram of the well-mixed borings into a casserole. Add gradually 25 cc of HNO₃, sp. gr. 1.2. When the metal is dissolved evaporate to dryness and bake for a few minutes; then take up in 10 or 15 cc of HCl and heat until the iron salts go into solution. Now add 10 cc of dilute H SO and evaporate until dense 2^{4} fumes of H2SO are given off, keeping the dish well covered, to avoid loss from spattering, and to avoid the formation of dry salts on the sides of the casserole. Cool and add 100 cc of water, and warm until all the ferric sulphate goes into solution. Transfer to a 500 cc flask, nearly neutralize with Na₂CO₂, cool and then add the zinc oxide, suspended in the water as described before, till the iron is precipitated. Dilute to 500 cc, mixing the contents of the flask thoroughly. Filter off 250 cc. Add to this two drops of HNO₃, sp. gr. 1.2, heat to boiling and titrate carefully with KMnO4 till a faint pink color is permanent.

Instead of taking 1 gram for analysis, it is usually more convenient to take such an amount of the sample as

- 29 -

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will make 1 cc of the permanganate equivalant to 1 percent of manganese. This amount can be calculated from the iron standard of the permanganate; thus, if 1 cc of the KNnO₄ equals .01 Fe take 0.5892 grams of the sample for the process, then the half of the solution taken for the titration will contain 0.2941 grams of the sample and each cc of permanganate used will represent one per-cent * of manganese.

d. Sulphur

The Iodine Method

This is the most generally used of the evolution methods, and is extremly rapid. Its results will be lower than those given by the direct oxidation method with all metals containing considerable combined carbon, but as the results are concordant, they serve for foundry controll

PROCEDURE

Weigh out five grams of iron, dissolve with concentrated HCl. The H₂S evolved on dissolving the iron, is absorbed in an ammonical solution of cadmium. All sulphur not evolved as H₂S is necessarily lost in this process. The advantage in the use of the ammoniacal solution of cadmium lies in the fact that the H₂S is fixed in a visible form as CdS, and one which is not so subject to alternations on standing.

The reaction between HoS and a strongly ammoniacal

* From "Notes on Metalurgical Analysis" by Lord

- 30 -

solution of cadmium chloride is as follows:-

 $H_{2}S + CdCl_{2} + 2NH_{4}OH = CdS + 2H_{2}O + 2NH_{4}Cl.$

If the liquid containing the precipitate of CdS is diluted with a large volume of water and a sufficient excess of HCl added, H_2S is again liberated by the following reaction; CdS+2HCl = H_2S +CdCl₂.

A considerable excess of HCl is required to cause the complete disappearance of the yellow precipitate. The presence of a large volume of cold water is necessary so that the H_2S shall not escape but will remain in solution in the water. This solution of H_2S is then titrated by a standard solution of iodine. The iodine unites with the hydrogen, setting free the sulphur; $H_2S+2I = 2HI+S$. A little solution of starch is added to the liquid and the least excess of iodine is shown by an intense blue color. The liberated sulphur causes the liquid to become curiously opalescent and shows the various colors, but this does not at all obscure the end reaction which is very sharp.

The solutions needed are standard iodine and starch. PREPARATION of the STARCH SOLUTION

Stir 5 grams of starch in 200 cc of cold water. Heat the liquid to boiling with constant stirring until the starch is thoroughly dissolved. Now dilute the liquid with cold water to about a litre and add 10 grams of crystallized ZnCl₂. Let the solution settle for some time, and pour off for use the clear part. This solution is very sensitive and keeps indefinitely.

- 31 -

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PREPARATION of the IODINE SOLUTION

Weigh on a watch glass 3.956 grams of pure resublime iodine. Put it into a small beaker, add about 6 grams of pure potassium iodine (free from iodate) and locc of water. Let stand in the cold until all the iodine dissolves, Then transfer to a graduated flask and dilute to one litre.

One cubic centimeter of this solution is equivalant to 0.0005 grams of sulphur. If five grams of metal are taken for the analysis each cc of iodine solution consumed will be equivalant to 0.01 percent of sulphur.

e. Phosphorus

Process for Cast Iron

Weigh out 1.0 gram of well-mixed borings into a four-inch covered dish or casserole. Add cautiously 35 cc of HNO_3 , sp. gr. 1.2, boil to dryness, then bake for thirty minutes on a hot iron plate at $200^{\circ}C$. Dissolve in 20 cc of HCl and if silicon is present again evaporate to dryness and dissolve a second time. Now add 20 cc of HCl and digest till all of the iron is dissolved. Add 30 cc of water, boil, let settle and filter into a beaker of 200 cc capacity; wash with small portions of water, letting each run through before adding the next. The volume of filtrate and washings need not exceed 70 to 30 cc. Now add 35 cc of concentrated HNO₃ and boil down rapidly until the volume of the

* From "Notes on Metallurgical Analysis" by Lord

- 32 -
liquid is 15 cc. Take off the hot plate, wash off the cover and add water. Altogether at least from 15 to 20 cc of water should be added. Stir the solution and add from a pipette 40 cc of molybdic acid solution which should be at a temperature of not less than 25° C so that the mixed solution shall be at a temperature of not less than 40° C. Stir vigorously for two to three minutes, set in a warm place, not on a hot plate or water bath, until the precipitate has settled and the liquid is perfectly clear. This will take 30 to 60 minutes according to circumstances. Filter and dissolve the yellow precipitate in NH₄OH with a very considerable excess of H₂SO₄. All of the phosphorus remains in solution. If this solution is warmed with metalic zinc, the zinc dissolves, hydrogen is given off, and the molybdic acid is rapidly reduced, giving first a dark red and finally a green solution containing, if the reduction is complete, Mo203. If this solution is rapidly filtered from any undissolved zinc, it can be titrated with a solution of potassium permanganate which promptly oxidizes the Mo20, back to Mo03. The solution becomes colorless and finally when oxidation is complete is colored pink by the least excess of permanganate.

PREPARATION of the POTASSIUM PERMANGANATE SOLUTION.

To compute the strength of this solution assume that the reduction of the yellow precipitate gives $Mo_{12}O_{19}$. Then the permanganate solution must furnish 17 atoms

- 33 -

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of oxygen for each twelve of MoO_3 present before reduction. The yellow percipitate contains 24 MoO_3 to one of P_2O_5 or twelve of MoO_3 to one of P, hence the permanganate solution must furnish 17 atoms of 0 for every atom of P present in the percipitate.

It is convenient to make the permanganate solution of such a strength that one cc equals 0.01 percent of phosphorus when one gram of iron is analyzed. Therefore to make such a solution, dissolve 3.46 grams of pure crystallized potassium permanganate in one litre of water. Allow the solution to stand some time before using. The soultion should be standardized on a piece of metal in which the phosphorus is known.

- 34 .

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Chemical Composition and Average Shrinkage of the One-half Inch Square Bar Cast

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Heat No.	Si.	S.	Phos.	Mn.	Ca: C.C.	rbon G.C.	Shrinkage
1	2.76	.074	1.100	.39	.75	2.46	.12
2	2.87	.073	1.123	.26	.72	2.34	.09
3	2.80	.088	1.159	.39	.02	3.09	.12
4	2.77	.068	.887	.35	.57	2.65	.12
5	2.60	.085	.940	.11	.56	2.49	.13
6	2.64	.050	.470	.22	.52	2.81	.13
7	2.59	.099	•940	.17	.94	2.21	.14
8	2.51	.097	1.09	.17	.90	2.27	.15
9	2.50	.100	.81	.22	•55	2.65	.14

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V. Effects of Various Elements

1. Carbon

Carbon is the most important element in cast iron and is present in larger amounts than any other element. The amount of carbon varies from 2.5 to 4%, depending upon the character of the ore and the condition of the cupola while the iron was being melted. Without carbon iron could not be melted readily and made into castings; the correct degree of hardness and the strength, which determine the grade of the casting, could not be obtained. Iron has a great affinity for carbon and absorbs a great part of the total carbon in the furnace, when it is in the liquid state. Remelting cast iron increases the combined carbon which gives the castings greater strength and makes them harder. The percent of total carbon determines the melting point of cast iron.

A large amount of silicon reduces the power of the iron to absorb carbon. Manganese increases this power but it also increases the shrinkage. The more carbon the iron contains, the greater will be the influence of silicon and manganese in changing the grade of the casting.

When iron is solidified, the carbon will remain in the combined state unless some influence is present to change it. The combined state is the natural form for carbon in cast iron. If the iron has not absorbed more carbon than it can hold when cold the casting will be

- 36 -

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white. If the iron absorbed more carbon in the cupola while it was being melted, than it could hold when cold, the excess will be crystallized and separated as graphitic carbon lying between the iron crystalls which form on cooling. Rapid cooling prevents the formation of graphitic carbon by not allowing it sufficient time in which to crystallize, while slow cooling has the opposite effect. Rapid cooling will result in hard castings having a light, close-grained fracture, while slow cooling will result in soft castings having a darker fracture and a coarser grain, hence small castings are harder and stronger than large castings which contain the same amount of combined carbon to begin with.

Remelting cast iron or introducing into the molten iron, wrought iron or scrap iron, lowers the graphite a this increases the strength of cast iron so long as the carbon is not reduced sufficiently to make unsound castings. Cast iron should always contain a small amount of graphitic carbon in order to prevent blow holes: it need not be sufficient to make the casting gray.

Remelting cast iron increases the total carbon. The amount of increase depending upon the amount of fuel used and the time that the iron remains in the cupola. Little fuel and a quick melt may reduce the carbon slightly, while enough fuel to produce hot iron with a

* From " Cast Iron " by Keep.

- 37 -

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slow melt will increase the carbon. Generally in remelting iron, the combined carbon is increased and the graphitic carbon is decreased.

The great variation in the cooling of castings and the great variation in the amount of carbon absorbed while the iron is being melted, make it impracticable to attempt to regulate mixtures by regulating the amount of carbon. The regulation by means of controlling the silicon which is more positive, is much more to be advocated.

2. Silicon

Silicon is found in all of the iron ores and in the ash of fuel, therefore more or less silicon is always found in cast iron. Its presence is necessary, if cast iron is to be used for ordinary casting purposes. Being as bulky as carbon, silicon will increase the fusibility of cast iron and make a smooth casting by depositing graphite on the surface of the iron, thus preventing sand from burning to the casting. Turner in his research work showed that silicon plays an important part in cast iron, namely that when it is added to white iron, it will turn the iron gray: furthermore that by varying the amount of silicon ,the physical properties of cast iron can be controlled at will.

Silicon is the controlling element of cast iron and may vary in amount from one to four percent. Its in-

* From " Cast Iron " by Keep.

- 38 -

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fluence is indirect since it acts through the carbon which the iron contains. The chief office of silicon in cast iron is that of a softener. It accomplishes this result by reducing the total amount of carbon which the iron may take up and by causing the carbon to remain in the graphitic form instead of combining with the iron. Owing to this fact, silicon is a great element in cheapening iron mixtures, as by using a small quantity of high silicon iron, a large amount of scrap iron may be used and a soft iron may be obtained. In mixing iron, silicon should be considered the basis for changing the character of the iron: manganese and sulphur should not be neglected: phosphorus remains practically constant under given conditions.

Silicon acts through the carbon contained in the iron, and its effect is modified by various conditions attending the remelting and cooling of the iron. The greater the amount of total carbon or the less the amount of combined carbon, the less silicon will be required to produce a given effect. Little silicon acting a long time, as in large slowly-cooling castings, will have an effect similar to that of a large amount of silicon acting quickly as in small rapidly-cooling castings. From 2 to 4% of silicon will change all of the combined carbon into graphite which can be changed, hence it is not advantagous to have more than 4% of silicon in small rapidly cooling castings. An increase in silicon up to 3% is usually accompanied by an increase in fluidity

- 39 -

and a decrease in shrinkage. Remelting cast iron decreases the silicon by oxidation. The amount of decrease depends upon the strength of the blast and the temperature of the fire. The loss of silicon in remelting varies from .1 to .3%.

3. Sulphur

Sulphur in iron is mainly derived from the fuel used in melting it in the cupola. It is the most uncontrolable injurious element which the founder has to deal with. In the ordinary cast iron, sulphur seldom appears quantities than 0.08 of one percent as the carbon seems to prevent its absorption. There are three qualities sometimes commendable in it; one is its influence in increasing the fusibility of iron, another its tendency to increase the strength of the iron, and the third is its ability to harden iron by reason of its promoting combined carbon. The latter effect may as well be obtained by using iron low in silicon or high in manganese, since with these, there is less injury from unyielding strains due to the contracting of the iron. Silicon counteracts sulphur, and an increase of 0.01 of one percent of sulphur will neutralize from 0.10 to 0.15% of silicon. With the exception of the three qualities mentioned above, the effects of sulphur are greatly for evil making light castings hard and molten iron sluggish, and giving rise to blow holes in iron which solidifies rapidly.

* From " Metalurgy of Cast Iron " by West

- 40 -

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4. Phosphorus

Phosphorus, as it occurs in cast iron, is obtained from the phosphoric acid which is found in the fuel and in the iron ore. It retards the saturation of iron for carbon, prevents blow holes, adds fluidity and life to the metal, and tends to keep the liquid iron in the molten state for a longer period of time than if no phosphorus were present, hence small, thin castings which are not subject to shock, should contain a large amount of phosphorus. It has a bad effect in weakening iron if it is present in large quantities, over 1.0%. Phosphorus is a very essential element in successful foundry work and therefore it needs to be watched as closely as silicon. Mr. Beckett in his experimental work with phosphorus has found it to greatly counteract the tendency of sulphur to increase combined carbon, and by increasing the phosphorus from 0.5 to 0.75%, he has made hard castings soft. He also shows that an increase of 0.1% of phosphorus will produce the same effect as an increase of 0.25% in silicon, other conditions not varying until the phosphorus has reached the safety limit of 1.0%. If phosphorus is added to the molten iron in the form of sticks it increases its strength as much as two fold. Phosphorus is not affected by remelting and when present in cast iron in greater quantities than 17 makes the castings brittle and weak. It acts in a very decided manner in decreasing the shrinkage, and consequently iron high in phosphorus and silicon

- 41 -

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will have but little shrinkage. Phosphorus has a very peculiar effect upon the fracture of cast iron, imparting a yellow tinge to the crystals. This yellow tinge is sometimes seen on the outside of high phosphorus iron which has been rapidly cooled. It has been noticed thin that the foundry iron which makes the best castings, as a general rule, contains over 1.0% of phosphorus. The ordinary grade of American pig iron will rarely impart to castings more than this percent.

5. Manganese

Manganese combines with iron in almost any proportions. If iron which contains manganese is remelted, a part of the manganese will escape by volitilization and by oxidation with other elements, especially sulphur. For this reason manganese can be used to counteract the evil effect of large quantities of sulphur as it is very effective in reducing the latter. When manganese is present in cast iron in large quantities, it increases the saturation of iron for carbon, it increases the combined carbon, it increases the shrinkage and slightly increases the strength, but it does not affect the depth of the chill. The influence of the manganese on the physical properties of the iron depends a great deal upon the original state of the manganese in the pig, yet if the amount of manganese is below 0.75% or even 1.0%, its effects cannot be detected, therefore the small percentages of manganese which

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are found in commercial pig, will have no influence on # the strength of the casting.

- 43 -

When making iron, a hot furnace will send the manganese into the pig, as a high temperature is required to make it combine with the iron, while a cold furnace will send it into the slag. The higher the temperature in the cupola, the greater will be the decrease in manganese.

The effects of adding manganese to molten iron in sticks, as ferro- manganese, a moment before pouring it, are as follows: - The transverse strength of the metal is increased from 30 to 40%, the shrinkage is decreased from 20 to 30%, the depth of chill is decreased nearly 25%, while nearly one-half of the combined carbon is changed into free carbon. The percentage of manganese in the iron is not sensibly increased by the addition of the ferro-manganese. The small proportions of manganese which have been added are found in the form of oxide in the scoria. When a casting which has been treated in this way and artificially softened, is remelted, the effects of the ferro-manganese disappear, and hard iron results. In high manganese iron, an increase in silicon can to a large extent neutralize the effects of the manganese.

* From " Cast Iron " by Keep.

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Explanation of Curves

The following curves show the variation in the strength of cast-iron bars. Each curve shows the strength of test bars which were made from the same heat and which were supposed to have the same chemical composition. The ordinate of each point on the curve represents the strength of a certain bar in pounds per square inch. Since the comparative strength of the bars was plotted in the same order as the bars were broken, the variation in the strength of cast iron, due to its non-homogeneous composition, can be seen at a glance. The horizontal line represents the average of all the bars made from that heat.

The last curve shows the relation between the strength of cast iron and the amount of silicon in per cent. The ordinate of any point represents the average strength of the cast iron in that heat, and the abscissa represents the average amount of silicon in the same heat. The curve is drawn so as to bring out the average relation and take in the majority of points. If the other elements are kept constant, the strength of cast iron increases as the silicon increases and since the shrinkage depends upon the amount of silicon, we have a direct method of determining the strength of cast iron by measuring the shrinkage of standard test bars.

- 44 -



CURVES SHOWING VARIATIONS IN THE STRENGTH OF CAST-IRON BARS MADE OF THE SAME CHARGE





CURVES SHOWING VARIATION IN THE STRENGTH OF CAST-IRON BARS MADE OF THE SAME CHARGE





CURVES SHOWING VARIATION IN THE STRENGTH OF CAST-IRON BARS MADE OF THE SAME CHARGE.

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ABS. = AMOUNT OF SILICON IN PERCENT

CURVE SHOWING RELATION BETWEEN THE PERCENT OF SILICON AND THE STRENGTH OF CAST- IRON.



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Size	Si.	Phos.	Mn.	S
1/4"	3.25	1.00	•40	.025
1/2"	2.75	.80	.40	.040
3/4"	2.50	.75	.50	.050
1"	2.00	.70	.60	.060
1 1/2"	1.75	.65	.70	.070
2"	1.50	.60	.80	.080
2 1/2"	1.25	.55	.90	.090
3"	1.00	.50	1.00	1.00

Relation Between the Chemical Composition

and Thickness of Metal

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VIII. Conclusions

1. Disadvantages of Chemical Analysis

In order to make a chemical analysis a chemist must be employed and a laboratory provided. If the scheme is to be successful even in a small foundry, the chemist must give his whole time to the analysis of the iron and coke. It takes time to make the chemical analyses, especially if other elements than silicon are to be determined. For this reason, it is necessary to purchase pig far in advance, hence extra space for storage must be provided and special means for its transportation, all of which add to the operating expense of the foundry. Furthermore, mixing iron by chemical analysis is only approximate and since cast iron is not homogeneous, mechanical tests must be made in addition to the chemical analysis to determine the characteristic properties of the iron.

If pig whose chemical composition is known can be purchased on the market, the foregoing relation between the chemical composition and the size of the casting gives satisfactory results, and is used in a great many commercial foundries.

2. Advantages of Physical Tests

Mechanical analyses of cast iron are made use of in order to obtain information regarding the

* From "Foundry" March 1906

- 50 -
physical characteristics of any perticular iron, either for comparison and use in connection with the chemical analysis of that iron, or as a direct guide for the regulation of iron mixtures without making chemical analysis. Since the strength of cast iron upon the silicon and since silicon controlls the shrinkage, as shown in the previous tests, the strength of cast iron and a great many of its physical properties can be controlled by regulating the shrinkage. This can be accomplished by means of shrinkage tests which are conducted by moulding bars between chills or yokes a definite distance apart. To get reliable results, one-half inch square bars ought to be used rather than one inch square bars, because the one-half inch bar cools more rapidly and the heat has no influence on the chemical composition. The bars must be cast between chills or vokes whose projections are smooth and not covered with rust so that the shrinkage can be measured exactly.

The following directions are given by Keep for the regulation of the chemical analysis of iron by means of mechanical tests based on the facts, which apply in this case, that the condition of carbon depends on the proportion of silicon and that the shrinkage varies in-versely as the silicon.

"1. Measure the shrinkage of a 1/2" test bar from your mixture and when you consider the castings satisfactory, use this shrinkage as your standard.

"2. For stove plate and small castings the shrinkage

- 51 -

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should be .115" to .195", for ordinary machine castings from .150" to .160".

"3. If the shrinkage is greater than your standard use more soft iron thus increasing the silicon, if it is less use more scrap or cheap iron.

"4. With high shrinkage and high strength of a 1/2" bar, heavy castings will be strong but thin castings, 1/8" to 1/4", are likely to be brittle.

"5. With low shrinkage and high strength of a 1/2" bar, large castings will be weak and thin castings strong.

"6. With uniform shrinkage, an increase in the strength of a 1/2" bar will denote a proportional or corresponding increase in all the castings.

"7. The strength of a 1/2" bar broken transversely on supports 12" apart ought to be over 400#".

Mechanical analysis of this kind shows the physical properties of cast iron, the combined influence of each element in the chemical composition, and in addition to the influence this it shows of fuel and of every varying condition attending the melting. By this method of analysis the founder can determine the physical properties of the castings in a few minutes. Since this method of analysis does not require skilled labor and costly apparatus, it can be made in the ordinary foundry and when made, independent of chemical analysis, it will give all the necessary information.

- 52 -



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